Strategic Entry in Dynamic Markets*

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Abstract

In developing industries firms have to decide whether and when to enter the market depending on the state of demand, the existing firms in the industry, and the firm capabilities. This paper investigates a model of increasing demand, where firms decide when to enter the market anticipating the strategic behavior of other potential entrants, and the effects of entry on future potential entrants. The paper shows that the ability of early entry to deter future competitors' entry leads to firms to enter the market at a rate faster than demand is expanding. If firms enter the market depending on their fixed capabilities rather than depending on the firm circumstances at each moment in time, firms end up entering the market at a faster rate in the early periods. If there is the potential for many firms in the market, firms may be less likely to enter because of future competitor entry to correct any market opportunities.

1. Introduction

In growing industries potential entrants have to decide whether and when to enter the industry. For example, in the satellite radio industry in the late 90's, XM, Sirius, and other potential competitors had to decide when to enter the industry.[1] If a firm enters too early, the return of its investments may end being too late to recoup the entry costs, but the firm is guaranteed a place in the industry. If the firm delays its entry, the present value of the entry costs will be lower, but the firm may end up preferring not to enter because other firms entered earlier. This trade-off yields an equilibrium timing for the firm to enter the market. In the satellite radio industry case, both XM and Sirius (and only XM and Sirius) decided to enter, and later eventually merged.

One question that arises is whether in equilibrium firms enter the market at a faster rate than the rate at which the market expands. This could result from firms entering the market earlier in order to deter entry from the competitors. We find that this will indeed occur, because of the strategic interaction among potential entrants and the entry deterrence effects. We show that this effect is stronger the greater the opportunity for firms to enter prior to the market expansion.[2] We also investigate the effect of private information of entry costs and of the number of potential entrants on the strength of this entry deterrence effect.

A firm's decision to enter the market, in addition to depending on the state of demand and the existing firms in the market, may depend on the firm's conditions (e.g., entry costs) and its beliefs about what the other firms might do. If the firms' conditions are stable through time, non-entry of a potential entrant may partially reveal that that potential entrant has poor conditions to enter the market. This may potentially lead the remaining potential entrants to delay entry as they fear too much entry in the next period. On the other hand if the firms' conditions vary from period

[1] See, for example, some information on the satellite radio industry in "XM Satellite Radio (A)," HBS case 9-504-009.

[2] In the late 90's E Ink entered aggressively into the electronic ink category partially in order to get demand and potentially deter other competitors' entry ("E Ink," HBS case 9-800-143).

to period the overall (stochastic) characteristics of the pool of potential entrants does not change through time, and the firms may have an incentive to delay entry in the hope of better future entry conditions.

Agarwal and Bayus (2003) show empirically that across several industries the takeoff of the number of firms occurs before the takeoff of demand. This paper explores theoretically a rationale for this empirical regularity based on firms being forward looking and the entry deterrence effects of the existence of firms in the industry. Shen (2008) investigates empirically this rationale with a dynamic structural model of firm entry and exit of the early years of the clothes washer industry. Klepper (1996) discusses several empirical facts related to the evolution of demand and of the number of firms in the developing phases of new industries, and Jovanovic (1982) presents a model with perfect competition of the evolution of the number of firms in a new industry. Related to Jovanovic (1982) this paper considers the case of imperfect strategic competition where entry of one firm may deter other potential firms from entering. This paper can also be seen as related to the literature on preemption and adoption of a new technology (e.g. Gilbert and Harris, 1984, Fudenberg and Tirole, 1985) where firms have to decide whether to adopt a new technology with benefits for the first adopter. That literature focuses on preemption and rent equalization effects while this paper looks at the dynamics of entry as it relates to demand evolution. Also related is the work on dynamic "grab the dollar" games (entry into a natural monopoly), knowing that entry is only profitable if only one firm enters the market, and leads to losses if both firms enter the market (see Fudenberg and Tirole, 1995, pp. 127-128). In relation to that literature this paper looks at the dynamic case where firms can decide to enter in advance of the market growth, and explores how such possibility affects the rate of market entry, and the firms strategic behavior. Also related to this paper, the literature on firm exit in a declining industry (war of attrition, e.g., Fudenberg et al., 1983, Ghemawat and Nalebuff, 1985) study how firms may exit a market through time, if too many firms end up in the market. Londregan (1990) considers the case in which two asymmetric potential firms can enter, exit, and re-enter through the industry life cycle. Narasimhan and Zhang (2000) consider the effect of strategic firm entry to capitalize on potential pioneering advantages depending on heterogenous firm capabilities, and argue why the market dominant firm may be the late entrant.[3] One potential issue in some markets that is not considered here is that early entrants may engage in market activities that may dissuade potential future entrants of entering the market (e.g., Bourguignon and Sethi, 1981, Milgrom and Roberts, 1982).

The paper considers strategic entry of firms as demand develops exogenously, and argues that firms being forward-looking and competition leads to firms coming into the market before demand take-off. In some markets one may argue that entry of firms may itself lead to increased demand, that is, the evolution of demand is endogenous to firm entry (see, for example, Agarwal and Bayus, 2003, for arguments in this regard). With respect to this possibility this paper can be seen as showing that we can observe in the data firm take-off before demand take-off independent of firm entry causing demand to develop. Note also that if firm entry causes demand to grow, the results presented here will continue to hold. In fact, if firm entry causes demand to grow at a decreasing rate, firms may enter to exhaust the returns to entry, and the analysis presented here can be seen as approaching the case when several firms are already in the market (such that we are already in the part of the curve where more firms leads to lower profits per firm). If firm entry affects future demand, another potential issue is that firms may free ride on other firms entering first, which may give incentives for firms to delay entry. This could potentially generate a force against firms entering before demand take-off. Also related to this work, in some cases one can see firms being more likely to enter a market if firms with similar characteristics are already there (Debruyne and Reibstein, 2005). This could be seen as firms with similar characteristics entering the market at the same time, or as a firm entering a market providing information to potential entrants of the

[3]For analysis on the market effects of order of entry see, for example, Robinson et al. (1993) and Golder and Tellis (1993).

market profitability (Ofek and Turut, 2008). These issues, although important in some markets, are not considered in this paper to concentrate on the main point of the paper of strategic entry under demand evolution.

The remainder of the paper is organized as follows. The next section presents a two-period two-potential entrants model of the market interaction. Section 3 extends this model to the case of more than two periods. The case of potential entrant specific firm conditions being private information is explored in Section 4, and Section 5 considers the case of more than two firms. Section 6 presents concluding remarks and directions for future research.

2. A Basic Model

Consider a market with two periods, where two potential entrants can decide to enter the market in each of the periods by paying an entry cost F. Demand is only realized in the second period, with the payoff per firm depending on the number of firms in the market. This feature tries to capture the effect that firms may have the ability to enter the market prior to demand growth. One could also consider the case where demand is small but positive in the first period but this possibility does not affect the main insights in this two potential entrants case. Denote the payoff for a firm in the second period given that n symmetric firms are in the market as $\pi(n)$. We make the natural assumption that the total profit in the industry weakly decreases with the number of competitors, as more competitors may lead to more intense competition. That is, $n\pi(n) \geq n'\pi(n')$ if $n < n'$. Note that this implies that the payoff for a firm is decreasing in the number of firms in the market, $\pi(n)$ is decreasing in n. For the two firm case in this Section we just worry about the profit of only one firm in the market, $\pi(1)$, the profit of two firms in the market, $\pi(2)$, and we have $\pi(1) > 2\pi(2)$. Firms discount future payoffs with the discount factor $\delta < 1$ per period. We consider the case of δ sufficiently close to one such that $\delta\pi(1) > \pi(2)$.

To make the problem interesting we consider the case in which $\delta\pi(1) > F > \pi(2) > 0$. This allows for the possibility that a single firm entering in the first period can be profitable, and yields that two firms entering in the second period would lead to both firms being unprofitable.[4] We concentrate the analysis on the symmetric equilibria. As a threshold for comparison, note that if there is only one potential entrant, it optimally waits until the second period to enter the market.

To solve for the subgame perfect equilibrium let us consider first the equilibrium actions in the second period given the state of the market after the first period. Suppose no firm entered in the first period. Then, in the second period the symmetric equilibrium would be in mixed strategies, as a firm prefers to enter if the competitor does not enter the market, and prefers to stay out if the competitor enters the market. Denoting as p_i the probability of entry in period i, the equilibrium probability of entry for a firm is determined by indifference between entry and staying out, $p_2\pi(2) + (1 - p_2)\pi(1) - F = 0$, which yields $p_2 = \frac{\pi(1)-F}{\pi(1)-\pi(2)}$. The expected profit in the second period after no entry in the first period is zero.

Consider now that only one firm entered in the first period. Then, the other firm will not want to enter, as $F > \pi(2)$, and the incumbent firm has a payoff of $\pi(1)$, as the entry cost was paid in the first period. Finally, if both firms entered in the first period, then each firm gets a profit of $\pi(2)$.

Let us now look at the first period. Note that by the argument above if only one firm enters the market in the first period then the other firm will stay out in the second period. Then, if the competitor entered in the first period a firm would have preferred to stay out as $F > \delta\pi(2)$, and if the competitor does not enter in the first period the firm would prefer to enter then as by waiting the firm gets an expected profit of zero, while by entering the market the firm gets $\delta\pi(1) - F > 0$. This

[4] If $\pi(1) > F > \delta\pi(1)$ no firm would ever enter in the first period; if $\pi(1) < F$ no firm would ever enter the market; if $\pi(2) > F$, both firms would always enter the market. For completeness, note also that if $\delta\pi(1) < \pi(2)$ then either firms enter the market in the second period or they never enter the market.

represents the effect that a firm gains from coming in early as it deters entry from the competitor.

Then, the equilibrium in the first period is again in mixed strategies, where the payoff of staying out is zero as noted above. The equilibrium probability of entry for a firm in the first period, p_1 is determined by indifference between entry and staying out, $p_1\delta\pi(2) + (1 - p_1)\delta\pi(1) - F = 0$, which yields $p_1 = \frac{\delta\pi(1)-F}{\delta(\pi(1)-\pi(2))}$.

This yields two interesting results. First, with strategic entry and forward-looking firms, firms enter in the first period, in advance of demand realization, with positive probability. This is because by entering earlier a firm can deter the entry of the competitor. Note that without competition (just one potential entrant) the firm only enters the market in the second period, and there is no entry in the first period. Note also that with two myopic potential entrants there is no entry in the first period, and in the second period each firm enters with the probability p_2. This result illustrates how the existence of competing forward-looking potential entrants may yield that the takeoff of the number of firms occurs prior to the takeoff of demand, an empirical fact presented in Agarwal and Bayus (2003).[5]

Second, for the equilibrium probabilities of entry, one can immediately obtain that, conditional on no prior entry, firms are more likely to enter when demand is present than prior to demand being realized, $p_1 < p_2$. This results from discounting of the entry payoffs if a firm enters in the first period, while the entry costs are paid in the period of entry.

Because of the importance of these two results we state them in the following proposition.

PROPOSITION 1: *Consider a two-period two-potential entrants market. In such a market firms enter prior to the demand realization with positive probability, and, conditional on no entry, each*

[5]Another potential factor of firms early entry, not explored here, is that firms may not be sure which product characteristics are the preferred ones, and may enter early to learn to get the product right. This could potential lead to the existence of early entry followed by an industry shake-out of the inefficient product designs.

firm enters with greater probability in the period in which demand is realized than prior to demand realization.

Computing the expected number of firms in the market in the second period one obtains $2p_1 + 2p_2(1-p_1)^2$. Comparing it with the case in which firms are myopic one obtains that if the discount factor δ is sufficiently close to one, then the expected number of firms is larger with forward-looking firms. As δ is close to one the equilibrium strategies in the first period are close to the ones in the second period, and therefore, with two periods for firms to enter, the expected number of firms in the second period is larger than if firms only enter in the second period, as it is the case when firms are myopic. Note, however, that if p_2 is close to one, i.e., firms enter almost for sure in the second period, then if δ is sufficiently less than one, we have a smaller expected number of firms when firms are forward looking than when firms are myopic as with myopic firms we have almost always two firms in the market while with forward-looking firms, with probability $2p_1(1-p_1)$, only one firm enters the market.

Comparing with the case when there is only one potential entrant, note that if the entry costs are low, the expected number of firms in the second period is greater with two potential entrants than with one, and that the threshold of entry costs for this greater number of firms is higher when firms are forward-looking. This is because the cost of entry is considered low, and with multiple opportunities to enter the market (the forward-looking case) more firms may end up in the market. We note these results on the number of firms in the following proposition.

PROPOSITION 2: *Consider a two-period two-potential entrants market. Then, if the discount factor δ is sufficiently close to one, the expected number of firms in the market in the second period is greater when firms are forward-looking than when firms are myopic. If entry costs are low the expected number of firms in the market is greater with two potential entrants than with one potential* *entrant, and the entry costs threshold for this to occur is greater when firms are forward-looking than when firms are myopic.*

3. Large Number of Periods

3.1. General Case

Consider now the case with a large number of periods and two potential entrants. Suppose that starting in period zero two potential entrants decide when and if to enter the market in each of an infinite number of periods.[6] Demand is zero until period τ and positive at a constant level from period τ onwards.[7] Denoting by $\pi(n)$ the profit per period after period τ if n firms are in the market, the present value of profits from period $t > \tau$ onwards if the market has n firms from period t onwards is $\frac{\pi(n)}{1-\delta}$. In order to make the problem interesting we assume that the entry costs F are such that $\frac{\pi(2)}{1-\delta} < F < \frac{\pi(1)}{1-\delta}$, so that ex-ante, it is advantageous to be the only firm in the market, and disadvantageous to be one of two firms in the market. That is, if there is already another firm in the market, a firm chooses always not to enter.

Define $\underline{\tau}$ by $\delta^{\tau-\underline{\tau}+1}\frac{\pi(1)}{1-\delta} < F < \delta^{\tau-\underline{\tau}}\frac{\pi(1)}{1-\delta}$. Then, $\underline{\tau}$ is the first period when it is profitable for a firm to enter the market. Note that for any τ, if δ is high enough we have $\underline{\tau} = 0$.

By the analysis above we can then obtain the probability of entry p_t at period t given that the

[6] Entry is defined here as the observable commitment of a firm to enter the market and sinking entry costs. Given delay in coming to market, in several industries entry can potentially occur before a firm actually starts selling in a market.

[7] It would be interesting to investigate what happens if τ is uncertain. The main ideas presented here should continue to hold, but there may be additional interesting effects related to potential greater incentives to early entry.

other firm has not entered as

$$p_t = \begin{cases} 0 & \text{if } t < \underline{\tau} \\ \frac{\delta^{\tau-t}\pi(1)-F(1-\delta)}{\delta^{\tau-t}(\pi(1)-\pi(2))} & \text{if } \underline{\tau} \leq t < \tau \\ \frac{\pi(1)-F(1-\delta)}{\pi(1)-\pi(2)} & \text{if } t \geq \tau. \end{cases} \quad (1)$$

Note that this probability is increasing through time as the closer we are to when demand increases the more profitable it is for firms to enter the market. Note that if demand were expect to decrease at some point in the future, the probability of entry given no entry by the other firm would decrease after period τ. Note also that the probability p_t is increasing at a decreasing rate for $t < \tau$, as the increase in the benefit of entry is smaller the closer the firms are to the period in which demand increases. Figure 1 illustrates the evolution of p_t for $\tau = 20, \delta = .97, F = 20$, and $\pi(n) = 1/n$. For this case, $\underline{\tau} = 4$.

Note that the probability of entry for $t < \tau$ is increasing in the discount factor δ. This is because a higher discount factor increases the present value of profits when demand increases, both the profit at the moment of when demand increases, and also the total present value of profits after demand increases.

The expected number of firms in the market at period $t > \underline{\tau}$, En_t, can be obtained as

$$En_t = 2p_{\underline{\tau}} + 2p_{\underline{\tau}+1}(1-p_{\underline{\tau}})^2 + 2p_{\underline{\tau}+2}(1-p_{\underline{\tau}})^2(1-p_{\underline{\tau}+1})^2 + ... + 2p_t\Pi_{i=0}^{t-\underline{\tau}-1}(1-p_{\underline{\tau}+i})^2 = \sum_{t'=0}^{t} 2p_{\tau}\Pi_{i=0}^{t'-1}(1-p_i)^2. \quad (2)$$

Figure 1 shows the evolution of the expected number of firms for a set of parameter values. Note that the expected number of firms in the market increases through time with an increase in period t of $2p_t\Pi_{i=0}^{t-1}(1-p_i)^2 > 0$. Note that after some period this increase is smaller and smaller as the term $\Pi_{i=0}^{t-1}(1-p_i)^2$ decrease in t. Note that for $t > \underline{\tau}$ but small we can have that the increases in the expected number of firms are increasing in t as the term p_t increases in t, which may overcome the decreases resulting from $\Pi_{i=0}^{t-1}(1-p_i)^2$.

Comparing with the one firm case note that prior to τ the expected number of firms in the market is greater than zero, while it is zero in the one firm case. That is, competition leads to more firms entering the market prior to τ. Note also that with probability one at least a firm enters the market when there are two potential entrants, and with positive probability two firms enter the market. Then, the expected number of firms that ever is in the market is strictly greater than one firm.

For δ close to one (with F changing such that $\pi(2) < F(1-\delta) < \pi(1)$) we have p_t close to $\frac{\pi(1)-F(1-\delta)}{\pi(1)-\pi(2)}$ for all t. We can then obtain that the expected number of firms in the market at period τ is close to $2p_\tau \frac{1-(1-p_\tau)^{2\tau}}{1-(1-p_\tau)^2}$ which is greater than one if $\tau > \frac{1}{2}\frac{\log(p_\tau/2)}{\log(1-p_\tau)}$. For this case, the expected number of firms that is ever in the market can be obtained to be close to $\frac{2}{2-p_\tau} > 1$. The probability of two firms ending up in the market can be obtained to be close to $\frac{p_\tau}{2-p_\tau}$.

Comparing with the case when firms are myopic, while assuming that $\frac{\pi(2)}{1-\delta} < F < \frac{\pi(1)}{1-\delta}$, we see that if $\delta > \frac{\pi(1)-\pi(2)}{\pi(1)}$ then there are no values for the entry costs F such that firms enter when they are myopic. In other words, if firms are myopic they are less likely to enter as they do not fully value the future potential profits.

3.2. Time Between Periods

Consider now that the time period between periods is reduced while maintaining the same time value of money. That is, denoting as Δ the time interval between periods, we have that the discount factor as a function of Δ is $\delta(\Delta) = e^{-r\Delta}$ where r is the continuous time constant interest rate. If demand increases at moment in time τ, then there are τ/Δ periods from time zero until time τ. Defining $\widetilde{\pi}(n)$ as the profit rate when there n firms in the market, we have that the present value of the profit for a period as a function of the interval of time between periods is $\pi(n;\Delta) = \int_0^\Delta \tilde{\pi}(n)e^{-rt}\,dt = \frac{\tilde{\pi}(n)}{r}(1-e^{-r\Delta})$.

Denoting $\underline{\tau}$ as the moment in time at which it becomes potentially profitable to enter the market, $e^{-r(\tau-\underline{\tau})}\pi(1)/r - F = 0$, the number of periods in which firms have a probability of entry prior to time τ is $\frac{\tau-\underline{\tau}}{\Delta}$ up to an integer approximation. The probability of entry for a period i between time $\underline{\tau}$ and time τ can be written as

$$p_i = \frac{e^{-r(\tau-i\Delta)}\pi(1) - rF}{e^{-r(\tau-i\Delta)}(\pi(1)-\pi(2))}.$$

The probability of at least one firm entering prior to time $\tau' < \tau$ can then be written as

$$1 - \Pi_{i=\underline{\tau}/\Delta}^{\tau'/\Delta}(1-p_i)^2. \tag{3}$$

For any $\tau' \in (\underline{\tau}, \tau]$ one can then obtain that when the time interval between periods converges to zero, $\Delta \to 0$, the probability of entry of at least one firm prior to time τ' converges to one. This can be obtained by noting that when $\Delta \to 0$ there is an infinite number of periods between $\underline{\tau}$ and τ' with $p_i > 0$, which leads the expression in (3) to converge to one. To see what happens in the market when the time interval between periods converges to zero, $\Delta \to 0$, consider a time interval $[\tau'', \tau']$ with $\underline{\tau} < \tau'' < \tau' < \tau$, under the condition that no firm entered the market prior to τ''. By the argument above when $\Delta \to 0$, and for any (τ'', τ') at least one firm enters in the interval $[\tau'', \tau']$ with probability converging to one. Furthermore, we have that if $\tau' \to \tau''$, the probability of two firms entering in the interval $[\tau'', \tau']$ converges to $q_{\tau''}^2 + q_{\tau''}^2(1-q_{\tau''})^2 + q_{\tau''}^2(1-q_{\tau''})^4 + \ldots = \frac{q_{\tau''}}{2-q_{\tau''}}$, where $q_{\tau''} = \frac{e^{-r(\tau-\tau'')}\pi(1)-rF}{e^{-r(\tau-\tau'')}(\pi(1)-\pi(2))}$. Then we have that if in addition $\tau'' \to \underline{\tau}$ the probability of two firms entering the market converges to zero. That is, if $\Delta \to 0$, the equilibrium converges to only one firm entering the market at time $\underline{\tau}$ with probability approaching one. This is the result discussed in Fudenberg and Tirole (1985) in the context of adoption of a new technology, with a continuous time specification that yields the same result.

4. Private Information, Pure Strategies, and Firm Capabilities

4.1. Private Information Independent Through Time

The analysis above considered the case where firms were symmetric and there was no private information, with the equilibrium being in mixed strategies in every period. We now consider the existence of private information by each potential entrant, and discuss how this possibility leads to pure strategy equilibria, as noted by Harsanyi (1973) for the static case. Here we consider the case when the private information of each firm is only relevant in the period under consideration, and in the next subsection we allow the private information to have effect throughout the lifetime of the potential entrant.

In the model of the previous section consider now that in each period t each potential entrant i draws a fixed cost F_{it} from a uniform distribution with support $[\underline{F}, \overline{F}]$, with $\frac{\pi(2)}{1-\delta} < \underline{F} < \overline{F} < \frac{\pi(1)}{1-\delta}$, and with F_{it} being independent across i and t.[8]

Then, denoting as $V(t)$ the expected net present value of payoffs for a firm if no firm has entered the market at the beginning of period t, before the realization of F_{it}, we have that the threshold $\widehat{F}_t$, such that a firm enters in period t if and only if it draws a fixed cost less or equal to $\widehat{F}_t$, is determined by

$$-\widehat{F}_t + p_t\delta^{\tau-t}\frac{\pi(2)}{1-\delta} + (1-p_t)\delta^{\tau-t}\frac{\pi(1)}{1-\delta} = (1-p_t)\delta V(t+1), \tag{4}$$

[8] Alternatively, the private information of the firms could be a signal over the market demand level. This possibility is discussed in the Conclusion section.

where the probability of entry by a firm in period t is $p_t = \frac{\widehat{F}_t - \underline{F}}{\overline{F} - \underline{F}}$.

By the definition of $V(t)$, we also have that with probability p_t the firm enters the market in period t with an expected entry cost of $\frac{\widehat{F}_t + \underline{F}}{2}$ and with probability $(1-p_t)^2$ no firm will enter the market and the firm can get $V(t+1)$ in the next period. That is,

$$V(t) = p_t[-\frac{\widehat{F}_t + \underline{F}}{2} + p_t \delta^{\tau - t}\frac{\pi(2)}{1-\delta} + (1-p_t)\delta^{\tau - t}\frac{\pi(1)}{1-\delta}] + (1-p_t)^2 \delta V(t+1), \tag{5}$$

which can be re-written, using (4), as

$$V(t) = (1-p_t)\delta V(t+1) + p_t^2 \frac{\overline{F} - \underline{F}}{2}. \tag{6}$$

This representation generates several points of difference with respect to the analysis in the previous section. Now firms have a strictly positive expected payoff, in comparison to zero in the previous section. This is because firms can draw a lower fixed cost of entry which is smaller than the threshold fixed cost of entry that yields entry being optimal. Furthermore, now firms can choose to wait because of the option value of drawing a low fixed cost of entry in future periods. This option value is included in $V(t)$, the expected net present value of profits at the start of period t if both firms have not yet entered, and before the entry cost is observed.

In order to proceed consider $\delta \to 1$ with $\overline{F} = \frac{\widetilde{\overline{F}}}{1-\delta}$, and $\underline{F} = \frac{\widetilde{\underline{F}}}{1-\delta}$ for fixed $\widetilde{\overline{F}}$ and $\widetilde{\underline{F}}$, such that the condition above for $\overline{F}$ and $\underline{F}$ remains satisfied. Note also that for $\delta \to 1$, we have $V(t) - V(t') \to 0$ and $p_t - p_{t'} \to 0$ for any t and t'. Define $\lim_{\delta \to 1}(1-\delta)V(t) = \widetilde{V}$, the average per period expected payoff, and $\lim_{\delta \to 1} p_t = p$. Substituting this into (6) one obtains

$$\widetilde{V} = p\frac{\widetilde{\overline{F}} - \widetilde{\underline{F}}}{2} \tag{7}$$

which can be substituted into (4) to obtain

$$p^2 \frac{\tilde{\overline{F}} - \underline{\tilde{F}}}{2} - p[\pi(1) - \pi(2) + \frac{3}{2}(\tilde{\overline{F}} - \underline{\tilde{F}})] + \pi(1) - \underline{\tilde{F}} = 0, \tag{8}$$

from which one can obtain the equilibrium p, and then, substituting in (7), $\tilde{V}$. From this one can then obtain the following proposition.

PROPOSITION 3: *Suppose that the draws of entry costs are independent through time and the discount factor $\delta \to 1$. Then, the probability of entry p_t converges to being constant through time, increasing in the firms' profits, $\pi(1)$ and $\pi(2)$, and decreasing in the average entry cost, $\frac{\tilde{\overline{F}}+\underline{\tilde{F}}}{2}$, and in the spread of possible average per period entry costs, $\tilde{\overline{F}} - \underline{\tilde{F}}$.*

When the discount factor converges to one, and entry costs are drawn independently through time, firms count the future as much as the present, and at each moment of time where no entry has occurred the market conditions are the same, whether or not demand is realized. We obtain then that the probability of entry converges to being constant through time, whether the firms are or are not in a period where demand has started. As expected, the greater the profits that firms can obtain, whatever the market structure, the more likely firms are to enter. Also as expected, as the average entry cost increases the less likely firms are to enter the market, as they try to avoid entering at the same time as the competitor.

More interestingly, the greater the spread of possible entry costs (variance of entry costs), the less likely firms are to enter the market, and more likely they are to wait for another period. The intuition is that the greater the variance of entry costs the more it pays off to wait for the option value of drawing a very low entry cost in the next period. Note also that this implies that the probability of entry is lower in this case than when there is no private information of entry costs.

One can also obtain that the probability of entry is greater than the case of no private information and entry costs equal to $\widetilde{\overline{F}}/(1-\delta)$, and lower than the case of no private information and entry costs equal to $\underline{\widetilde{F}}/(1-\delta)$.

In terms of expected present value of profits, as expected, we can obtain that it is increasing in the profits that firms can obtain with either one or two firms in the market, and decreasing in the average entry cost. Again, more interestingly, the expected present value of profits is increasing in the variance of possible entry costs. The reason is that, with a greater variance of entry costs, firms tend to wait for the option of a low value of entry costs, and enter the market only then.

One can also compute the expected costs of entry conditional on firms entering the market. These can obtained to be equal to $\underline{\widetilde{F}} + p\frac{\widetilde{\overline{F}}-\underline{\widetilde{F}}}{2}$. One can then obtain that the expected costs of entry conditional on firms entering the market are increasing in the firms' profits, $\pi(1)$ and $\pi(2)$, increasing in the average unconditional entry cost, $\frac{\widetilde{\overline{F}}+\underline{\widetilde{F}}}{2}$, and decreasing in the spread of possible average per period entry costs, $\widetilde{\overline{F}} - \underline{\widetilde{F}}$, as expected, given the arguments above.

Comparing with monopoly (only one firm potential entrant), note that competition leads again to a positive probability of firms entering the market before demand is realized, and that this is more likely to occur the greater the discount factor and the greater the number of periods in which entry is possible and demand is still not in place. Note also that with competition there is a positive probability of two firms being in the market, which does not happen under monopoly.

Note that from a social welfare point of view firms may enter too fast in equilibrium. This could be the case if the benefits in terms of consumer surplus of competition are not too large. In competition a second firm considering entering compares π_2 with the entry costs, while in terms of social welfare the comparison is of $2\pi_2 - \pi_1$ with the entry costs (and note that $\pi_2 > 2\pi_2 - \pi_1$). Note also that if the social welfare optimum is for only one firm to enter the market, and if a regulator can set a threshold entry cost for entry, then it may be optimal for the regulator to set a lower threshold entry cost than the equilibrium one considered above, so that the likelihood of too many firms entering the market is lower. A regulator could change the firms' entry costs by, for example, affecting the cost of licenses to enter a market (e.g., licenses in mobile telecommunications markets).

4.2. Private Information Fixed Through Time

Consider now the case in which firms draw independently an entry cost that is fixed throughout the lifetime of the market. That is, the drawn entry cost could be seen as the firm capability for that market. However, a firm might not know the entry cost drawn by the competitor, the firm capability of the competitor. But, the decision of a firm not to enter gives some information to the competitor about the firm's entry cost.

The equilibrium involves then firms waiting to enter in a certain period if the other firm has not entered previously. In each period t there will be then a threshold $\widehat{F}_t$ of the last firm's type entering in that period. Firms with $F \in (\widehat{F}_{t-1}, \widehat{F}_t]$ enter in period t if the other firm has not entered before, with firms with a lower F entering earlier. The probability of a firm entering in period t given no prior entry is $p_t = \frac{\widehat{F}_t - \widehat{F}_{t-1}}{\overline{F} - \widehat{F}_{t-1}}$. Denoting $V(t+1)$ as the expected present value of a firm with type $\widehat{F}_t$ delaying entry until period $t+1$ we have that the condition for the threshold $\widehat{F}_t$ entering in period t, prior to the period τ when demand starts, is

$$-\widehat{F}_t + p_t \delta^{\tau-t} \frac{\pi(2)}{1-\delta} + (1-p_t)\delta^{\tau-t} \frac{\pi(1)}{1-\delta} = (1-p_t)\delta V(t+1) \tag{9}$$

where

$$V(t+1) = -\widehat{F}_t + p_{t+1} \delta^{\tau-t-1} \frac{\pi(2)}{1-\delta} + (1-p_{t+1})\delta^{\tau-t-1} \frac{\pi(1)}{1-\delta}. \tag{10}$$

Consider now $\delta \to 1$ with the notation, similar to above, that $F = \frac{\widetilde{F}}{1-\delta}$. The expressions above become then

$$-\widetilde{F}_t + p_t\pi(2) + (1-p_t)\pi(1) = (1-p_t)[-\widetilde{F}_t + p_{t+1}\pi(2) + (1-p_{t+1})\pi(1)] \tag{11}$$

which yields

$$\frac{p_{t+1}}{p_t} = \frac{\widetilde{F}_t - \pi(2)}{(1-p_t)(\pi(1)-\pi(2))}. \tag{12}$$

From this, given that the threshold $\widetilde{F}_t$ increases in t, we have that the probability of entry p_t changes through time, contrarily to the case in which the entry costs are drawn independently through time. Furthermore, as stated in the next proposition, one can show that the probability of entry is decreasing through time.

PROPOSITION 4: *Suppose $\delta \to 1$, and that the entry costs drawn by each firm are fixed through time. Then, the probability of entry of each firm, conditional on no prior entry, is decreasing through time.*

The proof is presented in the Appendix. In equilibrium, firms can never enter with probability zero or one, which means that $\widetilde{F}_t$ is strictly increasing in t without ever reaching $\overline{\widetilde{F}}$. This means that, at least at some point, $\widetilde{F}_t$ increases at a decreasing rate, which then implies that the probability of entry is decreasing through time.

As above, note that with several potential entrants, firms come into the market before demand is realized with positive probability. Note also that if one calibrates the support of entry costs such that the average entry probability conditional on no prior entry is the same with independent draws as with fixed capabilities, entry occurs faster under fixed capabilities.

Note also that all types have to have a chance of entering the market, because otherwise the probability of entry would become at some point so low, that it would pay off for a firm with a high entry cost to enter the market. This means that $\widetilde{F}_t$ converges to $\overline{\widetilde{F}}$ when t goes to infinity. At the limit one can obtain the probability of entry as $\frac{\pi(1)-\tilde{\tilde{F}}}{\pi(1)-\pi(2)}$, which is the probability of entry in the case without private information and entry costs $\tilde{\tilde{F}}/(1-\delta)$.

Given that the probability of entry is decreasing in t, $\frac{p_{t+1}}{p_t} < 1$, we have, from (12), that $p_t < \frac{\pi(1)-\tilde{F}_t}{\pi(1)-\pi(2)}$. That is, for each threshold $\tilde{F}_t$ the probability of entry with private information and fixed capabilities is lower than the probability of entry without private information at that entry cost. Given that the firms with entry costs such that they enter in period t have lower entry costs than $\tilde{F}_t$ then the probability of entry in period t is also lower than if there was no private information at any entry cost of the firms that enter in period t under private information and fixed capabilities. This also then implies that if there is no private information with fixed capabilities at the average entry costs, the probability of entry is greater than with private information and fixed capabilities. With private information, firms are strategic in delaying entry as there is an expected positive payoff of delaying entry.

Comparing the case of entry costs independently drawn though time with the case of fixed capabilities, one can obtain that after some time with no entry, the probability of entry when entry costs are independently drawn through time is greater than in the case of fixed capabilities.

5. Large Number of Firms

5.1. Introduction

Consider now the base two-period model presented above but suppose that there are several potential entrants, and that the market may ultimately have in equilibrium more than one firm. We denote as N the number of potential entrants, and as n the number of firms that have entered the market. Furthermore, let n_1 be the number of entrants in the first period and n_2 be the number of entrants in the second period. We assume N and n to be large, with N much greater than n in equilibrium.

Denote $\pi(n)$ as the profit per firm in the second period if there are n firms in the market. As mentioned above, we assume $n\pi(n)$ decreasing in n, the industry profits go down with the number of firms. Furthermore, we assume that $\pi(n)$ is convex, that is, the decrease in profits when increasing n is greater for a small number of firms than for a large number of firms. Both of these assumptions hold for the case of price competition or Cournot competition under some general assumptions. The later means, for example, that the profit per firm goes down more when the industry goes from two to three firms in the market, than from three to four firms. In order to get sharper results we assume $\pi(n) = e^{-\beta n}$, with $\beta \geq 1$ for $n\pi(n)$ to be decreasing in n for all $n > 1$. In the first period the profit per firm is $\alpha\pi(n)$ if there are n firms in the market, with $0 < \alpha < 1$.

In a symmetric equilibrium in a given period each firm will enter with some probability p. Given that N firms follow independently this strategy, the actual number of firms that enter the industry is a random variable. With N large the probability distribution of the number of firms that enter the market can be approximated by a normal distribution with mean pN and variance $p(1-p)N$, which we will use in what follows. Let $g(x;\mu,\sigma^2)$ and $G(x;\mu,\sigma^2)$ be, respectively, the density and cumulative distribution function of a normally distributed random variable x with mean μ and variance σ^2.

Finally, define m as the number of firms such that the profit per firm in the second period is equal to the fixed cost of entry, $\pi(m) = F$.

5.2. Second Period

Consider the equilibrium strategies in the second period of the market assuming that n_1 firms entered in the first period. If $n_1 + 1 \geq m$, no firm will enter the market in the second period, and even it was only one firm entering the market, the profit obtained, $\pi(n_1 + 1)$, would be less than the fixed cost of entry F.

Suppose now that $n_1 + 1 < m$. Then the probability of entry in the market should be such that the expected profit of entering is equal to the expected profit of staying out, zero. The probability of entry in the second period p_2 would be then defined by

$$-F + \int e^{-\beta(n_1+n_2)} g(n_2; p_2(N-n_1), p_2(1-p_2)(N-n_1))\, dn_2 = 0. \tag{13}$$

Given that $g()$ is the normal density, this expression can be reduced to

$$-F + e^{-\beta(n_1+p_2(N-n_1))+\frac{\beta^2}{2}p_2(1-p_2)(N-n_1)} = 0 \tag{14}$$

which defines the equilibrium probability of entry p_2 as

$$p_2 - \beta p_2(1-p_2) = \frac{-\log F - \beta n_1}{\beta(N-n_1)}. \tag{15}$$

From this one can obtain directly that the probability of entry is decreasing in the number of potential entrants N, the number of incumbents, n_1, and in the entry costs F.[9] A greater number of potential entrants makes each firm be more conservative with their probability of entry, for fear of too many firms in the market. However, one can obtain that the expected number of firms in the market $n_1 + p_2(N - n_1)$ is increasing in the number of potential entrants, N. A greater number of incumbents, n_1, or a greater entry cost F, makes the market less attractive to entry, and the probability of entry is lower.

Interestingly, note that the expected number of firms in the market, $n_1 + p_2(N - n_1)$, is greater than the number of firms m whose entry would lead to a profit per profit equal to the entry cost. That is, with probability greater than 1/2, there are too many firms in the market. The intuition for this result follows from the convexity of the profit function $\pi(n)$. With a convex profit function,

[9] Note that $F < 1$, and $\log F < 0$ for some number of firms to want to enter the market.

firms like uncertainty, and are more likely to enter, enticed by the possibility of the market having very few firms and generating high profits per firm.

5.3. First Period

Consider now the decisions by the firms in the first period. In a symmetric equilibrium, firms will enter with some probability p_1 in the first period. If the actual number of firms that enter the market in the first period is greater than m we have from above that no other firm will enter in the second period. If the actual number of firms that enter the market in the first period is less than m then firms enter in the second period with a probability of entry $p_2(n_1)$ which is determined by (15). Making the expected present value of profits equal to the entry costs F one obtains

$$-F + \int \alpha\pi(n_1)g(n_1; p_1N; p_1(1-p_1)N)\,dn_1 + \delta\int_0^m [\int \pi(n_1+n_2)g(n_2; p_2(n_1)(N -n_1), p_2(n_1)(1-p_2(n_1))(N-n_1))\,dn_2]g(n_1; p_1N, p_1(1-p_1)N)\,dn_1 + \delta\int_m^N \pi(n_1)g(n_1; p_1N, p_1(1-p_1)N)\,dn_1 = 0 \tag{16}$$

where the second term is the expected profit in the first period, the third term is the part of the second period expected profit when few firms enter in the first period, such that there is more entry in the second period, and the fourth term is the part of the second period expected profit when too many firms enter in the first period, such that there is no more entry in the second period.

Using (15) and g() being a normal density one can then obtain the equilibrium condition for p_1 as

$$-F + e^{-\beta p_1 N + \frac{\beta^2}{2}p_1(1-p_1)N}\{\alpha + \delta[1 - G(m; p_1N - \beta p_1(1-p_1)N, p_1(1-p_1)N)]\} + \delta F G(m; p_1N, p_1(1-p_1)N) = 0. \tag{17}$$

This expression yields several observations. First, as in the second period analysis, given the convexity of the profit function there is a force for more firms to enter in the first period than the number of firms that would lead to zero ex-ante profits.

Second, if δ is close to one, we have that the number of firms that enter in the first period that would lead to zero ex-ante profits is greater than the number of firms that would lead to zero ex-ante profits for firms entering in the second period. This is because firms that enter in the first period also benefit from the first-period profits. This is a force for the number of firms to increase prior to demand increasing.

Third, as the number of potential entrants N increases, following up on the analysis of the second period, there is a force for the expected number of firms in the market to increase.

Fourth, and interestingly, with a large number of firms, the possibility of later entry in the second period is a force towards less entry in the first period. To see this note that if too few firms enter in the first period, then they know that firms will come in the second period such that the expected second period profits is F. On the other hand, if too many firms enter in the first period, then potential entrants in the second period stay out. That is, first period entrants, will have a limit on much they can gain if the entry uncertainty results in too few firms, but have no limit on their losses if the entry uncertainty results in too many firms.

Note that if the market has not many firms (integer problems), then the effects of the previous sections start coming into play with entering firms knowing that by being in the market they will deter entry by the competitors, and have positive profits.

6. Conclusion

This paper considers strategic entry in a market where firms may be able to enter the market prior to demand growth. We show that competition leads to entry prior to demand growth, as presented in empirical studies, and discuss how the probability of entry increases through time prior to demand realization.

The paper also shows that with private information one may replicate the complete information mixed strategy equilibria with pure strategies, but that there are additional important effects. Private information can be considered as either independent draws of entry costs through time, or as fixed draws at the beginning of the market. In both cases, the expected profits are greater than in models of complete information. In the independent draws case, the greater expected profits result from the option value of waiting a lower draw of entry costs. In the fixed draws case the greater expected profits result from the potential to strategic delay in entering by firms that do not have a very low entry cost.

For the case of potentially many firms in the market, the paper shows two additional important effects. First, given the convexity of the profit function, firms like uncertainty, and the expected number of firms is greater than the one that would lead to zero ex-ante profits. Second, forward looking firms understand that if too few firms enter in the beginning of the market, other firms will come into the market later to correct any potential rents, and this reduces the benefit of entering the market early.

The paper considers private information by firms on their entry costs. Alternatively, one could consider that firms receive a private signal on the demand conditions. This possibility could be important in several markets, and it would be interesting to investigate. The analysis in such a case would be similar to the one considered here with the difference that a positive signal would likely mean that the competitor also received a positive signal, and therefore, the news of a positive signal have to be tempered by the increased likelihood of entry by competitors. Another issue that could be potentially interesting to investigate in future research is that in several market situations entry costs are invested gradually rather than the discrete entry decision considered here.

APPENDIX

PROOF OF PROPOSITION 4: Given that $\widetilde{F}_t$ is strictly increasing in t we have that $\frac{p_{t+1}(1-p_t)}{p_t}$ is strictly increasing in t, and therefore p_t cannot be constant in t. For any t this means $\frac{p_{t+2}(1-p_{t+1})}{p_{t+1}} > \frac{p_{t+1}(1-p_t)}{p_t}$ for all t. The proof is by contradiction. Suppose $p_{t+1} > p_t$. Then, we have $p_{t+2} > p_{t+1}$, and iterating we have $p_{t'+1} > p_{t'}$, for all $t' > t$. Furthermore, for any $t' > t$ we have $\frac{p_{t'+2}}{p_{t'+1}} > \frac{p_{t'+1}}{p_{t'}}$ which means that p_t goes to its maximum one at an accelerating rate. But then in the period in which p_t goes to one, entering is not optimal for any of the firms as the other firm is entering with probability one, a contradiction.

REFERENCES

Agarwal, R., and B.L. Bayus (2003), "The Market Evolution and Sales Takeoff of Product Innovations," *Management Science,* **48**, 1024-1041.

Bourguignon, F., and S.P. Sethi (1981), "Dynamic Optimal Pricing and (Possibly) Advertising in the Face of Various Kinds of Potential Entrants," *Journal of Economic Dynamics and Control,* **3**, 119-140.

Debruyne, M., and D.J. Reibstein (2005), "Competitor See, Competitor Do: Incumbent Entry in New Market Niches," *Marketing Science,* **24**, 55-66.

Fudenberg, D., R. Gilbert, J. Stiglitz, and J. Tirole (1983), "Preemption, Leapfrogging, and Competition in Patent Races," *European Economic Review,* **22**, 3-31.

Fudenberg, D., and J. Tirole (1985), "Preemption and Rent Equalization in the Adoption of New Technology," *Review of Economic Studies,* **52**, 383-402.

Fudenberg, D., and J. Tirole (1995), *Game Theory,* The MIT Press: Cambridge, Massachusetts.

Ghemawat, P., and B. Nalebuff (1985), "Exit," *RAND Journal of Economics,* **16**, 184-194.

Golder, P.N.,and G.J. Tellis (1993), "Pionner Advantage: Marketing logic or Marketing Legend?," *Journal of Marketing Research,* **30**, 158-170.

Harsanyi, J. (1973), "Games with Randomly Disturbed Payoffs: A New Rationale for Mixed-Strategy Equilibrium Points," *International Journal of Game Theory,* **2**, 1-23.

Jovanovic, B. (1982), "Selection and the Evolution of Industry," *Econometrica,* **50**, 649-670.

Klepper, S. (1996), "Entry, Exit, Growth, and Innovation over the Product Life Cycle," *American Economic Review,* **86**, 562-583.

Londregan, J. (1990), "Entry and Exit over the Industry Life Cycle," *RAND Journal of Economics,* **21**, 446-458.

Milgrom, P., J. Roberts (1982), "Limit Pricing and Entry under Incomplete Information: An Equilibrium Analysis," *Econometrica,* **50**, 443-459.

Narasimhan, C., and Z.J. Zhang (2000), "Market Entry Strategy under Firm Heterogeneity and Asymmetric Payoffs," *Marketing Science,* **19**, 313-327.

Ofek, E., and O. Turut (2008), "To Innovate or Imitate? Entry Strategy and the Role of Market Research," *Journal of Marketing Research,* **45**, 575-592.

Robinson, W.T., G. Kalyanaram, and G.L. Urban (1993), "First-Mover Advantages from Pioneering New Markets: A Survey of Emprical Evidence," *Review of Industrial Organization,* **8**, 669-678.

Shen, Q. (2008), "Industry Evolution with Endogenous Entry and Exit," *working paper,* The Wharton School, University of Pennsylvania.

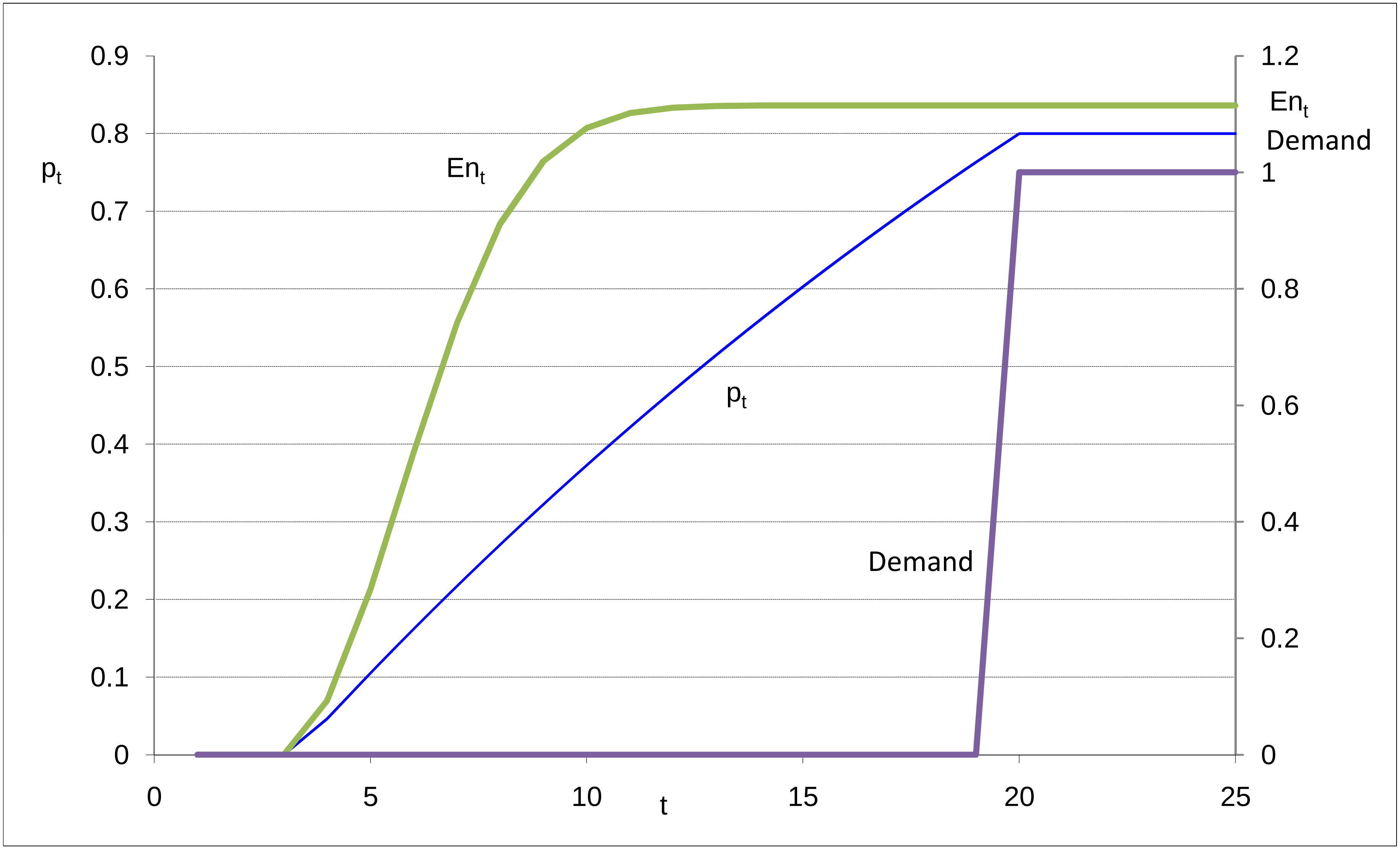

Figure 1: Probability of entry conditional on no prior entry and the expected number of firms in the market for N=2, τ=20, δ=.97, F=20, and $\pi(n)=1/n$.