When More Alternatives Lead to Less Choice*

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Abstract

One of the crucial decisions a firm or a government agency must make is the selection of the alternatives, e.g., the characteristics and/or the number of products, to offer to consumers or citizens. The selection of the set of alternatives to offer should not only take into account the potential preferences of the consumers and firms, but also the evaluation costs of the economic agents. This may lead to offering more alternatives not always being better for the firms than offering fewer alternatives. This paper shows that search/evaluation costs may lead consumers not to search and not to choose if too many or too few alternatives are offered. If too many alternatives are offered the consumer may have to engage in many searches/evaluations to find a satisfactory fit, which may be too costly and may dissuade the consumer from making a choice altogether. If too few alternatives are offered, a consumer may doubt that a satisfactory alternative is present, and therefore, also decide not to engage in search, and not to choose. These two forces may result in the existence of an interior optimal number of alternatives that should be offered in order to maximize the probability that a choice actually occurs. The number of alternatives offered may also signal the importance of fit in the market if there is asymmetric information about the importance of fit.

Keywords: product line, search costs, uncertainty, game theory

1. Introduction

A manager of an Internet Service Provider (ISP) was considering which DSL suppliers to provide as alternatives to its customers. When a customer would be signing up to the service, he/she would have to choose a DSL supplier out of the list the ISP offered as alternatives on a web page. The ISP had a list of fifteen DSL suppliers that it could present, and wondered what was the optimal number of DSL suppliers to actually present to their customers. The ISP experimented with lists of different sizes, from the list of all the fifteen DSL suppliers down to lists of only one DSL supplier, and found that the number of DSL suppliers presented to the ISP customers that led to the most orders was four. The manager of the ISP conjectured that with too many alternatives customers were overwhelmed, while if only a few alternatives were provided customers may have felt that they did not have enough choice. Again, the answer was that four alternatives (not the minimum, not the maximum number of available alternatives) maximized the number of orders. The actual four alternatives that the ISP offered varied across markets.[1]

Examples where firms (or other economic agents) have to decide on how many alternatives to offer to their customers are frequent. Salespeople are often told not to offer too many alternatives. As quoted in the New York Times (Jan. 26, 2004, p. A22), from a shoe salesman: "*As a neophyte shoe salesman, I was told never to show customers more than three pairs of shoes. If they saw more, they would not be able to decide on any of them.*" Iyengar and Lepper (2000) show in an experimental setting that when the number of flavors of jams offered to consumers increases, consumers are less likely to make a choice. Boatwright and Nunes (2001) show that reducing the assortment size in a grocery store does not reduce and often increases sales. In the first half of the 1990's Procter and Gamble cut its number of products by one third, often with gains in market share (Business Week, Sept. 9, 1996, p. 96), possibly due to consumers having a simpler choice problem. As recognized in marketing practice, one way in which a new product may affect the other products is by creating consumer confusion about which product best fits the consumer preferences. As noted by Keller (1998, p. 464), "Different varieties of line extensions may confuse and perhaps even frustrate consumers as to which version of the product is 'the right one' for them." Iyengar et al. (2004) show that employees are more likely to enroll in 401(k) retirement plans when the plans offered fewer funds. Bertrand et al. (2005) find that in loan mail offers in South Africa, loan take-up increased when fewer loans were described in the offer, the same effect as reducing the monthly interest rate by 2.3% (in a range from 3.25% to 11.75%).

[1] This example was described to the authors in an interview with the manager of the ISP.

In general, the problem of how many alternatives to offer by a firm or a government agency can be present in many economic environments: What is the optimal number of vacation options a travel agency should present in an email or a brochure? How many brands/models should a retailer carry? How many options should a salesperson mention to a customer? How many health (or retirement) plans should a firm offer to its employees? As the examples above suggest, there seems to be an optimal number of alternatives to offer to maximize choice, although this optimal number may vary from one situation to another.

This paper considers the implications of the possibility that consumers have costs of the evaluation of alternatives, and shows how such costs may explain the above phenomenon. Let us call the economic agent selecting an alternative the consumer of the alternative. If too many alternatives are offered, the consumer may have to engage in many searches to find a relatively good fit (Stigler, 1961). This may be too costly, and lead to no choice. If too few alternatives are offered, a consumer may believe that he will not find an alternative that is a relatively good fit, and therefore, decide also not to choose. These two forces may then result in the existence of an optimal and interior number of alternatives to be offered in order to maximize the probability that choice actually occurs. This paper models the economic environment endogeneizing both the search by the consumer and the optimal number and design of the alternatives offered by the supplier with the above forces at play.[2]

The model allows us to make predictions about what affects the optimal number of alternatives, and how consumer search behavior changes depending on where a consumer's preferences are in relation to the distribution of preferences of the total population. For example, it shows that the search behavior as a function of how extreme the preferences are may not be monotonic, but it could be that both extreme and average consumers do not search, while moderately extreme ones do. The number of alternatives offered may also signal the importance of fit in the market if there is asymmetric information, but may still lead consumers with high search costs, or low general valuation for the product, not to search. The result of the existence of an optimal interior number of alternatives to offer requires that the consumers believe that the provider of alternatives does not offer alternatives at random, but is strategic about it. This suggests possible qualifiers for when limiting the number of choices can be more profitable. We concentrate on the choice effects without including price-setting effects in order to simplify the presentation.[3]

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[2]If there is a fixed cost per alternative offered, the optimal number of alternatives is also obviously finite. The paper shows that even without costs of providing an additional alternative, costly evaluation of alternatives yields a finite optimal number of products offered, and may be a force towards a lower number of alternatives.

[3]This case can be seen as relevant for some situations where prices are not present, when the supplier of alternatives does not have influence over prices, or when the price cannot be informative about the attribute under consideration

As noted above, some experimental work (e.g., Iyengar and Lepper, 2000) has found that consumers are more likely to purchase and make a choice when confronted with a smaller choice set than when facing a larger choice set.[4] This work has argued that consumers may feel overwhelmed or overloaded with too many alternatives or information. More recently, Salgado (2005) shows that individuals may prefer fewer options because of contemplation costs. This idea goes back to the information overload literature (see, for example, Jacoby, 1977, and the references listed there), and the idea that decision-makers may only be able to process a limited amount of information (e.g., Simon, 1955, Miller, 1956, Shugan, 1980). In this regard, Hauser and Wernerfelt (1990) have argued that consumers may strategically limit their consideration sets (with search under fixed sampling) in order to limit evaluation costs at each consumption occasion.[5] In relation to those papers, this paper formalizes the costly product evaluation process. In this setting, consumers do not infer their preferences from the alternatives offered, but rather the number of alternatives offered affects the costs and benefits of that evaluation process.

This paper also contributes to the existing literature by presenting the link between the optimal number of products and the market situation (in particular, the consumer evaluation costs and the importance of product fit). The extant literature related to the product line design considered how the optimal number of products (alternatives) may be restricted by the adverse selection problem in the case of vertically extended product line (e.g., Moorthy, 1984, and Desai, 2001), by the costs of maintaining the product line (e.g., Shugan, 1989), or by the costs of communication (e.g., Villas-Boas, 2004). This paper adds to the above literature by considering how the consumer product evaluation costs may lead to the shorter optimal length of the product line.

The perspective presented in this paper is that the decision-maker is overwhelmed with alternatives because of search costs in evaluating the fit of each alternative. Due to this dependence, whether the consumer will appear to be "overwhelmed" or not is not exogenous, but depends on the search costs and on the importance of fit. If many alternatives are offered the decision-maker may have to look through many alternatives in order to find one that provides a satisfactory fit, and therefore, the decision-maker may become discouraged of searching, and end up not searching, and not choosing. If fewer alternatives are provided (but not too few), the decision-maker may find that it is reasonable to engage in product evaluation, and that within a "small" number of searches

due to other attributes present (although in that case the model would have to be extended with further attribute dimensions). The case with endogenous prices is further discussed in Section 7.

[4]See also the market evidence in Boatwright and Nunes (2001).

[5]Madrian and Shea (2001) show an example of when a firm automatically enrolls employees in the company 401(k) plan, there is an increase in retirement savings even though employees can easily opt out of the enrollment. See also the evidence in Benartzi and Thaler (2002), Choi et al. (2004), and Iyengar et al. (2004). This could potentially be seen as employees avoiding "active" choice.

the decision-maker will find an alternative that provides a relatively good fit.

As discussed in Section 4, it is crucial for this argument that the alternatives provided span the space of consumer preferences; that is, the locations of different alternatives in the space of consumer preferences are not independent, and an alternative does not have other alternatives located close by. This happens when the supplier of alternatives is strategic, which is a natural condition in choice situations faced by human beings. Note that even if this condition is not implemented in an experimental setting, the decision-makers may behave as if this condition is being implemented because they may perceive that the experimental setting is replicating the reality that they are familiar with.[6]

Note that the supplier of alternatives being strategic about which alternatives are offered is to be expected in economic environments. Firms choose which products to offer, such that they may cover the market that they target. For example, a bank offers a number of checking accounts to cover the potential preferences in the market. An automobile manufacturer offers different car models to cover its target market. A firm offers a variety of retirement or health plans to cover the potential preferences of its employees.

By providing an explanation for the phenomenon that individuals may be less inclined to make a choice when too many alternatives are offered, this paper helps understand the market interactions in situations where choice of varieties is an important component of firm decision-making, and may provide a framework for government agencies to think of which and how many alternatives to offer in a particular policy.

Van Zandt (2004) considers competition where firms communicate about their products and consumers evaluate a limited and fixed number of alternatives, and finds that there is too much communication in equilibrium, as a firm communicating about its product does not consider the negative externality on consumer information processing affecting the other firms. In relation to that paper this paper concentrates on the optimal consumer sequential evaluation process when a firm offers multiple products and models the firm's decisions on the number and location of products.[7] In independent work, Kamenica (2005) and Norwood (2005) consider the possibility of the products offered affecting the preferences of consumers. In contrast to this paper, Kamenica considers a firm that is better informed than some consumers about which are the products that are most popular.

[6]Another potential factor for consumers being overwhelmed with a greater number of alternatives may just be that a large number of alternatives creates overall confusion, independent of the search costs of evaluating each alternative. Note that if the locations of different alternatives were independent, this effect would still be present while the search costs effect would not have any impact.

[7]For product line competition without evaluation costs see, for example, Klemperer and Padilla (1997). For models of competition with differentiated products and search costs see, for example, Anderson and Renault (1999).

By offering a smaller set of (the most popular) products, the uninformed consumers are more likely to purchase (at random) a more popular product. Norwood considers free-entry price competition among fixed products that are vertically differentiated, one product per firm, under the assumption that only the most popular products are offered, and includes an approximation to the consumer sequential evaluation process.

Finally, it may also be that a greater number of alternatives may lead the decision-maker to delay choice (and not choose when the choice set is first presented) in order to gather more information on the choice problem (e.g., Dhar and Simonson, 2003). Possibly, this new information could come at a lower cost than the cost of evaluating different alternatives at the present. Note that this explanation could then be seen again as consistent with a search costs explanation for no choice when many alternatives are presented.[8]

The rest of the paper is organized as follows: The main idea is presented in (the next) Section 2. Section 3 presents the model and Section 4 shows that the optimal number of alternatives is "small" (and finite if there is an infinite number of types of consumers). Section 5 presents the search equilibrium for the case in which the firm offers up to three alternatives and presents conditions under which offering two alternatives is better than offering either three or just one alternative. Section 6 discusses the results and their potential applications to examples in market interactions, and Section 7 presents some extensions, in particular, the case when the supplier of alternatives has private information. Section 8 concludes. The Appendix presents the proofs.

2. Main Idea

Consider a supplier of horizontally differentiated alternatives deciding on how many and which alternatives to offer to a set of decision-makers with diverse tastes. Let us call the supplier of alternatives the firm, and the decision-makers the consumers. The firm has no cost of introducing product varieties and consumers have positive evaluation/search costs. This is a problem about the optimal design of a pool of alternatives when search without replacement is costly. If the consumers have zero search costs the firm would want to offer all possible products so that every consumer finds his/her ideal product. This paper shows that with positive search costs the firm instead wants to offer a limited number of product varieties.

[8]Note also that consumers could potentially prefer smaller choice sets because of self-control problems (see, for example, the discussion in Gul and Pesendorfer 2001, Bénabou and Tirole 2004, Fudenberg and Levine 2005) or regret preferences (see, for example, Loomes and Sugden 1987, Irons and Hepburn 2005, and Sarver 2005).

It is clear that in order to make search less costly, the firm may just want to offer alternatives that are likely to be close to the consumers' preferences. The question is then whether the firm would find it optimal to offer as many alternatives as there are types of consumer preferences. This paper shows that the answer to this question is negative, that the firm may want to offer fewer alternatives than the number of consumer types (and a finite number of alternatives if there is a continuum of consumer types). The fundamental reason is that search without replacement (as opposed to with replacement) gives less benefits when there are many alternatives close to each other.

In order to get some intuition for this result let us consider the following introductory model. Let consumers incur an evaluation/search costs c per alternative evaluated. A consumer's preference is determined by the pair (v, x), the consumer's type, with v and x assumed independently distributed in the population, with $v \in [0, \overline{v}]$ being a general utility level of the ideal alternative, and $x \in [0, 1]$ being the location of the consumer ideal preferences. The alternatives are characterized by locations $z \in [0, 1]$. A consumer's utility of choosing alternative z is $v - t|x - z|$. The term t is a dis-utility parameter of the ideal point of the consumer being different from alternative located at z

Note first that given that the search is without replacement, the firm should not have products that just duplicate any other product. Once a product is evaluated and discarded, a duplicate would just slow down the subsequent search for a better product. To see this consider the following example with just two types of consumers on x, A and B, with the firm offering the ideal alternative for each consumer type, a and b, respectively, one of each. In this case, each consumer only needs to do one search to find his ideal alternative, as if the alternative first evaluated is not the ideal one, the consumer knows that the other alternative is it. If the firm puts many copies of each alternative into the menu of alternatives, then search without replacement becomes equivalent to search with replacement. Under search with replacement, the expected number of searches is $\sum_{i=1}^{\infty} i/2^i = 2$, greater than the one search under search without replacement.

To illustrate the situation with many consumer types, consider now that there are many consumer x types clustered around types A and B. Having only two alternatives, a and b, allows most consumers to find an alternative that is relatively good, although not the deal for each consumer (that is, a type near A finds alternative a), with just one search. If the firm offered the ideal alternative for every consumer type it would be similar for consumers to be searching among many copies of a and b, which has the disadvantage of search with replacement. The expected search costs of finding an alternative close to one's ideal alternative would then be $2c$. If the search costs are zero (or small enough) it is better for the firm to offer all the consumer types' ideal alternatives. But for high enough search costs, it is better for the firm to offer just two alternatives. As the paper shows below, if there is a continuum of consumer types and search costs are positive, then the firm never finds it optimal to offer an infinite number of alternatives.

The remaining of the paper formalizes these ideas, fully modelling the information structure of consumers and firm, the optimal search behavior of the consumers, and the equilibrium decisions by the firm, and discussing several extensions.

3. The Model

Consider the problem of the supplier of alternatives presented in the previous section. Before deciding whether to search, consumers observe the number n of alternatives being offered by the firm. The firm decides the number n of alternatives to offer, and the location of each of these alternatives on the segment $[0, 1]$.[9] We denote the location of alternative i by z_i, and without loss of generality assume that $z_i \leq z_{i+1}$.

After observing the number of alternatives offered (but not their locations), the consumer decides to evaluate an alternative if the expected utility of starting the search process is greater than both choosing an alternative at random (without evaluation) and not choosing any alternative. The utility of not choosing any alternative is normalized to zero. After evaluating one alternative, the consumer may decide to choose one of the alternatives already evaluated, choose one of the alternatives not yet evaluated, decide to evaluate an additional alternative, or decide not to choose.

The utility for a consumer of choosing an alternative i after evaluating m alternatives is

$$U = v - t|x - z_i| - mc - K \qquad (1)$$

where $v, c, t,$ and c were introduced above, and K is the cost incurred to enter the market.[10] Once K is paid consumers learn x. Consumers are heterogenous in v and x with v and x being independently distributed in the population, without mass points, and with supports $[0, \overline{v}]$ and $[0, 1]$, respectively. Furthermore, we assume that x is distributed uniformly in order to simplify the analysis. All consumers have the same search cost $c > 0$ except for a mass ε of consumers, with ε close to zero,

[9] Several of the results presented here can be easily obtained for consumers distributed in a circle. That case would not allow us to consider what happens at the extreme points of the distribution of consumers (there are no extreme points), and the location of products would not be unique.

[10] More generally, the cost of evaluating an additional alternative could vary with the number of alternatives evaluated. The main messages of the paper would also go through in such setting. The supplier of alternatives can also potentially choose to structure the presentation of alternatives to lower the consumers' evaluation costs. This possibility is not considered here in order to focus on the number of alternatives/evaluation costs effects, but is interesting to explore in further research (possibly with more attribute dimensions).

who have zero search costs (and have $K = 0$). We focus on the case in which $\varepsilon \to 0$, and therefore, the expected profit for the firm, or expected payoff for the consumers, are always presented at the limit. The role of this assumption, as discussed in the next section, is to yield a unique equilibrium outcome in product positions.[11] The consumers with zero search costs will always optimally choose the product that generates the best fit.

The consumer starts by knowing his own v, but not his own x. After observing v the consumer decides whether to incur the cost K, learn x, and go ahead with the search process, choosing one alternative, or not choosing any alternative. We assume K to be large enough, so that if in equilibrium a consumer incurs this cost, that consumer will end up choosing an alternative. From (1) the maximum net of K that a consumer can get is v, so that for a consumer to buy in equilibrium we must have $v > K$. Then, since we allow consumers to choose an alternative at random without evaluation, if $K > t$, after incurring the cost K, a consumer always ends up choosing an alternative. The role of this assumption is to simplify the analysis so that whether a consumer decides not to choose depends only on v and not on both v and x. This cost K could be seen as the cost of a consumer starting to think about the potential alternatives in the product space. In Section 7 we look at the case $K = 0$, when consumers know both v and x before deciding whether to search. While consumers do not observe the locations of the products, they must form rational beliefs about their locations. This implies that consumers are able to infer the distribution of the product locations from the number of products offered. However, consumers remain uncertain about which product is which.

Suppose that a consumer observes the firm offering n alternatives and infers from this offer the location of the n alternatives being offered, $\{z_i, i = 1, \ldots, n\}$. Consider the situation where the consumer has evaluated m alternatives, comprising a set which we denote by I. Then, the expected payoff of a consumer with preference characteristics (v, x) after having evaluated the alternatives in this set I and incurring the cost K and search costs mc is

$$V(v,x,I,n) = \max\{0, \max_{i\in I} v - t|x-z_i|, \sum_{i\notin I} \frac{1}{n-m}(v-t|x-z_i|), -c+\sum_{i\notin I} \frac{1}{n-m} V(v,x,I\cup\{i\},n)\} \quad (2)$$

The right hand side of this equation represents the four possible options available to a customer who is searching. The first element in the max function represents the option of dropping out of the search process, and not choosing any alternative. The second element represents the option

[11] Alternatively, in order to get a unique product positioning equilibrium, one could make the assumption that consumers observe the distribution of the products offered without knowing which product is which. This could potentially be justified with a reputation or social learning argument.

of stopping the search process and choosing the best alternative among the ones that have been evaluated. The third element represents the option of choosing an alternative at random among the alternatives that were not evaluated yet. Finally, the fourth element represents the option of evaluating one more alternative. All these options have the sunk cost of having evaluated m alternatives, $mc + K$. As noted above, if $K > t$ the option of dropping out of the search process is never optimal for a consumer that incurs the cost K.

Note that the problem represented by (2) is a search problem with a finite and non-independent number of alternatives, with the possibility of recall, and with no replacement.

The expected utility net of K for a consumer of preference characteristic v of starting the search process can be written as

$$\int_0^1 V(v, x, \emptyset, n)\, dx = v - d(n), \tag{3}$$

where the function $d(n)$ represents the expected dis-utility of a consumer given that the firm offered n alternatives. This expected dis-utility given n offered alternatives is composed of the expected costs of searching plus the expected mis-fit of settling on an alternative that does not match the consumer preferences exactly. The marginal consumer $\widehat{v}$ is then determined by $\widehat{v} = K + d(n)$.

Suppose now that the payoff for the firm is equal to a fixed margin times the number of consumers that end up choosing one alternative. Then, the payoff is proportional to $\text{Prob}[v \geq \widehat{v}] = \text{Prob}[v \geq K + d(n)]$, which is strictly decreasing in $d(n)$. Hence, the problem of the firm reduces to

$$\min_n d(n). \tag{4}$$

4. The Benefit of Fewer Alternatives

This section shows that the optimal number of alternatives offered is "small" in the sense that it may be smaller than the number of consumer preference types. In particular, in the context of the model above this section shows that the number of alternatives is finite, while the number of consumer preference types is infinite. That is, there is a finite number of alternatives such that if the firm offers more, or less, alternatives the firm is worse off. To see this, we argue first that as the number of products n tends to infinity, the distribution of the products tends to the uniform distribution on $[0, 1]$. Then, we characterize the optimal search process of consumers under this continuous distribution of alternatives. Finally, we show that the firm can do better with a certain finite number of alternatives that we define.

4.1. Optimal Distribution of Alternatives

Suppose the firm offers n alternatives. Since K was assumed to be greater than t, all consumers that incur the cost K choose an alternative. On the other hand, before incurring cost K, uninformed consumers cannot find out the locations of any products. Therefore, the locations of the alternatives chosen by the firm do not determine whether the consumers with positive search costs decide to buy. That means that the locations chosen by the firm only affect the purchase decision whether to buy of the consumers with zero search costs. The number of zero search costs consumers that choose an alternative is maximized when the alternatives are equidistant on the segment $[0, 1]$, so that the subsegment of points that are closer to any given product than to any other product is of equal size across products. Therefore, the n products are located at $\frac{2i-1}{2n}$ with $i = 1, 2, ..., n$.[12] When $n \to \infty$ this distribution of locations of the alternatives converges to the continuous uniform distribution in the interval $[0, 1]$. That is, the limit when the number of products goes to infinity is a continuum of alternatives with a uniform distribution. This analysis shows that the uniform distribution is the only distribution for which the discrete approximation may consist of optimally-located products, given their number.

The next subsection considers the optimal search process when the firm offers a continuous and uniform distribution of alternatives.

4.2. Optimal Consumer Search Under a Continuous Uniform Distribution of Alternatives

Because there is an infinite number of products, the optimal search process is the same as a search process with replacement (e.g., Diamond, 1971).

The problem, generally defined, is the following: Let the alternatives be distributed with density $f(x)$ (and cumulative distribution $F(x)$) on the line. It is well known that in such problems the optimal search process involves a stopping rule where the decision-maker keeps on searching until he finds an alternative that provides a utility which is greater or equal to some reservation utility. In this particular set-up the problem is for the decision-maker to find a product that is sufficiently close to his ideal point. This means that a consumer located at x will have a reservation alternative located to his left, $R_L(x) \leq x$, and a reservation alternative located to his right, $R_R(x) \geq x$. If the product searched falls in $[R_L(x), R_R(x)]$, the consumers stops searching and buys that alternative; otherwise,

[12] It can be shown that these locations also minimize $d(n)$ for a given n, if the products are such that for any given product there is a positive mass of uninformed consumers that continues to evaluate products until they find that particular product.

the consumer keeps on searching. Note that under this search strategy the expected dis-utility of a consumer located at x is $t\int_{R_L(x)}^{R_R(x)} |y-x|\,dF(y)/[F(R_R(x))-F(R_L(x))]+c/[F(R_R(x))-F(R_L(x))]$.

Coming back to our problem of search with alternatives distributed uniformly on $[0,1]$, one can see that for a consumer located close to the center of the segment $[0,1]$ (where "close" is defined below) there are two reservation products located, one to the left and one to the right of the consumer's location. The reservation product is defined by the condition that the marginal cost of searching an extra product, c, is equal to the marginal expected benefit (in terms of better fit) of that search, given that a reservation product has just been found. This condition can be written for a consumer located at x as

$$c=\int_{x-\delta}^{x+\delta} t \mid x-y \mid dy \tag{5}$$

where δ is the distance of the reservation product to the consumers location. This yields $\delta=\sqrt{c/t}$; that is, if $\sqrt{c/t}<x<1-\sqrt{c/t}$, the consumer can achieve the reservation utility with a product either on the right or the left of its location. This expected search costs plus dis-utility, for $\sqrt{c/t}<x<1-\sqrt{c/t}$, is then $t\delta=\sqrt{ct}$. We should compare this with the expected dis-utility if the consumer does not search and buys at random, which is $\int_0^1 t\mid y-x\mid\ dy=t(x^2-x+\frac{1}{2})$. Note then that if $\frac{c}{t}<\frac{1}{16}$, all consumers $\sqrt{c/t}<x<1-\sqrt{c/t}$ engage in the search process. This means that if the search costs are low enough, or the importance of alternative fit is high enough, all consumers in the center of market that choose one alternative engage in the search process.

Consider now the case of $x<\sqrt{c/t}$ or $x>1-\sqrt{c/t}$. Then the reservation utility may only be obtained on one side of the consumers location. Consider the case of $x<\sqrt{c/t}$. Then, the reservation product on the right of x is defined by

$$c=\int_0^x t(x-y)\ dy+\int_x^{x+\widetilde{\delta}(x)} t(y-x)\ dy \tag{6}$$

where $x+\widetilde{\delta}(x)$ is the reservation product for the consumer located at x. From this, one can obtain $\widetilde{\delta}(x)=\sqrt{2c/t-x^2}$, which is decreasing in x, and such that, $\widetilde{\delta}(\sqrt{c/t})=\delta$.

If the consumers engage in the search process, the expected search costs plus dis-utility of the product bought for a consumer located at $x<\sqrt{c/t}$ is then $t\widetilde{\delta}(x)=t\sqrt{2c/t-x^2}$ (the case of $x>1-\sqrt{c/t}$ is symmetric). Note that this expected search costs plus dis-utility of the product bought is concave in x. That is, for x close to $\sqrt{c/t}$, when the location of the consumer moves away from the center of the distribution of preferences, the expected search costs plus dis-utility of product bought increases "steeply" because the consumer is willing to accept a product that is relatively far away. However, when x is close to zero, when moving away from the center of the market, the expected dis-utility of searching and product mis-fit increases less steeply because the consumer is less and less willing to accept a product much further away.

In comparing with the expected dis-utility of choosing a product at random, $t(x^2 - x + \frac{1}{2})$, one can obtain that for $\frac{c}{t} < \frac{1}{16}$, all consumers engage in the search process. Therefore, we have the following proposition:

PROPOSITION 1: *Suppose that the firm offers an infinite number of products and that $c/t < 1/16$. Then the consumer search strategy is the following: consumers with $x \in \left(\sqrt{c/t}, 1 - \sqrt{c/t}\right)$ search until they find a product at most $\sqrt{c/t}$ from them; consumers with $x < \sqrt{c/t}$ search until they find a product at most $\sqrt{2c/t - x^2}$ from them; finally, consumers with $x > 1 - \sqrt{c/t}$ search until they find a product at most $\sqrt{2c/t - (1-x)^2}$ from them.*

Consider now what happens with greater search costs (the complete analysis is presented in the Appendix). Comparing the expected dis-utility of buying a product at random with searching, we can obtain that if $\frac{c}{t} > \frac{1}{8}$ then no consumer engages in the search process and all consumers buy at random.

If $\frac{c}{t} \in [\frac{1}{16}, c^*]$, where c^* is defined in the Appendix and is close to 0.078, we have a situation where consumers located in the center of the market buy at random, while the rest of the consumers engage in the search process. This implies an interesting search strategy for the consumers depending on their preferences: Consumers that have relatively specific preferences (in the model, with a location close to zero or one) search, and consumers that have more generic preferences (around the center of the market) do not evaluate a product prior to purchase and buy at random.

If $\frac{c}{t} \in (c^*, \frac{1}{8}]$, the set of consumers that choose at random is not convex. That is, starting from the center of the market and going to either extreme, we have first consumers that choose at random, then consumers that engage in the search process, then consumers that choose at random, and, finally, again consumers that engage in the search process. In order to have some intuition for this possibility note that for x small the expected dis-utility is decreasing in x for both when a consumer engages in the search process and when a consumer buys at random. As argued above, when a consumer engages in the search process, the expected search costs plus dis-utility of the product bought is concave in x. On the other hand, the expected dis-utility of the product bought when a consumer buys at random is convex in x; that is, the further away a consumer is from the center of the market the consumer is in an increasingly worse situation. This then allows for the possibility that for some search costs, there is an intermediate low region of x where buying at random is better than engaging in the search process. Figure 1 illustrates this possibility with the comparison of the consumer payoffs under search and under choice at random.

Returning to the case $\frac{c}{t} < \frac{1}{16}$ and integrating over all x, we can get the expected dis-utility of the product bought as[13]

$$d(\infty) = 2t[(\frac{1}{2} - \sqrt{c/t})\sqrt{c/t} + \frac{c}{4t}(2+\pi)] = \sqrt{ct} + c\frac{\pi - 2}{2}. \tag{7}$$

As noted previously, for $\frac{c}{t} > \frac{1}{8}$ all consumers choose at random, and the expected utility across all consumers is $t\int_0^1 (x^2 - x + \frac{1}{2})\, dx = \frac{t}{3}$. For $\frac{c}{t} \in (\frac{1}{16}, \frac{1}{8})$ the expression for $d(\infty)$ is more complicated because some consumers engage in the search process while other consumers choose at random. However, by the principle of the optimum, it is easy to see that $d(\infty)$ is weakly increasing in c and t as c or t just represent a cost for the decision-maker. Figure 2 shows $d(\infty)$ as a function of c for $t = 1$.

4.3. Optimality of a Finite Number of Alternatives

We now show that a firm offering a finite number of alternatives will do better than the firm offering an infinite number of alternatives. Consider $\frac{c}{t}$ small ($\frac{c}{t} < \frac{1}{16}$),[14] and suppose that the firm offers $\widehat{n}$ products located at $z_i = \frac{2i-1}{2\widehat{n}}$, for $i = 1, 2, ..., \widehat{n}.$, where $\widehat{n}$ is the smallest integer that is greater or equal to $\frac{1}{2\sqrt{c/t}}$, that is, $\widehat{n} - 1 < \frac{1}{2\sqrt{c/t}} \leq \widehat{n}$. We already know that these locations are uniquely optimal, and therefore will be expected by consumers who see that $\widehat{n}$ products are offered. To show that offering these $\widehat{n}$ products is better than offering an infinite number of products, it suffices to show that this set of products would generate lower consumer dis-utility as consumer search is restricted to some (not necessarily optimal) search rule. Specifically, consider the consumer's search process which is to search until the consumer finds the alternative that is closest to him.

Let us denote the expected dis-utility under this search process by $\widehat{d}(\widehat{n}) \geq d(\widehat{n})$, and compare it with the expected dis-utility $d(\infty)$ given an infinite number of products.

With $\widehat{n}$ alternatives and with the proposed consumer search process the expected dis-utility of fit is $\frac{t}{4\widehat{n}}$. The expected search costs for each consumer are

$$c\frac{1}{\widehat{n}} + 2c\frac{\widehat{n}-1}{\widehat{n}}\frac{1}{\widehat{n}-1} + 3c\frac{\widehat{n}-1}{\widehat{n}}\frac{\widehat{n}-2}{\widehat{n}-1}\frac{1}{\widehat{n}-2} + \cdots + (\widehat{n}-1)c\frac{1}{\widehat{n}} + (\widehat{n}-1)c\frac{1}{\widehat{n}} = (\widehat{n}-1)(\frac{1}{2} + \frac{1}{\widehat{n}})c. \tag{8}$$

We then have $\widehat{d}(\widehat{n}) = \frac{t}{4\widehat{n}} + (\widehat{n}-1)(\frac{1}{2} + \frac{1}{\widehat{n}})c$. By the definition of $\widehat{n}$ one can then obtain that $d(\widehat{n}) \leq \widehat{d}(\widehat{n}) < \frac{3}{4}\sqrt{ct} + c < \sqrt{ct} < d(\infty)$, because $\frac{c}{t} < \frac{1}{16}$. That is, the expected dis-utility under

[13] In order to get this expression, the Appendix shows that $\int_0^{\sqrt{\frac{c}{t}}} \sqrt{2\frac{c}{t} - x^2}\, dx = \frac{c}{4t}(2+\pi)$.
[14] The case of $\frac{c}{t} > \frac{1}{16}$ is considered in Section 5.

$\widehat{n}$ alternatives is lower than in the expected dis-utility under an infinite number of alternatives. This means that the firm strictly prefers to offer a finite number of alternatives rather than infinite. Since as the number of alternatives increases to infinity the firm's outcome tends to the inferior outcome of infinitely many alternatives, we obtain that there is a N such that it is not optimal to have more than N alternatives. Therefore, the optimal number of products is finite, and we have the following proposition:

PROPOSITION 2: *When search costs are positive, the firm strictly prefers to offer a finite number of alternatives. That is, there is an optimal finite number of alternatives for a firm to offer.*

The intuition is that by spreading out the location of the alternatives, and by offering a small number of alternatives, the firm allows the consumers to save on search costs. This is because, by having fewer alternatives to search through, a consumer can rule out areas of the product space that are less appealing for him, because of search without replacement. When $\frac{c}{t}$ approaches zero, $\widehat{n}$ alternatives allows the consumers to save, in expected value, half of the search costs in the case of an infinite number of alternatives. This result also shows that if too many alternatives are offered the consumers realize that they will incur too many search costs, and will therefore prefer not to search – that is, not to choose.

When choosing the number of alternatives to offer, the firm faces the trade-off between potentially providing consumers with a better fit, and complicating their search process. The positive effect of better fit only holds when the number of products is not very large, while the search process is more and more costly as the number of alternatives increases to infinity. This is because when the number of alternatives is very large, consumers adopt a reservation rule strategy that never involves an exhaustive search for all alternatives, but rather search until the first product satisfying the reservation rule. When consumers adopt such a rule and the number of products is high enough so that consumers do not search exhaustively, increasing the number of the products does not increase the expected fit between the first product found to satisfy the reservation rule and the consumer preferences. On the other hand, increasing the number of products, keeps increasing the expected search costs until the first product that satisfies the reservation rule. The intuition for that is that search with replacement is less efficient than search without replacement, and as the number of products tends to infinity, the search process approaches search with replacement. Therefore, when the number of alternatives is large, there is only the negative effect and no positive effect of increasing the number of alternatives.

While we considered a particular distribution of consumers (uniform) and a particular functional form of the utility cost of misfit (linear travelling cost), the above intuition suggests that the result about the optimality of the finite number of alternatives would hold for a more general distribution of consumer preferences and when the utility cost of misfit is any decreasing function of the distance between the consumer's ideal point x and the product location.

5. Equilibrium Search and Optimal Behavior with Up to Three Alternatives

In order to gain further intuition on the results above, we consider in this Section the equilibrium search and optimal behavior with up to three alternatives, and compare them with the payoffs of the case with a "large" number of products. Furthermore, we consider an approximation to the optimal number of alternatives to offer.

5.1. One Alternative

When the firm offers only one alternative, consumers do not need to search. The optimal product location for the firm is the center of the market, which also yields the lowest expected dis-utility. The dis-utility ranges from $t/2$ for type $x = 0$, to $t/4$ for type $x = 1/4$, and to zero for type $x = 1/2$. The expected dis-utility of the product bought across consumers is

$$d(1) = t \int_0^1 \mid x - \frac{1}{2} \mid \ dx = \frac{t}{4}. \tag{9}$$

Note that this is below the expected dis-utility in the case of random choice with an infinite number of products. In that case the expected dis-utility ranges from $t/2$ for type $x = 0$ to $5t/16$ for type $x = 1/4$, to $t/4$ for type $x = 1/2$, for an expected dis-utility across all consumers of $t/3$. The expected dis-utility for one alternative is lower than for random choice with an infinite number of alternatives because with only one alternative consumers in the center of the market get dis-utility close to zero, and this is not the case with random choice with an infinite number of products. This example of one alternative illustrates one reason the provider of alternatives may want to restrict the number of alternatives to offer: when offering a smaller number of alternatives, the more "generic" (i.e., fitting more customers) ones will be chosen. This reason becomes less important when more alternatives are offered, and consumers engage in active search. To see this note that this advantage of one alternative over many alternatives disappears when the consumer types are distributed on a circle (no "generic" product), but the results of active search with more than one alternative offered (remaining of this section, previous section) still hold in that case.

5.2. Two Alternatives

In the case of only one product being offered no consumer incurs the search cost. Consider now the case of the firm offering two products, such that a positive mass of consumers will actually incur the search costs. As discussed above, for $\varepsilon > 0$ (as assumed), the optimal locations for the two products are locations $\frac{1}{4}$ and $\frac{3}{4}$. This pair of locations is also the location of the products that minimizes the expected dis-utility for consumers (see Appendix). In this case, consumers in the middle are more indifferent between the two products that consumers at the either end. Therefore, the consumers in the middle find it optimal not to search, and buy at random, while the more extreme consumers search. We have the following proposition:

PROPOSITION 3: *Suppose that the firm offers two alternatives and that $c/t < 1/4$. Then consumers with $x \in (1/2 - c/t, 1/2 + c/t)$ choose an alternative at random. The rest of consumers search once and choose the alternative that fits them better. Furthermore, the expected consumer dis-utility is*

$$d(2) = \frac{t}{8} + c - \frac{c^2}{t}. \tag{10}$$

To gain intuition on the results above it is interesting to go through the first search of a consumer located at $x = 0$ under this case of two alternatives and the case of an infinite number of alternatives. Under an infinite number of alternatives the expected distance of the first alternative searched is $1/2$, and choosing any other alternative after the first search gets also an expected distance of $1/2$. However, with just two alternatives, the expected distance of the chosen alternative with the first search is $1/4$. Either the alternative searched is at $1/4$, in which case it is the chosen one, or the alternative searched is at $3/4$, and then the consumers knows that the other alternative is located at $1/4$. This illustrates the advantage of search without replacement (with just two alternatives) discussed above.

5.3. Three Alternatives

Consider now the case where the firm offers three alternatives. This case allows us to consider a situation where all consumers that buy a product engage in the search process (in contrast with the two alternative case above where some consumers that bought a product chose at random). As argued above, the locations for the three products will be $z_1 = \frac{1}{6}, z_2 = \frac{1}{2}$, and $z_3 = \frac{5}{6}$. As in the two-alternative case, it can be shown that these locations are also the ones that minimize the expected dis-utility of the consumers.

The Appendix completely characterizes the search process in a setting where all consumers buying a product engage in the evaluation of at least one product, and shows that all consumers do search at least one alternative if $\frac{c}{t} < \frac{2}{21}$ (see Proposition 5 in the Appendix). The proposition illustrates again that consumers that are located in a certain range in between two alternatives are willing to take either alternative. The proposition also shows that this range is different whether the consumer has searched one of these two alternatives, or if the consumer has searched all alternatives except these two alternatives. In the former, the consumer would benefit from searching by twice the distance to the mid-point. In the latter, the consumer buys at random between these two alternatives, and would benefit from searching by the distance to the expected product location, the mid-point between the two alternatives.

Figure 3 shows the expected dis-utility for each x when the firm offers three alternatives and the consumers search the first alternative. As expected, the consumers located close to the alternatives' locations do better. However, consumers could also decide not to search the first alternative, and just to buy at random among the three alternatives.

Note that, if $\frac{c}{t} \in (\frac{2}{21}, \frac{1}{6})$, when faced with three alternatives, some consumers would buy at random while other consumers would engage in the evaluation of a first alternative. Figure 3 shows the expected dis-utility when consumers buy at random and when they engage in the evaluation of the first alternative for $c = \frac{1}{16}$ and $t = 1$, in which case all consumers engage in the evaluation of at least one alternative. Figure 4 shows the same curves when $c = \frac{1}{9}$ and $t = 1$, in which case some consumers engage in the evaluation of at least one alternative, while other consumers buy at random.

For the case in which all consumers that buy a product engage in the evaluation of the first alternative, $\frac{c}{t} < \frac{2}{21}$, one can compute the expected dis-utility across all consumers as

$$d(3) = \frac{t}{12} + \frac{5}{3}c - \frac{c^2}{t}. \tag{11}$$

5.4. Comparison Across Number of Alternatives

Comparing the expected dis-utility of one product with the case of an infinite number of products one can see that

$$d(1) < d(\infty) \text{ if and only if } \frac{c}{t} > \frac{1}{2(\sqrt{\pi} + \sqrt{2})^2} \tag{12}$$

where $\frac{1}{2(\sqrt{\pi}+\sqrt{2})^2}$ is close to $.05 < \frac{1}{16}$. If consumer search costs are sufficiently large, the firm is better off offering only one alternative and saving on the consumer search costs than offering an infinite number of alternatives. This also illustrates the result in Proposition 2 that a finite number of alternatives (not necessarily just one alternative) is optimal with positive search costs.

Note again that for $\frac{c}{t} > \frac{1}{8}$ consumers facing an infinite number of alternatives choose at random and get an expected dis-utility of the product bought, $\frac{t}{3}$, that is greater than when the firm offers only one alternative. Because $d(\infty)$ is weakly increasing in c, it can also be immediately seen that, for $\frac{1}{16} < \frac{c}{t} < \frac{1}{8}$, we have $d(1) < d(\infty)$.

Comparing the case of two alternatives with the case of one alternative we have

$$d(2) < d(1) \text{ if and only if } \frac{c}{t} < \frac{2-\sqrt{2}}{4}, \tag{13}$$

that is, offering two products is better for the firm than offering one product if the search costs are low enough.

Comparing two products with the case of an infinite number of products we can see that

$$d(2) < d(\infty) \text{ if and only if } \frac{c}{t} > \widehat{c}, \tag{14}$$

where $\widehat{c}$ satisfies $\widehat{c}^2 - (2 - \frac{\pi}{2})\widehat{c} + \sqrt{\widehat{c}} - \frac{1}{8} = 0$. It can be easily seen that $\widehat{c}$ is uniquely defined, and is close to $.017 < \frac{1}{2(\sqrt{\pi}+\sqrt{2})^2}$.

Comparing three alternatives with an infinite number of alternatives, one can get that $d(3) < d(\infty)$ if and only if $\frac{c}{t} > c^{**}$ where c^{**} defined by ${c^{**}}^2 - (\frac{8}{3} - \frac{\pi}{2})c^{**} + \sqrt{c^{**}} - \frac{1}{12} = 0$ is below $\frac{1}{16}$ and close to .009.

Comparing the optimal strategy for the firm among an infinite number of products, three products, two products, and just one product, we have then the following result:

PROPOSITION 4: *Suppose that the options for the firm are to offer one, two, three, or an infinite number of products. Then, if $\frac{c}{t} < c^{**}$, the firm prefers to offer an infinite number of products. If $\frac{c}{t} \in [c^{**}, \frac{1}{16}]$, the firm prefers to offer three products. If $\frac{c}{t} \in [\frac{1}{16}, \frac{2-\sqrt{2}}{4}]$, the firm prefers to offer two products. Finally, if $\frac{c}{t} > \frac{2-\sqrt{2}}{4}$, the firm prefers to offer a single product.*

This comparison illustrates that offering a finite number of products is optimal if the search costs are not too low, and suggests that lower search costs should lead the firm to offer more alternatives.

The above proposition also shows that if a firm offers too many alternatives some consumers may not make any choice and stay out of the market, because they understand that it will be too costly (in terms of search costs) for them to find the alternative that best fits their preferences (that is, there would be a higher threshold $\widehat{v}$). Similarly, if the firm offers too few alternatives, some consumers may also stay out of the market because they feel that the alternatives that are available may not fit well their preferences. In sum, given the existence of search costs there is an optimal (finite) number of alternatives to offer.

5.5. "Approximate" Optimal Number of Products

The analysis above suggests an approach to try to get at an approximate optimal number of products. Suppose that the firm chooses a number of products such that a set of consumers close to a product always keep on searching until they find the product closest to them. In addition, consider the approximation where all consumers are in this situation. That is, no consumer settles with the product that is not their most preferred product, or buys at random. As seen above, this may not be the optimal search process for some consumers.

Given this approximation, the expected dis-utility across all consumers of the firm offering n products is

$$\widetilde{d}(n) = \frac{t}{4n} + (n-1)(\frac{1}{2} + \frac{1}{n})c.$$

The optimal number of products can then be obtained to be (without worrying about integer issues)

$$n = \sqrt{\frac{t}{2c} - 1}.$$

Figure 5 illustrates the optimal number of products as a function of $\frac{c}{t}$ which illustrates that the optimal number of products reduces quickly when $\frac{c}{t}$ increases. Figure 6 illustrates the expected dis-utility as a function of the number of products, and shows that the expected dis-utility increases beyond the optimal number of products, the information overload effect because of search costs, and that the expected dis-utility decreases for a lower number of products, because of less fit between the products offered and the consumer preferences.

This approximation does not account for the fact that consumers in a range in between two alternatives may be willing to accept either of the two alternatives. In the Appendix we derive an approximation for the optimal number of products (which is close to the one above) that accounts for this possibility.[15]

[15] In particular, we show that when the search costs go to zero, the fraction of consumers that settle on the second-best alternatives is bounded away from zero, and derives an approximation of this fraction of consumers and the optimal number of the products under the condition that search costs are small.

6. Discussion and Applications

A formal model of consumer decision making as depending on the importance of fit and evaluation costs allows us to see how the optimal number of alternatives varies when model parameters or the distribution of preferences change. Consider, for example, the problem of how many options a computer operating system provider (e.g., Microsoft) should offer to consumers. The operating system may come with different software either pre-installed or not, and in some cases (e.g., Netscape vs. Internet Explorer vs. Firefox) in the case of software installed, can have a choice of several alternative softwares. If all the choice possibilities are presented at once, the consumers may be overwhelmed with choice alternatives. Can the problem be avoided by presenting a sequential choice on each software with only a few options (install or not, and if yes, a couple of choices to install)? What the model suggests is that when the decision becomes less important (the cost of misfit is low), consumers prefer not to have a choice at all rather than to have a few choices. Furthermore, the consumer may prefer this especially when he/she expects the firm to be better aware what would best satisfy an average consumer. For example, the consumer may want Microsoft to decide on which parts of the Microsoft software to install, but may prefer Microsoft to provide a choice when it is between a Microsoft and a competitor's product. This example also suggests that the model presented above could help us understand potential limits to versioning strategies in information good industries (see Shapiro and Varian, 1999, p. 67).

It may also be that the preference distribution is not as in the spacial Hotelling case, with some consumers having more "central" and some consumers having more "extreme" preferences, but rather be more binary ("equi-distant"). The simplest example is a two-segment consumer preference space. If the mass of consumers in each segment is the same, uninformed consumers (uninformed both about preferences before entering the market and about which alternative is which) have no reason to prefer the firm to provide one rather than two alternatives. In the same way, if the consumer distribution is two-point in m independent dimensions, the firm may offer up to 2^m different alternatives, and again, uninformed consumers can not be better off when the number of alternatives is smaller. If, however, the mass of consumers is bigger in some segments than in others, consumers could save in search costs if only the most popular alternatives are offered (again given that the strategic firm offers only the most popular alternatives).

One way to think about the problem of how much decision making should be done by the firm and how much given to the consumer is that (a) the firm may know the environment better (i.e., it knows exactly where all the possible alternatives are located), but (b) the objective function of a consumer is different than the one of the firm. The first point implies that more of the decision making (preselecting what to offer to consumers) should be done by the (more knowledgeable) firm, whereas the latter point suggests that the consumer, while with less information, may be able to make a decision more aligned with the consumer's objectives, and therefore, the consumer should do more decisions. The trade-off between the two forces leads to the optimal split of the choice done by the firm (restricting the number of alternatives offered) and the choice done by the consumers (which alternative to choose out of the set offered by the firm) in this regard.

7. Extensions

7.1. Private Information of the Supplier of Alternatives

In some markets, some consumers may face uncertainty about the importance of fit, while the firm may have more information. Then, consumers may try to infer the importance of fit from the number of alternatives offered by the firm. In the context of the model above, this would mean, for example, that the firm has private information on t. Suppose that consumers have a prior on t that is distributed with some cumulative distribution function with no mass points, that the firm is considering whether to offer one or two alternatives, and that the set of consumers with zero search costs, ε, is strictly greater than zero.

One can then show that if the firm has private information on t it only offers two alternatives (versus one alternative) if the importance of fit t is large enough. This illustrates the idea that when a consumer sees more alternatives offered, he may infer that alternative-fit is more important. Notice that, even though the firm prefers the consumer to believe that alternative-fit is not too important, the firm may still end up offering more alternatives, and revealing the importance of alternative-fit, because of the consumers that can learn about the importance of alternative-fit in some other way.

This intuition raises the question of whether the firm may be tempted to offer fewer alternatives under private information than under the case in which there is no private information. Indeed, one can show that the supplier of alternatives offers a lower number of alternatives when she has private information than when she does not have private information, in order for the consumers to believe that the importance of fit is not too large. That is, private information is a force towards offering a lower number of alternatives.

7.2. Knowledge of Preferences Prior to Deciding to Enter the Market

The analysis above is under the assumption that consumers need to invest a large amount K prior to searching, and only then they completely learn their preferences, learn x in addition to v. This simplified the analysis because whether a consumer decided to buy a product only depended on v, and not on both v and x. Consider now the case in which $K = 0$, consumers know both v and x prior to deciding whether to search, or purchase a product at random. Let us restrict attention to either one or two products.

Suppose first the case in which the firm offers only one product. Suppose also that the consumers infer the product location to be z_1. Without loss of generality consider $z_1 \leq \frac{1}{2}$. No consumer will search because there is nothing to learn from incurring the search costs. Consumers just decide to buy or not the product depending on their preference parameters, v and x. Then, consumers buy if $v \geq t|z_1 - x|$. The total demand is then great for consumers with x close to z_1, and one can then show that the z_1 that generates greater demand is $z_1 = 1/2$ (which is also the equilibrium location if there is a mass $\varepsilon > 0$ of consumers with zero search costs).

Consider now the case in which the firm offers two products, located at z_1 and z_2, with $z_1 < z_2$. From the analysis in Section 5.2, we know that if the consumer decides to buy the product, buying at random is better if and only if x is close to the mid-point between z_1 and z_2. It is also straightforward to see that consumers with x closer to z_1 and z_2 have lower dis-utility from engaging in the search process, and therefore are willing to search with a lower valuation v. One can then get that total demand is maximized if z_1 and z_2 are located symmetrically around $1/2$, $z_1 = 1 - z_2$. If the probability distribution on v is uniform then this results in $z_1 = \frac{1}{4}$, as in Section 5.2, and which is also the equilibrium if there is a mass $\varepsilon > 0$ of consumers with zero search costs. Note also that, as above, if the search costs c are low enough demand is greater, and the firm chooses to offer two products, while if the search costs are high enough the firm prefers to offer only one product.

For the case in which v is uniformly distributed, the comparison between offering one and two alternatives is exactly as presented in Section 5.4 for K large. This is because, with a uniform distribution for v, the density is constant for all v, and therefore demand is greater when the expected dis-utility, across all consumers, is smaller, which was exactly the decision criterion in the previous sections.

7.3. Consumer and Firm Uncertainty About the Market

In some markets, consumers, although knowing their own preferences, may not know where the preferences of the other consumers may be, while the firm may have information about the distribution of consumer preferences. In the context of the model, the firm could know where the unit segment of consumer ideal points is located, while each consumer knows only his own preference, the location of his own x. In such a case, a consumer does not know the location of the products with respect to his preferences. Consumers may then be willing to incur the search costs to evaluate a product in order to check if the product may be satisfactory. Some consumers could then engage in the search process, while ending up not purchasing any product.

Similarly, the firm may not know exactly where the consumers preferences lie, due to some non-exact market research. Then, consumers do not know where the products may be offered, which is a further incentive for consumers to incur search costs to evaluate products, and not be willing to purchase at random. Furthermore, some consumers may engage in the search process, only to find out that no product fits their preferences, and therefore, decide not to purchase any product.

7.4. Endogenous Prices

We considered the supplier of alternatives trying to maximize the number of customers choosing an alternative ignoring the possible implications for price. Considering the pricing decision for each alternative to be offered can lead to additional insights. Suppose that prices are the same across alternatives. This restriction may be practical, for example, when charging different prices is costly to the firm, as in the versioning of software example considered above, or when it could lead to further ambiguities, as when quality is also a potential decision variable, and consumers are afraid that a lower price may indicate lower quality. In this case a consumer will decide to enter the market and purchase a product if the expected utility (following his optimal search process) exceeds the price. In other words, the problem becomes: A consumer enters the market and buys the product if and only if $U(p,n) \equiv v - d(n) - p \geq 0$, and the firm maximizes $\pi(p) = p \times \text{Prob}(U(p,n) \geq 0)$ over the price p (and the number and location of the products). It is easy to see that given p, the decision problem of the firm reduces to the one we already considered: $\min_n d(n)$. In other words, all implications about the optimal number of products remain the same.

If consumers can observe the prices charged, and prices can be different across products, then different prices can potentially signal the location of the different alternatives. This signalling could be hindered by the firm's private information on other dimensions. If prices are also costly to evaluate then some of the insights mentioned above may also apply, with the additional effect that if too many products are offered, prices may be higher because of greater potential fit, which may lead to expectations of lower surplus, and therefore less search and choice. A firm may also want to limit the number of products offered in order to reduce search costs, as lower search costs may allow a firm to charge higher prices.

8. Conclusion

This paper considers the decision of the number of alternatives to be offered by a firm or government agency. Offering more alternatives may potentially allow each consumer to get an alternative that best fits his preferences. However, if more alternatives are offered it is more costly for the consumers to find a relatively acceptable alternative matching their preferences. This results in there being an optimal (finite) number of alternatives to be offered. If the firm offers more alternatives, consumers incur too high search costs if they decide to enter the market. This leads some consumers to be "overwhelmed" with too many alternatives, and to decide not to choose any alternative. This can be seen as an explanation for the information overload (alternative overload, more specifically) effect that, with more information (more alternatives), consumers are less likely to make a choice.

Some of the results presented here may extend to a setting where more information about an alternative may lead a consumer to be less likely to choose that alternative. Another interesting issue that should be explored in future work is allowing for non-perfect evaluation of alternatives, and consumers being able to decide on the intensity of their evaluation of each alternative. It would also be interesting to investigate what happens when there is competition and price learning in a context where product attributes have to be evaluated for consumer fit.

APPENDIX

The case of an Infinite Number of Products, and $\frac{c}{t} > \frac{1}{16}$: First, consider consumers with $x \in (\sqrt{c/t}, 1-\sqrt{c/t})$, i.e., those that are not too close to the ends of the segment. If $\frac{c}{t} \in (\frac{1}{16}, \frac{3-2\sqrt{2}}{2})$ consumers with

$$x \in (\frac{1-\sqrt{4\sqrt{c/t}-1}}{2}, \frac{1+\sqrt{4\sqrt{c/t}-1}}{2})$$

buy at random, while the other consumers with $x \in (\sqrt{c/t}, 1-\sqrt{c/t})$ engage in the search process as described in the text. If $\frac{c}{t} > \frac{3-2\sqrt{2}}{2}$, all consumers with $x \in (\sqrt{c/t}, 1-\sqrt{c/t})$ choose to buy at random. As noted in the text this implies an interesting search strategy for the consumers depending on their preferences: Consumers that have relatively specific preferences (in the model, with a location close to zero or one) search, and consumers that have more generic preferences (around the center of the market) do not evaluate a product prior to purchase and buy at random.

Now, consider consumers with $x < \sqrt{c/t}$ (consumers with $x > \sqrt{c/t}$ behave similarly to these). For $\frac{1}{16} < \frac{c}{t} < \frac{1}{8}$, the expected dis-utility is decreasing in x for both when a consumer engages in the search process and when a consumer buys at random. Note first that, as argued above, when a consumer engages in the search process, the expected search costs plus dis-utility of the product bought is concave in x. On the other hand, the expected dis-utility of the product bought when a consumer buys at random is convex in x, that is the further away a consumer is from the center of the market the consumer is in an increasingly worse situation.

Comparing the engaging in the search process strategy with buying at random, the condition on x that engaging in the search process is better than a random choice is

$$x^2 - x + \frac{1}{2} \geq \sqrt{2\frac{c}{t} - x^2}$$

which reduces to

$$f(x; \frac{c}{t}) \equiv x^4 - 2x^3 + 3x^2 - x + \frac{1}{4} - 2\frac{c}{t} \geq 0.$$

This polynomial function is convex and decreasing in $\frac{c}{t}$. Therefore, there is a c^* such that if $\frac{c}{t} < c^*$ then $f(x; \frac{c}{t}) = 0$ has no solutions, if $\frac{c}{t} > c^*$ then $f(x; \frac{c}{t}) = 0$ has two distinct solutions, and if $\frac{c}{t} = c^*$ then $f(x; \frac{c}{t}) = 0$ has exactly one solution (c^* is close to .078). We also know that $f(x; \frac{3-2\sqrt{2}}{2}) = 0$ at $x = \frac{2-\sqrt{2}}{2}$ and that $\frac{3-2\sqrt{2}}{2}$ is the only $\frac{c}{t} \in (\frac{1}{16}, \frac{1}{8})$ in which $x = \sqrt{\frac{c}{t}}$ satisfies $f(x; \frac{c}{t}) = 0$ and $f(\sqrt{c}, \frac{c}{t}) < 0$ for $\frac{c}{t} \in (\frac{3-2\sqrt{2}}{2}, \frac{1}{8})$. Furthermore, $f'(\frac{2-\sqrt{2}}{2}; \frac{3-2\sqrt{2}}{2}) > 0$ and $f(0; \frac{3-2\sqrt{2}}{2}) > 0$. Therefore, $f(x; \frac{3-2\sqrt{2}}{2}) = 0$ has another solution strictly greater than zero, and strictly smaller than $\frac{2-\sqrt{2}}{2}$. This implies that $c^* < \frac{3-2\sqrt{2}}{2}$. Checking that $f(x;\frac{1}{16}) - x^4$ is always positive for $x \in (0,\frac{1}{2})$ we have that $c^* > \frac{1}{16}$. We then can conclude the following. For $\frac{c}{t} \in (\frac{1}{16}, c^*)$ all consumers with $x \in (0, \sqrt{\frac{c}{t}})$ engage in the search process. For $\frac{c}{t} \in (c^*, \frac{3-2\sqrt{2}}{2})$ we have that $f(x;\frac{c}{t}) = 0$ has two solutions, $y_1(\frac{c}{t})$ and $y_2(\frac{c}{t})$, with $0 < y_1(\frac{c}{t}) < y_2(\frac{c}{t}) < \sqrt{\frac{c}{t}}$, consumers with $x \in [0, y_1(\frac{c}{t})) \cup (y_2(\frac{c}{t}), \sqrt{\frac{c}{t}})$ engage in the search process, and consumers with $x \in (y_1(\frac{c}{t}), y_2(\frac{c}{t}))$ choose at random. Finally, for $\frac{c}{t} \in (\frac{3-\sqrt{2}}{2}, \frac{1}{8})$ there is only solution to $f(x,\frac{c}{t}) = 0$ that is below $\sqrt{\frac{c}{t}}$, $y_1(\frac{c}{t})$, and consumers with $x \in [0, y_1(\frac{c}{t}))$ engage in the search process, and consumers with $x \in (y_1(\frac{c}{t}), \frac{c}{t})$ choose at random.

Putting all these results together we have that there are $\frac{c}{t} \in (c^*, \frac{3-2\sqrt{2}}{2})$ such that the set of consumers that choose at random is not convex. That is, starting from the center of the market and going to either extreme, we have first consumers that choose at random, $x \in (\frac{1-\sqrt{4\sqrt{c/t}-1}}{2}, \frac{1+\sqrt{4\sqrt{c/t}-1}}{2})$, then consumers that engage in the search process, $x \in (y_2(\frac{c}{t}), \frac{1-\sqrt{4\sqrt{c/t}-1}}{2}) \cup (\frac{1+\sqrt{4\sqrt{c/t}-1}}{2}, 1-y_2(\frac{c}{t}))$, then consumers that choose at random,

$$x \in (y_1(\frac{c}{t}), y_2(\frac{c}{t})) \cup (1 - y_2(\frac{c}{t}), 1 - y_1(\frac{c}{t})),$$

and, finally, again consumers that engage in the search process, $x \in [0, y_1(\frac{c}{t})) \cup (1 - y_1(\frac{c}{t}), 1]$.

Solution of the Integral $\int_0^{\sqrt{\frac{c}{t}}} \sqrt{2\frac{c}{t} - x^2}\, dx$:

Define the variable τ as $\sqrt{2\frac{c}{t} - x^2} = x\tau + \sqrt{2\frac{c}{t}}$ which yields $x = -\frac{2\tau\sqrt{2\frac{c}{t}}}{1+\tau^2}$. Note also that $\frac{dx}{d\tau} = -2\sqrt{2\frac{c}{t}}\frac{1-\tau^2}{(1+\tau^2)^2}$. Substituting variables one can then write

$$\int_0^{\sqrt{\frac{c}{t}}} \sqrt{2\frac{c}{t} - x^2}\, dx = \int_{1-\sqrt{2}}^{0} 4\frac{c}{t}\frac{(1-\tau^2)^2}{(1+\tau^2)^3}\, d\tau. \tag{i}$$

Noting now that $\frac{(1-\tau^2)^2}{(1+\tau^2)^3} = \frac{4}{(1+\tau^2)^3} - \frac{4}{(1+\tau^2)^2} + \frac{1}{1+\tau^2}$, and that $\int \frac{1}{1+\tau^2}\, d\tau = \arctan \tau$, $\int \frac{1}{(1+\tau^2)^2}\, d\tau = \frac{\tau}{2(1+\tau^2)} + \frac{1}{2}\arctan \tau$, and $\int \frac{1}{(1+\tau^2)^3}\, d\tau = \frac{5\tau+3\tau^3}{8(1+\tau^2)^2} + \frac{3}{8}\arctan \tau$, we have

$$\int_0^{\sqrt{\frac{c}{t}}} \sqrt{2\frac{c}{t} - x^2}\, dx = 4\frac{c}{t}[\frac{\tau(1-\tau^2)}{2(1+\tau^2)^2} + \frac{1}{2}\arctan \tau]_{1-\sqrt{2}}^{0} = \frac{c}{4t}(2+\pi). \tag{ii}$$

Proof of Proposition 3:

To show that the pair of locations $\{1/4, 3/4\}$ minimizes the expected consumer dis-utility, consider general locations for the two alternatives. Denote by z_1 the location of the product that is located closer to zero, and z_2 as the location of the other product, with $z_1 < z_2$ (the case of $z_1 = z_2$

being the case of one product presented above). Suppose that $z_2 - z_1 \geq 2\frac{c}{t}$ (Otherwise all consumers prefer to search at random, and we are back in the one alternative case). A consumer that searches one product gets his most preferred product (between the two products), because if the searched product is not the most preferred the consumer knows that the other product will be. Then, if a consumer located at x searches one product, he gets a total cost (search cost plus dis-utility of product bought) of $c + t\min[\mid x - z_1 \mid, \mid z_2 - x \mid]$. If a consumer chooses a product at random, he gets an expected dis-utility of $\frac{t}{2} \mid x - z_1 \mid + \frac{t}{2} \mid z_2 - x \mid$.

From this one can see that if $x \in (\frac{z_1+z_2}{2} - \frac{c}{t}, \frac{z_1+z_2}{2} + \frac{c}{t})$, then a consumer prefers to choose an alternative at random, if $x < \frac{z_1+z_2}{2} - \frac{c}{t}$, then a consumer searches to find product z_1 and gets a search cost plus dis-utility of the product bought equal to $c + t \mid x - z_1 \mid$, and finally, if $x > \frac{z_1+z_2}{2} + \frac{c}{t}$, then a consumer searches to find product z_2 and gets a search cost plus dis-utility of the product bought equal to $c + t \mid x - z_2 \mid$.

The expected search costs plus dis-utility of the product bought across all consumers as a function of z_1 and z_2 is then equal to

$$z_1(c + t\frac{z_1}{2}) + (1 - z_2)(c + t\frac{1 - z_2}{2}) + (\frac{z_1 + z_2}{2} - \frac{c}{t} - z_1)[c + \frac{t}{2}(\frac{z_1 + z_2}{2} - \frac{c}{t} - z_1)] + (z_2 - \frac{z_1 + z_2}{2} - c)[c + \frac{t}{2}(z_2 - \frac{z_1 + z_2}{2} - \frac{c}{t})] + 2\frac{c}{t}\frac{z_2 - z_1}{2}t \tag{iii}$$

which reduces to

$$t\{\frac{c}{t} - (\frac{c}{t})^2 + \frac{z_1^2}{2} + \frac{(1 - z_2)^2}{2} + (\frac{z_2 - z_1}{2})^2\}. \tag{iv}$$

Minimizing with respect to the location of the products (for the firm to offer the best pair of alternatives) one gets $z_1 = \frac{1}{4}$ and $z_2 = \frac{3}{4}$, which are also the optimal locations with two products and zero search costs. As argued in Section 3 for the general case these are also the locations chosen by the firm given K large.

Substituting the equilibrium $(z_1, z_2) = (1/4, 3/4)$ into equation (iv), one can obtain

$$d(2) = \frac{t}{8} + c - \frac{c^2}{t} \tag{v}$$

for $c < \frac{t}{4}$ (for $c > \frac{t}{4}$ consumers choose to buy at random, and then just having one product is optimal). QED

Statement and Proof of Proposition 5 (three alternatives):

Proposition 5: *Suppose that the firm offers three alternatives and $\frac{c}{t} < \frac{2}{21}$. Then all consumers* *search the first alternative. After that, the consumers' optimal search process is characterized by the following: If $x < \frac{1}{3} - \frac{c}{t}$ then the consumer keeps z_1 if he finds it first, and keeps on searching if he does not find z_1 first, for an expected dis-utility of $\frac{5}{3}c + t|x - \frac{1}{6}|$. If $x \in [\frac{1}{3} - \frac{c}{t}, \frac{1}{3} - \frac{c}{2t}]$ the consumer buys z_1 if he finds it first, keeps on searching if he finds z_2 first, and buys at random if he finds z_3 first, for an expected dis-utility of $\frac{4}{3}c + t(\frac{2x}{3} - \frac{1}{18})$. If $x \in [\frac{1}{3} - \frac{c}{2t}, \frac{1}{3} + \frac{c}{2t}]$, the consumers buys z_1 or z_2 if he finds one of these alternatives in the first search, and, if he finds z_3 first, he buys at random one of the other two remaining alternatives, for an expected dis-utility of $c + \frac{t}{6}$. If $x \in [\frac{1}{3} + \frac{c}{2t}, \frac{1}{3} + \frac{c}{t}]$, the consumer buys z_2 if he finds it first, keeps on searching (to find z_2) if he finds z_1 first, and buys at random one of the other two remaining alternatives if he finds z_3 first, for an expected dis-utility of $\frac{4}{3}c + t(\frac{7}{18} - \frac{2x}{3})$. Finally, for $x \in [\frac{1}{3} + \frac{c}{t}, \frac{1}{2}]$, the consumer buys z_2 if he finds it first, and keeps on searching (to find z_2) if he finds z_1 or z_3 first, for an expected dis-utility of $\frac{5}{3}c + t(\frac{1}{2} - x)$.*

PROOF: Consider the locations of consumers $x \leq \frac{1}{2}$ (the case $x > \frac{1}{2}$ is the symmetric case) and $\frac{c}{t} < \frac{1}{6}$ (for $\frac{c}{t} > \frac{1}{6}$, two alternatives is better than three alternatives as discussed below). Let us look first at the case where consumers buy at random. If $x > \frac{1}{6}$ the expected dis-utility as a function of x is $t[\frac{1}{3}(\frac{5}{6} - x) + \frac{1}{3}(\frac{1}{2} - x) + \frac{1}{3}(x - \frac{1}{6})] = t(\frac{7}{18} - \frac{x}{3})$. If $x < \frac{1}{6}$, and in the same way, we can find that the expected dis-utility as a function of x is $t(\frac{1}{2} - x)$.

Consider now the case in which consumers search first one of the alternatives (restricting for now attention to $x > \frac{1}{6}$). Suppose that the consumer finds alternative $z_1 = \frac{1}{6}$ first. Then, the consumer (i) can choose to buy this alternative, in which case the consumer gets a dis-utility of $c + t(x - \frac{1}{6})$, (ii) the consumer can choose to search once more, in which case the consumer can get his most other preferred alternative, $z_2 = \frac{1}{2}$, for a dis-utility of $2c + t(\frac{1}{2} - x)$, or (iii) the consumer can choose to buy at random from among the other two alternatives, in which case the consumer gets an expected dis-utility of $c + \frac{t}{2}(\frac{1}{2} - x) + \frac{t}{2}(\frac{5}{6} - x) = c + t(\frac{2}{3} - x)$. It can be easily seen that buying at random between the other two alternatives is worse that searching once more as long as $\frac{c}{t} < \frac{1}{6}$, which was assumed. Finally, buying the alternative just searched, z_1, is better than searching once more if and only if $x < \frac{1}{3} + \frac{c}{2t}$. Note that this means that there are some consumers that even though they prefer z_2 to z_1 (if $x > \frac{1}{3}$) still keep alternative z_1 if they find it first, because of the additional search costs of trying to find z_2. Note that this also implies that if $x < \frac{1}{6}$, if the consumer finds z_1 first, he will then naturally buy this alternative.

Suppose now that the consumer first finds alternative $z_2 = \frac{1}{2}$. If the consumer buys this alternative, he gets a dis-utility of $c + t(\frac{1}{2} - x)$. If the consumer searches once more, he finds his most preferred other alternative for a dis-utility of $2c + t(x - \frac{1}{6})$. It can be seen that the consumer buying at random between the other two alternatives is dominated by either buying z_2, or searching once more. One can then see that buying the alternative just searched is better than searching once more if and only if $x > \frac{1}{3} - \frac{c}{2t}$. Again, some consumers that prefer z_1 to z_2 will be happy to keep z_2 if they find it first.

Finally, suppose that the consumer first finds alternative $z_3 = \frac{5}{6}$. If the consumer buys this alternative, he gets a dis-utility of $c + t(\frac{5}{6} - x)$. If the consumer buys at random one of the other two alternatives, he gets an expected dis-utility of $c + \frac{t}{2}(\frac{1}{2} - x) + \frac{t}{2}(x - \frac{1}{6})$. If the consumer searches once more, he gets his most preferred alternative, with a dis-utility of $2c + t(\frac{1}{2} - c)$ if $x > \frac{1}{3}$ and a dis-utility of $2c + t(x - \frac{1}{6})$ if $x < \frac{1}{3}$. One can then obtain that buying alternative z_3 is always dominated by either buying at random or searching once more, and that buying at random of the two other remaining alternatives is better than searching once more if $x \in [\frac{1}{3} - \frac{c}{t}, \frac{1}{3} + \frac{c}{t}]$.

It remains to prove that all consumers prefer to search at least one alternative if $\frac{c}{t} < \frac{2}{21}$. In order to show this, we need to check the conditions under which the expected dis-utility of buying at random, $t(\frac{1}{2} - x)$ for $x < \frac{1}{6}$, and $t(\frac{7}{18} - \frac{x}{3})$ for $x > \frac{1}{6}$, is greater than the expected dis-utility of search a first alternative, as stated in Proposition 5. For $x < \frac{1}{6}$ the condition is $\frac{c}{t} < \frac{1}{5}$. For $x \in [\frac{1}{6}, \frac{1}{3} - \frac{c}{t}]$ the condition is $\frac{c}{t} < \frac{1}{3}$. For $x \in [\frac{1}{3} - \frac{c}{t}, \frac{1}{3} - \frac{c}{2t}]$ the condition results in $\frac{c}{t} < \frac{2}{15}$. For $x \in [\frac{1}{3} - \frac{c}{2t}, \frac{1}{3} + \frac{c}{t}]$ the condition results in $\frac{c}{t} < \frac{2}{21}$. Finally, for $x \in [\frac{1}{3} + \frac{c}{t}, \frac{1}{2}]$ the condition results in $\frac{c}{t} < \frac{1}{9}$. All these conditions are satisfied if $\frac{c}{t} < \frac{2}{21}$. QED

Computation of Equation (11): The expected dis-utility $d(3)$ across all consumers can be obtained to be $\frac{1}{3}(\frac{5c}{3} + \frac{t}{12}) + (\frac{1}{3} - 2\frac{c}{t})(\frac{5c}{3} + t(\frac{1}{12} - \frac{c}{2t})) + \frac{c}{t}(\frac{4c}{3} + t(\frac{1}{6} - \frac{c}{2t})) + 2\frac{c}{t}(c + \frac{t}{6}) + \frac{c}{t}(\frac{4c}{3} + t(\frac{1}{6} - \frac{c}{2t})) + (\frac{1}{3} - 2\frac{c}{t})(\frac{5c}{3} + t(\frac{1}{12} - \frac{c}{2t}))$ which reduces to equation (11).

Proof of Proposition 4: Comparing $d(3)$ with $d(2)$ for $\frac{c}{t} < \frac{2}{21}$ one obtains directly $d(3) < d(2)$ if and only if $\frac{c}{t} < \frac{1}{16}$. In order to complete the proof one has to consider what happens when $\frac{c}{t} \in [\frac{2}{21}, \frac{1}{6}]$. For $\frac{c}{t} \in [\frac{2}{15}, \frac{1}{6}]$ one obtains $d(3) = \frac{t}{12} + \frac{19}{9}c - \frac{14}{3}\frac{c^2}{t}$ and one obtains $d(3) < d(2)$ if and only if $\frac{c}{t} < \frac{20-\sqrt{202}}{132} < \frac{1}{16}$, a contradiction. Finally, for $\frac{c}{t} \in [\frac{2}{21}, \frac{2}{15}]$ one obtains $d(3) = \frac{t}{12} + \frac{16}{9}c - \frac{49}{12}\frac{c^2}{t}$ and one obtains $d(3) < d(2)$ if and only if $\frac{c}{t} < \frac{28-\sqrt{562}}{74} < \frac{1}{16}$, a contradiction. QED

"Approximate" Optimal Number of Products with the Possibility of Consumers Settling on the Second-Best Alternative: As we already know, the optimal distribution of n products is such that the distance between neighboring products is $\frac{1}{n}$. Here, we estimate the approximate expected dis-utility of a consumer engaging in the optimal search, when some consumers always search until finding their first-best alternative, and consumers can settle in their second-best alternative. We then minimize this approximate expected dis-utility over n. This approximation is exact as c goes to zero (and n goes to infinity).[16]

Since all but $\frac{1}{n}$ consumers are located between two products, we only consider the expected dis-utility of consumers conditional on them being located between some two products. We have assumed that n is such that some consumers close to a product search until they find that particular product. However, consumers who are located almost at the mid-point between two adjacent products will search only until they find one of these two products. In other words, all consumers search until they find the first or second-best product. If a consumer finds the first-best before the second-best, he chooses the first-best. However, if he finds the second-best before finding the first-best, he chooses the second-best if he is close to the mid-point between the first-best and the second-best so that the expected cost of searching further until he finds the first-best is too high relative to the reduction in dis-utility.

The expected total cost of searching until the first-best is found is approximately $\frac{n}{2}c$ (the exact value is in Equation (8)). The expected cost of searching for the first-best after the second-best is found depends on how many products have already been tried. Let k be the number of searches until the second-best is found and assume that the first-best was not yet found by the kth search, then the expected additional cost of searching until the first-best is found is $\frac{n-k}{2}c$.

The optimal consumer strategy is the following. If the consumer is further than $\frac{nc}{4t}$ from the mid-point between the first-best and second-best product, the consumer will search until the first-best is found, since even if he finds the second-best first, the cost of searching for the first-best (at most $\frac{n}{2}c$) is lower than the additional utility from the first-best, which is twice his distance from the mid-point times t.

Consider now a consumer located within $\frac{nc}{4t}$ of the mid-point between the first-best and second-best alternative. We have that this consumer searches at least until the first *or* second-best is found. The probability that he will find the first or second-best when at least q but less than $q+dq$ fraction of the products (where $q=\frac{k}{n}$ for some integer k and $dq=\frac{1}{n}$) are searched is asymptotically $2(1-q)dq$. This is because the probability of finding one of the two best on the kth search is

$$(1-\frac{2}{n})(1-\frac{2}{n-1})\cdot\ldots\cdot(1-\frac{2}{n-k+2})\frac{2}{n-k+1}=\frac{2}{n-1}(1-\frac{k}{n})\approx 2(1-q)dq.$$

Once he found the first or second-best, he has spent qnc on search, and with probability $1/2$ found the best product. The expected dis-utility of the best product across all consumers we are now

[16] We can compute the exact expected dis-utility and exact optimal number of products under the assumption that some consumers close to one product always keep on searching until finding it. This computation is, however, more complicated than the approximation presented here for n large, without yielding major new insights.

considering is $\frac{t}{2n} - \frac{n}{8}c$.

With probability 1/2, however, he has found the second-best product. In this case, considering further search until the first-best is found, he faces the benefit of twice her distance to the mid-point times t and the expected additional cost of $(1-q)\frac{n}{2}c$. Hence, he will not search further for the first-best if and only if he is at most $(1-q)\frac{nc}{4t}$ from the mid-point. Hence, in the case we are considering, the consumers who would decide to search further are located at a distance between $\frac{nc}{4t}$ and $(1-q)\frac{nc}{4t}$ from the mid-point. These $q\frac{n^2c}{2t}$ consumers will incur further search costs of $(1-q)\frac{n}{2}c$ and will have a product dis-utility from the first-best of, on average, $\frac{t}{2n} - (1-\frac{q}{2})\frac{nc}{4}$. The other $(1-q)\frac{n^2c}{2t}$ consumers, located closer than $(1-q)\frac{nc}{4t}$ to the mid-point, will not search further, and will have product dis-utility from the second-best product of, on average, $\frac{t}{2n} + (1-q)\frac{n}{8}c$.

Integrating the total expected dis-utilities of consumers located within $\frac{nc}{4t}$ from the mid-point between their first-best and second-best choice over all values of q, we obtain that the sum of their expected dis-utilities is

$$\begin{aligned}\frac{n^2c}{2t}\int_0^1 (ncq \quad &+ \quad \frac{1}{2}(\frac{t}{2n} - \frac{nc}{8}) \\ &+ \quad \frac{1}{2}(q(\frac{(1-q)nc}{2} + \frac{t}{2n} - (1-\frac{q}{2})\frac{nc}{4}) + (1-q)(\frac{t}{2n} + \frac{(1-q)nc}{8})))2(1-q)\,dq,\end{aligned}$$

where $\frac{n^2c}{2t}$ is the total number of such consumers. Let $a \equiv \frac{n^2c}{2t}$. Then, the above expression reduces to $at\frac{5a+4}{8n}$, and it is (asymptotically) the sum of dis-utilities from the fraction a of all consumers who can possibly stop the search process at the second-best. The remaining $1-a$ fraction of the consumers always search until the best alternative is found. They incur each $\frac{n}{2}c = \frac{at}{n}$ expected search cost and product dis-utility, on average, of $\frac{t}{4n} - \frac{n}{8}c = (1-a)\frac{t}{4n}$.

Hence, the total consumer dis-utility is $t\frac{2+8a-a^2}{8n}$. This expression is minimized at

$$n = \frac{\sqrt{24-6\sqrt{10}}}{3}\sqrt{\frac{t}{c}} \approx 0.7473\sqrt{\frac{t}{c}}.$$

When n is chosen as above, asymptotically,

$$\frac{n^2c}{2t}\int_0^1 \frac{1}{2}(1-q)\,2(1-q)\,dq = \frac{n^2c}{6t} = \frac{4-\sqrt{10}}{9} \approx 0.093$$

fraction of all consumers end up choosing the second-best product; the rest of the consumers end up choosing the first-best.

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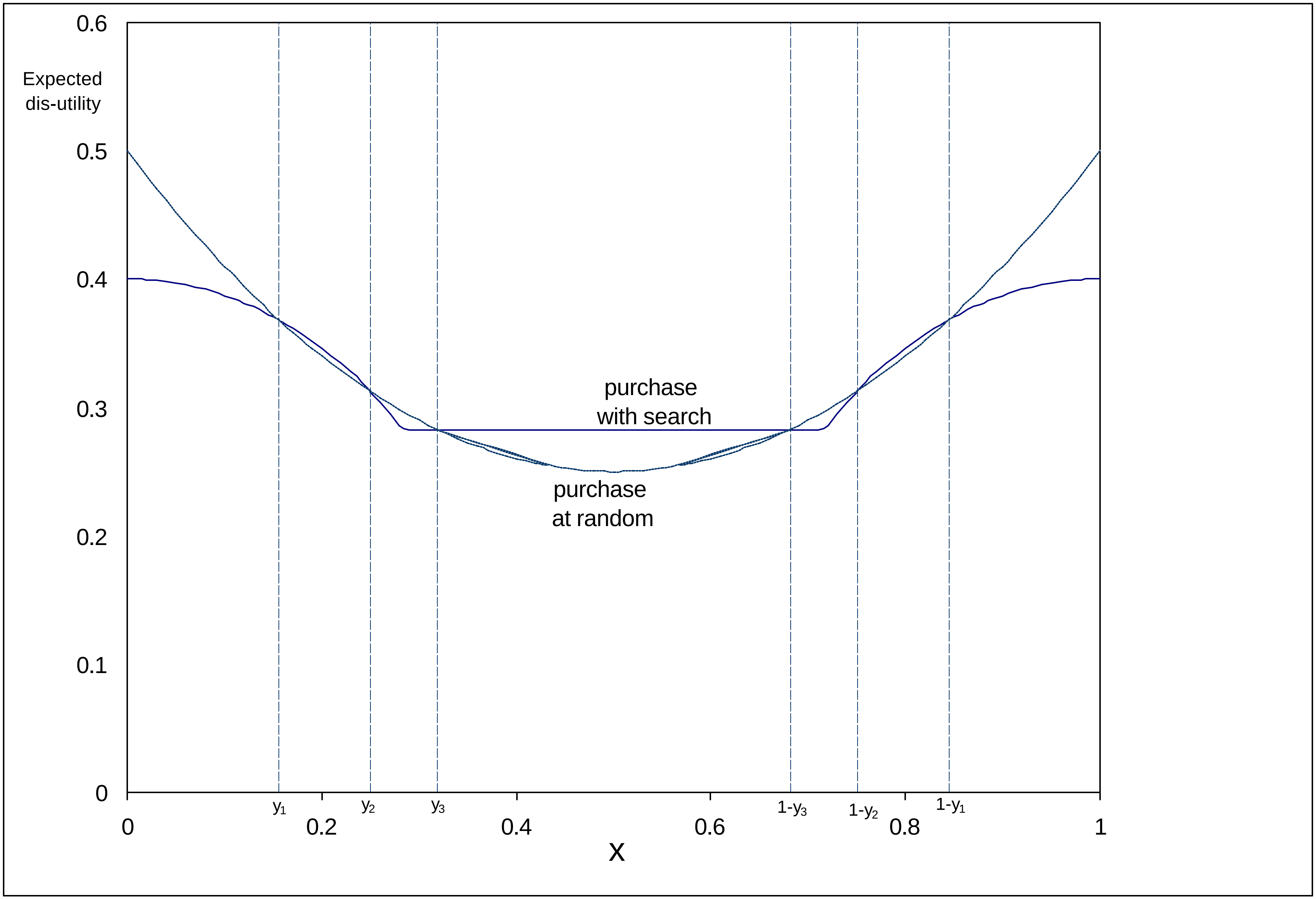

Figure 1: Expected search costs plus dis-utility of product bought for each location x for the case of an infinite number of products under purchase at random, and purchase with search for c/t=.08, t=1. $y_3=(1-(4c^{1/2}-1)^{1/2})/2$.

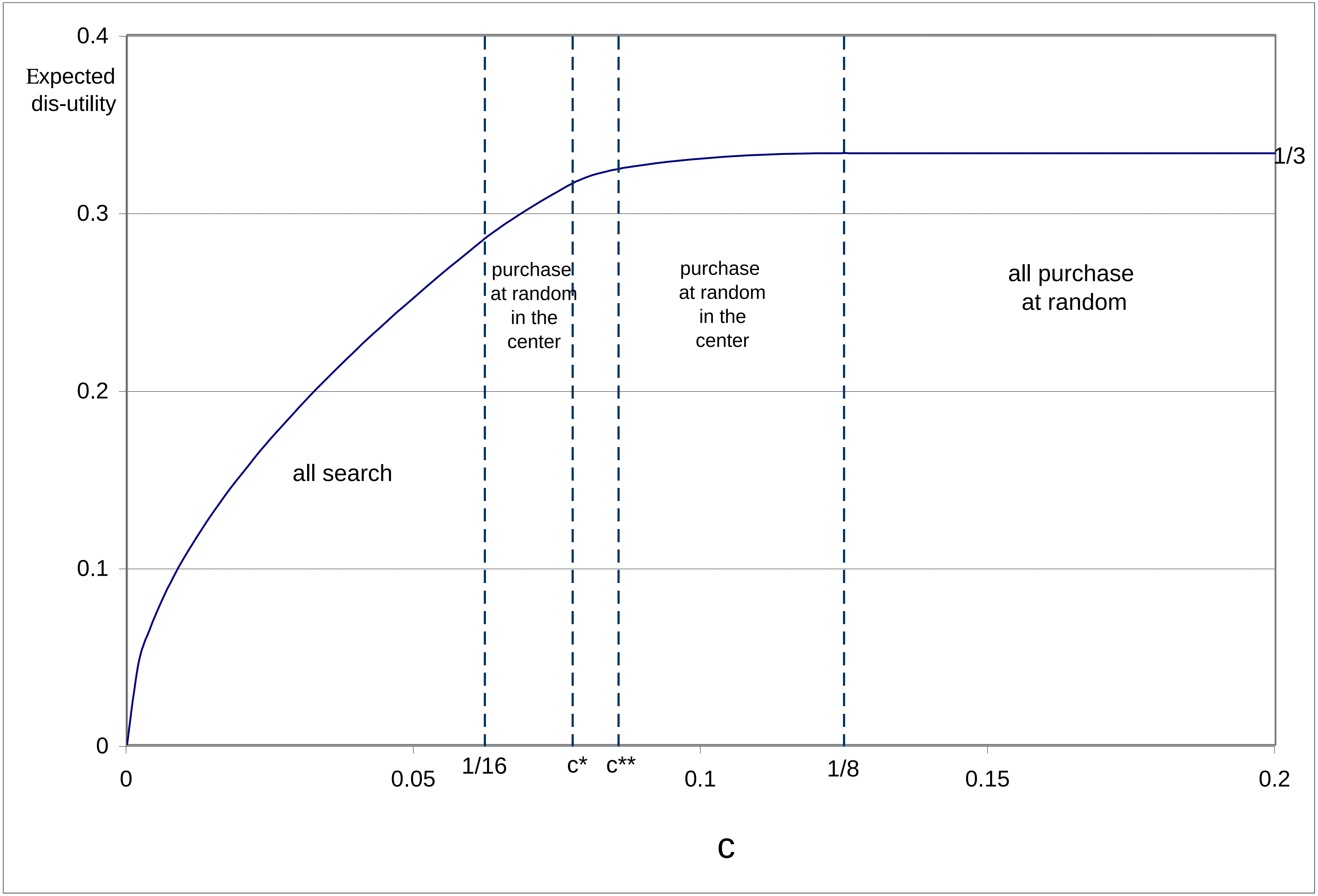

Figure 2: Expected search costs plus dis-utility of the product bought as a function of the search costs when there is an infinite number of products. c* is defined in the text, and $c^{**}=(3-8^{1/2})/2$. t=1.

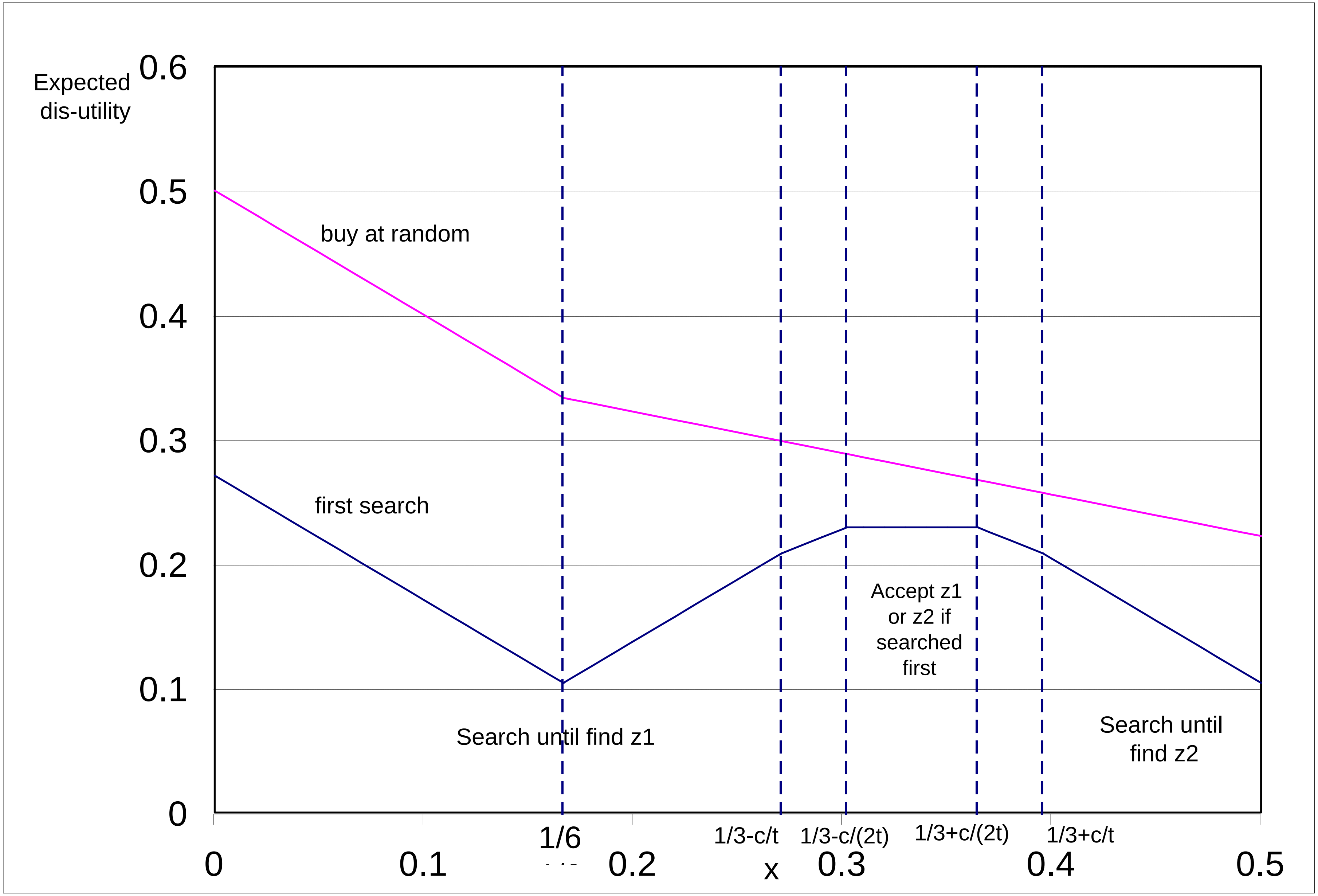

Figure 3: Expected dis-utility of search first and buy at random for each x, for three alternatives, t=1, and c=1/16. Note z1=1/6, z2=1/2, z3=5/6. If z3 searched first then buy at random if x in [1/3-c/t,1/3+c/t].

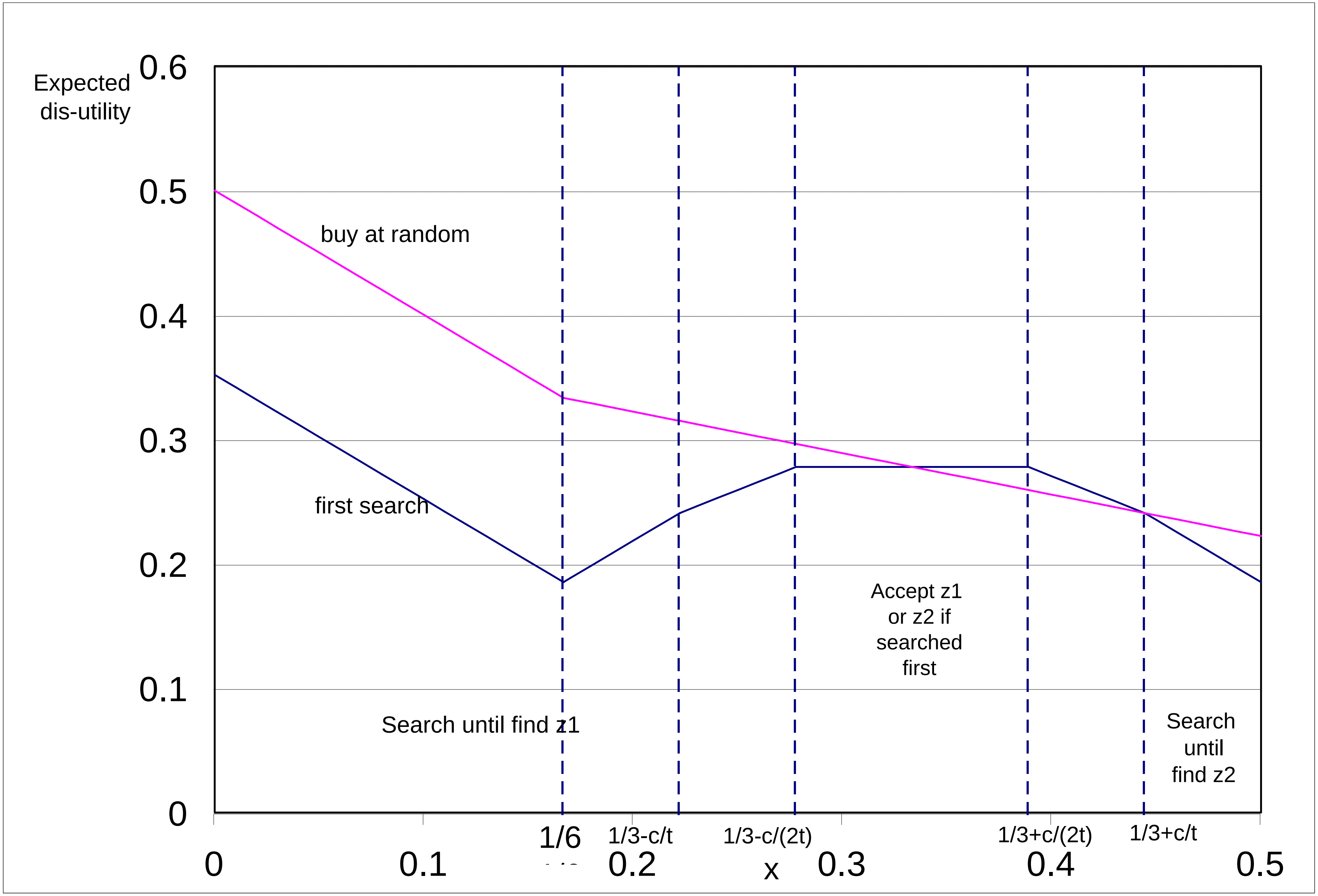

Figure 4: Expected dis-utility of search first and buy at random for each x, for three alternatives, t=1, and c=1/9. Note z1=1/6, z2=1/2, z3=5/6. Buy at random is better than search first for some x.

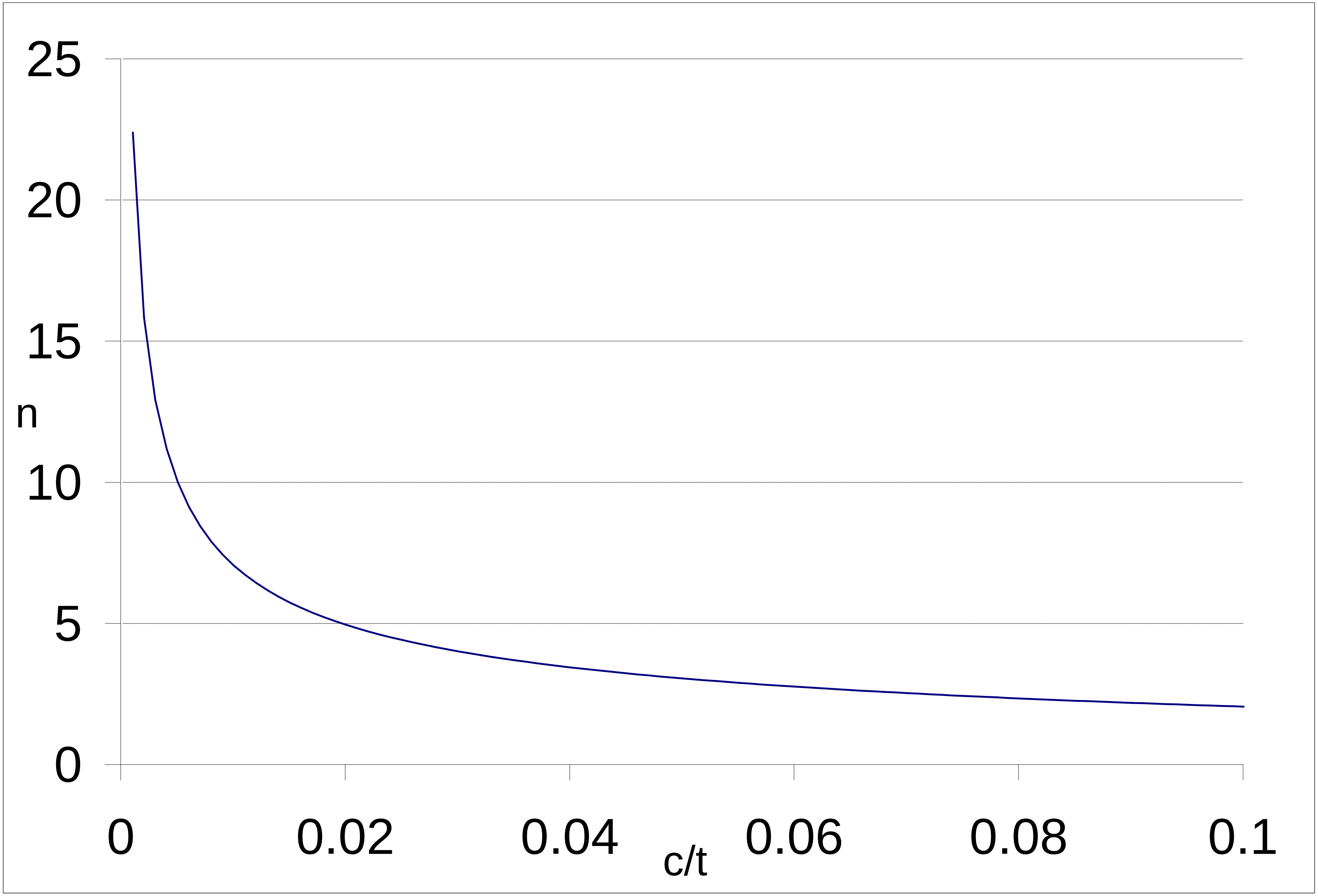

Figure 5: "Approximate" optimal number of product as a fucntion of c/t.

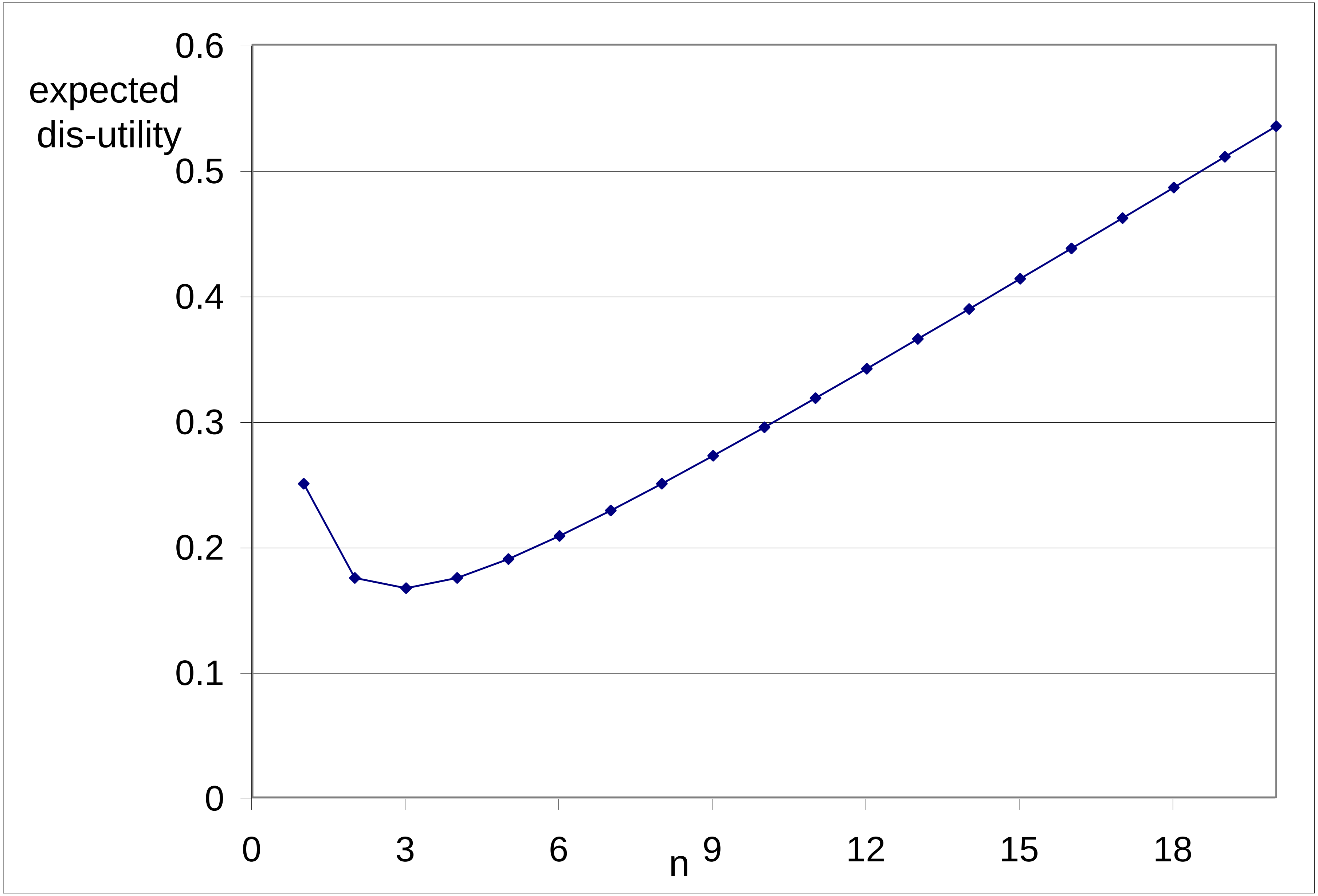

Figure 6: Expected dis-utility for each number of products under search until finding the closest product for t=1, c=05.