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Strategic Entry Before Demand Takes Off

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In developing industries, firms have to decide whether and when to enter the market depending on the state of demand, existing firms in the industry, and the firm's capabilities. This paper investigates a model of increasing demand, in which firms decide when to enter the market anticipating the strategic behavior of other potential entrants, and the effects of entry on future potential entrants. This paper shows that the ability of early entry to deter future competitors' entry leads firms to enter the market at a rate faster than demand is expanding. If there is the potential for many firms to enter the market, firms may be less likely to enter because of future competitor entry to correct any market opportunities. If firms enter the market depending on their fixed capabilities rather than depending on the firm's circumstances at each moment in time, firms end up entering the market at a faster rate in the early periods.

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1. Introduction

In growing industries, potential entrants have to decide whether and when to enter the industry. For example, in the satellite radio industry in the late 1990s, XM, Sirius, and other potential competitors had to decide when to enter the industry.[1] If a firm enters too early, the return on investment may come too late to recoup the entry costs, but the firm is guaranteed a place in the industry. If the firm delays entry, the present value of the entry costs will be lower, but the firm may finally prefer not to enter because other firms entered earlier. This trade-off yields an equilibrium timing for the firm to enter the market. In the satellite radio industry case, both XM and Sirius (and only XM and Sirius) decided to enter, but eventually merged.

One question that arises is whether in equilibrium firms enter the market at a rate faster than the rate at which the market expands. This can result when firms enter the market earlier in order to deter entry from competitors. We find that this will indeed occur because of the strategic interaction among potential entrants and the effects from entry deterrence. We show that this effect is stronger where the opportunity for firms to enter prior to the market expansion is greater.[2]

As a motivating example, consider the current case of the hybrid car category.[3] In the United States, this category's first main product launch was the Toyota Prius in 2000. Since then there have been 32 product introductions through the end of 2008 (http://en.wikipedia.org/wiki/List_of_hybrid_cars), with expectations or announcements of 40 additional product launches in the following three years. At the same time the total share of hybrid cars is expected to increase from 2.2% of the U.S. market in 2007 to 10% by 2015 and to 40% by 2030, with about 70% share of electric-drive vehicles.[4] That is, a large number of product launches, and announced product launches, exist in advance of the potential expected sales take-off in the category. Firms enter the market in advance of the sales take-off, because delaying early entry may mean that a firm loses substantial market share to the competitors and potentially be deterred from entry.

We also investigate the effect of the number of potential entrants and of private information of entry costs on the strength of this entry deterrence effect.

[1] See, for example, some information on the satellite radio industry in Godes and Ofek (2003).

[2] In the late 1990s E Ink entered aggressively into the electronic ink category partly in order to get demand and potentially deter other competitors' entry (Amabile and Archambault 1999).

[3] Several important aspects of this example are not considered in the model below. However, the example illustrates a situation of a significant number of product launches before the demand expansion, because not launching early would mean losing substantial market share to competitors and other potential entrants.

[4] See Heffner (2009) and other forecasts by HybridCars.com (2006).

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A firm's decision to enter the market, in addition to being dependent on the state of demand and the existing firms in the market, may depend on the firm's conditions (e.g., entry costs) and its beliefs as to what the other firms might do, as well. If the firms' conditions are stable through time, nonentry of a potential entrant may partially reveal that that potential entrant has poor conditions for entering the market. This may potentially lead the remaining potential entrants to delay entry as they fear too many entries in the next period. On the other hand, if the firms' conditions vary from period to period, the overall (stochastic) characteristics of the pool of potential entrants does not change through time, and the firms may have an incentive to delay entry in the hope of attaining better future entry conditions.

Agarwal and Bayus (2002) show empirically that across several industries the take-off of the number of firms occurs before the take-off of demand. One explanation presented for this empirical result is that supply creates demand. This paper shows that this explanation, although potentially important in some markets, is not essential to explain the pattern in the data. In fact, this paper theoretically explores a rationale for this empirical regularity based on firms being forward-looking and the entry deterrence effects of the existence of firms in the industry. Shen (2008) investigates empirically this rationale with a dynamic structural model of firm entry and exit in the early years of the clothes washer industry.[5] Klepper and Graddy (1990) and Klepper (1996) discuss several empirical facts related to the evolution of demand and the number of firms in the developing phases of new industries, and Jovanovic (1982) presents a model with perfect competition of the evolution of the number of firms in a new industry.[6] Related to Jovanovic (1982), this paper considers the case of imperfect strategic competition where the entry of one firm may deter other firms from entering. With imperfect competition, there is also a literature focusing on the effects of stochastic entry and exit due to firms' mixed strategies (e.g., Dixit and Shapiro 1986, Vettas 2000).[7] In relation to that literature, this paper considers the effect of dynamic entry prior to demand take-off and the effects of private information. This paper can also be seen as related to the literature on preemption and adoption of a new technology (e.g., Gilbert and Harris 1984, Fudenberg and Tirole 1985), in which firms have to decide whether to adopt a new technology with benefits for the first adopter. That literature focuses on preemption and rent equalization effects, whereas this paper looks at the dynamics of entry as it relates to demand evolution. Also related is the work on dynamic "grab the dollar" games (entry into a natural monopoly), knowing that entry is only profitable if only one firm enters the market, and leads to losses if both firms enter the market (see Fudenberg and Tirole 1995, pp. 127–128). In relation to that literature, this paper looks at the dynamic case where firms can decide to enter in advance of market growth and explores how such possibility affects the rate of market entry, and the firms' strategic behavior. Also related to this paper, the literature on firm exit in a declining industry (war of attrition, e.g., Fudenberg et al. 1983, Ghemawat and Nalebuff 1985) studies how firms may exit a market through time if too many firms end up in the market. Londregan (1990) considers the case in which two asymmetric potential firms can enter, exit, and reenter through the industry's life cycle. Narasimhan and Zhang (2000) consider the effect of strategic firm entry to capitalize on potential pioneering advantages depending on heterogeneous firm capabilities and argue why the market dominant firm may be the late entrant.[8] One potential issue in some markets that is not considered here is that early entrants may engage in market activities that may dissuade potential future entrants from entering the market (e.g., Bourguignon and Sethi 1981, Milgrom and Roberts 1982, Lal 1990). In this regard, the entry deterrence literature (e.g., Spence 1977, Dixit 1979) considers the possibility of firms investing in capacity to deter potential entrants from coming into the market. In relation to this literature, this paper looks at entry dynamics when demand evolves over time and firms are considered with an equal opportunity to enter the market. In addition, the paper considers the case of many firms and the role of different forms of private information.

The paper considers the strategic entry of firms as demand develops exogenously and argues that firms being forward-looking and competition lead to firms coming into the market before demand take-off. In some markets one may argue that entry of firms may itself lead to increased demand; that is, the evolution

[5] Ericson and Pakes (1995) present a framework for empirical work with the possibility of firm entry and exit in an industry. Gallant et al. (2009) present an empirical model of dynamic entry with cross-product spillovers. See also Doraszelski and Satterthwaite (2010) for computable Markov-perfect industry dynamics in a stable industry with private information independent through time.

[6] See also Hopenhayn (1992).

[7] See also Cabral (1993) for a similar framework investigating the effect of experience advantages. Amir and Lambson (2003) investigate the dynamics of entry and exit under stochastic market conditions, considering equilibria where the last firms to enter are the first firms to exit.

[8] For analysis on the market effects of order of entry see, for example, Robinson et al. (1993) and Golder and Tellis (1993). There could also be advantages for early entrants, for example, because of consumer search costs (see, for example, Kuksov 2004, for the effect of consumer search costs on product design under competition).

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of demand is endogenous to firm entry (see, for example, Agarwal and Bayus 2002, for arguments in this regard).[9] With respect to this possibility, this paper can be seen as showing that we can observe in the data firm take-off before demand take-off, independent of firm entry causing demand to develop. Note also that if firm entry causes demand to grow, the results presented here will continue to hold.[10] In fact, if firm entry causes demand to grow at a decreasing rate, firms may enter to exhaust the returns to entry, and the analysis presented here can be seen as approaching the case when several firms are already in the market (such that we are already in the part of the curve where more firms leads to lower profits per firm). If firm entry affects future demand, another potential issue is that firms may free-ride on other firms entering first, which may provide incentives for firms to delay entry. This could potentially generate a force against firms entering before demand take-off. Also related to this work, in some cases one can see firms being more likely to enter a market if firms with similar characteristics are already there (Debruyne and Reibstein 2005). This could be seen as firms with similar characteristics entering the market at the same time or as a firm entering a market providing information to potential entrants of the market profitability (Ofek and Turut 2008).[11] These issues, although important in some markets, are not considered in this paper so as to focus on the main point of the paper of strategic entry under demand evolution.[12]

The remainder of this paper is organized as follows. The next section presents a two-period, two-potential-entrants model of the market interaction. Section 3 extends this model to the case of more than two periods. The case of more than two firms is explored in §4, and §5 considers the case of potential entrants' specific firm conditions being private information. Section 6 presents concluding remarks and directions for future research.

[9] See also Economides (1996).

[10] Another possibility is that firms may want to enter early because of the lead time needed to develop some supply or demand side factors. However, in many of the industries considered, these factors do not seem crucial for the observed pattern of the take-off in the number of firms before the take-off of demand. Consumer uncertainty about preferences at the time of the first purchase (e.g., Guo 2006) could also potentially lead to early entry.

[11] See also Zhang (2010) on the possibility of the influence of actions of others on the update of beliefs about the market or products.

[12] The model considered here can also be seen as applying to an auction setting where bidders enter sequentially into an auction and have an entry cost of entering the auction. See, for example, Moreno and Wooders (2009) for an auction model with entry costs.

2. A Basic Model

Consider a market with two periods, where two potential entrants can decide to enter the market in each of the periods by paying an entry cost F. In each period, the decisions by the firms are simultaneous, which captures the idea that a firm makes a decision without knowing yet if the competitor has already decided to enter. Demand is only realized in the second period, with the payoff per firm depending on the number of firms in the market. This feature tries to capture the effect that firms may have the ability to enter the market prior to demand growth. One could also consider the case where demand is small but positive in the first period, but this possibility does not affect the main insights in this two-potential-entrants case. Denote the payoff for a firm in the second period, given that n symmetric firms are in the market as $\pi(n)$. We make the natural assumption that the profit per firm in the industry weakly decreases with the number of competitors because more competitors may lead to less demand per firm and more competition. That is, $\pi(n)$ is decreasing in the number of firms in the industry, n. For the two-firm case in this section, we are just concerned with the profit of only one firm in the market, $\pi(1)$, and the profit of two firms in the market, $\pi(2)$; and we have $\pi(1) > \pi(2)$.[13] Firms discount future payoffs with the discount factor $\delta < 1$ per period. We consider the case of δ sufficiently close to one such that $\delta\pi(1) > \pi(2)$.

To make the problem interesting, we consider the case in which $\delta\pi(1) > F > \pi(2) > 0$. This allows for the possibility that a single firm entering the market in the first period can be profitable and yields that two firms entering in the second period would lead to both firms being unprofitable.[14] We concentrate the analysis on the symmetric equilibria. As a threshold for comparison, note that if there is only one potential entrant, it optimally waits until the second period to enter the market.

To solve for the subgame perfect equilibrium, we will consider first the equilibrium actions in the second period given the state of the market after the first period. Suppose no firm entered in the first period. Then in the second period, the symmetric equilibrium would be in mixed strategies, because a firm prefers

[13] Because of more intense competition with a greater number of firms, we might have the stronger effect that the total industry profit is decreasing with the number of firms. That is, $n\pi(n) \geq n'\pi(n')$ if $n < n'$. For the two-firm case in this section, we would have $\pi(1) > 2\pi(2)$. This stronger assumption is only used in §4.

[14] If $\pi(1) > F > \delta\pi(1)$, no firm would ever enter in the first period; if $\pi(1) < F$, no firm would ever enter the market; if $\pi(2) > F$, both firms would always enter the market. For completeness, note also that if $\delta\pi(1) < \pi(2)$, then either firms enter the market in the second period or they never enter the market.

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to enter if the competitor does not enter the market and prefers to stay out if the competitor enters the market. Denoting as p_i the probability of entry in period i, the equilibrium probability of entry for a firm is determined by indifference between entry and staying out, $p_2\pi(2)+(1-p_2)\pi(1)-F=0$, which yields $p_2=(\pi(1)-F)/(\pi(1)-\pi(2))$. The expected profit in the second period after no entry in the first period is zero.

Now consider that only one firm entered in the first period. Then the other firm will not want to enter, as $F>\pi(2)$, and the incumbent firm has a payoff of $\pi(1)$, because the entry cost was paid in the first period. Finally, if both firms entered in the first period, each firm gets a profit of $\pi(2)$.

Let us now look at the first period. Note that by the argument above if only one firm enters the market in the first period then the other firm will stay out in the second period. Then, if the competitor entered in the first period, a firm would have preferred to stay out as $F>\delta\pi(2)$, and if the competitor does not enter in the first period, the firm would prefer to enter then, because by waiting the firm earns an expected profit of zero, whereas by entering the market the firm gets $\delta\pi(1)-F>0$. This represents the effect that a firm gains from coming in early because it deters entry from the competitor.

Then the equilibrium in the first period is again in mixed strategies, where the payoff of staying out is zero, as noted above. The equilibrium probability of entry for a firm in the first period, p_1, is determined by indifference between entry and staying out, $p_1\delta\pi(2)+(1-p_1)\delta\pi(1)-F=0$, which yields $p_1=(\delta\pi(1)-F)/(\delta(\pi(1)-\pi(2)))$.

This yields two interesting results. First, with strategic entry and forward-looking firms, firms enter in the first period, in advance of demand realization, with positive probability. This is because by entering earlier, a firm can deter the entry of the competitor. Note that without competition (just one potential entrant) the firm only enters the market in the second period, and there is no entry in the first period. Note also that with two myopic potential entrants there is no entry in the first period, and in the second period each firm enters with the probability p_2. This result illustrates how the existence of competing forward-looking potential entrants may yield that the take-off of the number of firms occurs prior to the take-off of demand, an empirical fact presented in Agarwal and Bayus (2002).[15]

[15] Another potential factor of firms' early entry, not explored here, is that firms may not be sure which product characteristics are the preferred ones and may enter early to learn to get their product right. This could potentially lead to the existence of early entry followed by an industry shake-out of the more inefficient product designs.

Second, for the equilibrium probabilities of entry, one can immediately obtain that, conditional on no prior entry, firms are more likely to enter when demand is present than prior to demand being realized, $p_1<p_2$. This results from discounting of the entry payoffs if a firm enters in the first period, whereas the entry costs are paid in the period of entry.

Because of the importance of these two results, we state them in the following proposition.

Proposition 1. *Consider a two-period, two-potential-entrants' market. In such a market, firms enter prior to the demand realization with positive probability, and conditional on no entry, each firm enters with greater probability in the period in which demand is realized than prior to demand realization.*

Computing the expected number of firms in the market in the second period, one obtains $2p_1+2p_2(1-p_1)^2$. Comparing it with the case in which firms are myopic, one obtains that if the discount factor δ is sufficiently close to one, then the expected number of firms is larger with forward-looking firms. As δ is close to one, the equilibrium strategies in the first period are close to those in the second period; therefore, with two periods for firms to enter, the expected number of firms in the second period is larger than if firms choose to enter only in the second period, as is the case when firms are myopic. Note, however, that if p_2 is close to one, i.e., firms enter almost for sure in the second period, then if δ is sufficiently less than one, we have a smaller expected number of firms when firms are forward-looking than when firms are myopic. With myopic firms we almost always have two firms in the market; whereas with forward-looking firms, with probability $2p_1(1-p_1)$, only one firm enters the market.

Now consider the comparison of the effect of one versus two potential entrants on the expected number of firms in the market in the second period. With only one potential entrant, that firm will always enter the market in the second period, and the number of firms in the market in the second period is always one. With two potential entrants, each firm can enter the market in each period with some probability, and the expected number of firms in the market in the second period can be greater than or less than one. In this comparison one can show that if the entry costs are low, the expected number of firms in the second period is greater with two potential entrants than with one and that the threshold of entry costs for this greater number of firms is higher when firms are forward-looking. This is because the cost of entry is considered low, and with multiple opportunities to enter the market (the forward-looking case) more firms may end up in the market. We note these results on the number of firms in the following proposition.

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Proposition 2. *Consider a two-period, two-potential-entrants' market. If the discount factor δ is sufficiently close to one, the expected number of firms in the market in the second period is greater when firms are forward-looking than when firms are myopic. If entry costs are low, the expected number of firms in the market is greater with two potential entrants than with one potential entrant, and the entry costs threshold for this to occur is greater when firms are forward-looking than when firms are myopic.*

The analysis above considered the case of two symmetric firms. Section 5 considers the case where firms may have different entry costs. If the profit per firm is also different across firms, similar results would follow if firms are not too different, with the firm with a higher expected profit more likely to enter in each period.

3. Large Number of Periods

Consider now the case with a large number of periods and two potential entrants. Suppose that starting in period zero, two potential entrants decide if and when to enter the market in each of an infinite number of periods.[16] Demand is zero until period τ and positive at a constant level from period τ onward.[17] Denoting by $\pi(n)$ the profit per period after period τ if n firms are in the market, the present value of profits from period $t > \tau$ onward, if the market has n firms from period t onward, is $\pi(n)/(1-\delta)$. To make the problem interesting, we assume that the entry costs F are such that $\pi(2)/(1-\delta) < F < \pi(1)/(1-\delta)$, so that ex ante, it is advantageous to be the only firm in the market and disadvantageous to be one of two firms in the market. That is, if there is already another firm in the market, a firm always chooses not to enter.

Define $\underline{\tau}$ by $\delta^{\tau-\underline{\tau}+1}\pi(1)/(1-\delta) < F < \delta^{\tau-\underline{\tau}}\pi(1)/(1-\delta)$. Then $\underline{\tau}$ is the first period when it is profitable for a firm to enter the market. Note that for any τ, if δ is high enough we have $\underline{\tau} = 0$.

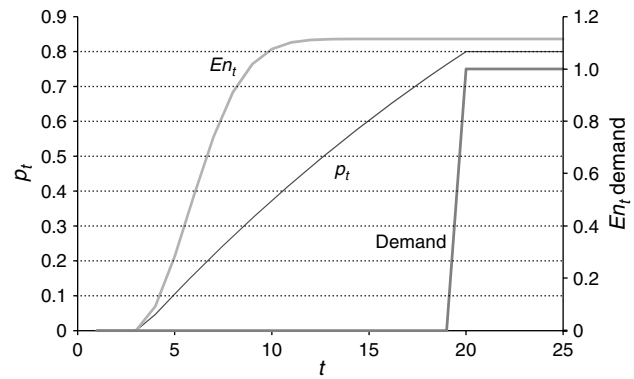
From the analysis above, we can then obtain the probability of entry p_t at period t, given that the other firm has not entered, as

$$p_t = \begin{cases} 0 & \text{if } t < \underline{\tau}, \\ \dfrac{\delta^{\tau-t}\pi(1) - F(1-\delta)}{\delta^{\tau-t}(\pi(1)-\pi(2))} & \text{if } \underline{\tau} \le t < \tau, \\ \dfrac{\pi(1) - F(1-\delta)}{\pi(1)-\pi(2)} & \text{if } t \ge \tau. \end{cases} \tag{1}$$

[16] Entry is defined here as the observable commitment of a firm to enter the market and its sinking of entry costs. Given delay in coming to market, in several industries entry can potentially occur before a firm actually starts selling in a market.

[17] It would be interesting to investigate what happens if τ is uncertain. The main ideas presented here should continue to hold, but there may be additional interesting effects related to potentially greater incentives to early entry.

Figure 1 **Probability of Entry Conditional on No Prior Entry and the Expected Number of Firms in the Market for $N = 2$, $\tau = 20$, $\delta = 0.97$, $F = 20$, and $\pi(n) = 1/n$**



Note that this probability is increasing over time as the closer we are to when demand increases, the more profitable it is for firms to enter the market. Note that if demand were expected to decrease at some point in the future, the probability of entry given no entry by the other firm would decrease after period τ. Note also that the probability p_t is increasing at a decreasing rate for $t < \tau$, because the increase in the benefit of entry is smaller the closer the firms are to the period in which demand increases. Figure 1 illustrates the evolution of p_t for $\tau = 20$, $\delta = 0.97$, $F = 20$, and $\pi(n) = 1/n$. For this case, $\underline{\tau} = 4$.

Note that the probability of entry for $t < \tau$ is increasing in the discount factor δ. This is because a higher discount factor increases the present value of profits when demand increases, both for the profit at the moment of when demand increases and also for the total present value of profits after demand increases.

The expected number of firms in the market at period $t > \underline{\tau}$, En_t, can be obtained as

$$\begin{aligned} En_t &= 2p_{\underline{\tau}} + 2p_{\underline{\tau}+1}(1-p_{\underline{\tau}})^2 + 2p_{\underline{\tau}+2}(1-p_{\underline{\tau}})^2(1-p_{\underline{\tau}+1})^2 \\ &\quad + \cdots + 2p_t\Pi_{i=0}^{t-\underline{\tau}-1}(1-p_{\underline{\tau}+i})^2 \\ &= \sum_{t'=0}^{t} 2p_\tau \Pi_{i=0}^{t'-1}(1-p_i)^2. \end{aligned} \tag{2}$$

Figure 1 shows the evolution of the expected number of firms for a set of parameter values. Note that the expected number of firms in the market increases over time because firms have had more opportunities to enter the market. The increase in the expected number of firms in period t is given by $2p_t\Pi_{i=0}^{t-1}(1-p_i)^2 > 0$. Note that after some period this increase becomes smaller and smaller as the term $\Pi_{i=0}^{t-1}(1-p_i)^2$ decreases in t. Note that for $t > \underline{\tau}$ but t small, we can have that the increases in the expected number of firms is increasing in t as the term p_t increases in t, which may overcome the decreases resulting from $\Pi_{i=0}^{t-1}(1-p_i)^2$.

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Compared with the one-firm case, note that prior to τ the expected number of firms in the market is greater than zero, whereas it is zero in the one-firm case. That is, competition leads to more firms entering the market prior to τ. Note also that with probability one, at least one firm enters the market when there are two potential entrants and with positive probability, two firms enter the market. Then, the expected number of firms that is ever in the market is strictly greater than one firm.

For δ close to one (with F changing such that $\pi(2) < F(1-\delta) < \pi(1)$), we have p_t close to $(\pi(1) - F(1-\delta))/(\pi(1) - \pi(2))$ for all t. We can then obtain that the expected number of firms in the market at period τ is close to $2p_\tau(1-(1-p_\tau)^{2\tau})/(1-(1-p_\tau)^2)$, which is greater than one if $\tau > \frac{1}{2}(\log(p_\tau/2))/\log(1-p_\tau)$. For this case, the expected number of firms that is ever in the market can be obtained to be close to $2/(2-p_\tau) > 1$. The probability of two firms ending up in the market can be obtained to be close to $p_\tau/(2-p_\tau)$.

Comparing with the case when firms are myopic and assuming that $\pi(2)/(1-\delta) < F < \pi(1)/(1-\delta)$, we see that if $\delta > (\pi(1) - \pi(2))/\pi(1)$, then there are no values for the entry costs F such that firms enter when they are myopic. In other words, if firms are myopic, they are less likely to enter because they do not fully value the future potential profits.

Another interesting question is what happens when the time between periods is reduced. One can show that when the time between periods goes to zero, only one firm enters the market at time $\underline{\tau}$ because there are then many opportunities to enter close to $\underline{\tau}$, and the probability of entry there is close to zero, making it unlikely that both firms will enter the market at the same time. The appendix provides further discussion on this case.

The results above consider the existence of mixed strategies with more than one firm potentially entering in each period (a firm making the decision of whether to enter without knowing whether competitors are also entering the market). As noted above, the limiting case of the time between periods converging to zero addresses this issue of mixed strategies because the equilibrium has only one firm entering the market. Another way to address this issue is to include some degree of private information by firms, as discussed in §5, such that a strategy by each firm is a pure strategy to enter or not to enter the market depending on its private information. Finally, still another possibility is to consider a situation where in each period only one firm can make the decision whether to enter the market. Suppose, for example, that one firm can enter only in the odd periods, and the other firm can enter only in the even periods.

Then one can obtain that if we are in the period immediately before τ and no firm has entered the market, then the firm that can enter in that period enters the market. But then the firm that has the opportunity to enter the market two periods before τ would choose to do so. By iteration one can obtain that in this situation the firm that has a chance to enter the market at period $\underline{\tau}$ chooses to do so. The equilibrium has then only one firm in the market that enters at period τ, before the sales take-off, which is exactly the same outcome as when the time between periods converges to zero.

4. Large Number of Firms

4.1. Introduction

Consider now the base two-period model presented above but suppose that there are several potential entrants and that the market may ultimately have in equilibrium more than one firm. We denote as N the number of potential entrants and as n the number of firms that have entered the market. Furthermore, let n_1 be the number of entrants in the first period and n_2 be the number of entrants in the second period. We assume N and n to be large, with N much greater than n in equilibrium.

Denote $\pi(n)$ as the profit per firm in the second period if there are n firms in the market. We assume that $n\pi(n)$ is decreasing in n, the industry profits go down with the number of firms. Furthermore, we assume that $\pi(n)$ is convex, that is, the decrease in profits when increasing n is greater for a small number of firms than for a large number of firms. Both of these assumptions hold for the case of price competition or Cournot competition under some general assumptions. The latter means, for example, that the profit per firm goes down more when the industry expands from two to three firms in the market than when it expands from three to four firms.[18] To get sharper results, we assume $\pi(n) = e^{-\beta n}$, with $\beta \geq 1$ for $n\pi(n)$ to be decreasing in n for all $n > 1$. In the first period the profit per firm is $\alpha\pi(n)$ if there are n firms in the market, with $(1-\delta)Fe^{\beta} < \alpha < 1$. There is always such an α if F is small enough such that several firms would like to be in the market. We think also of δ close to one, such that α can be close to zero. The inequality $\alpha > (1-\delta)Fe^{\beta}$ is not present in the previous section and is necessary here because now one single firm entering the market is not pivotal in deterring entry in the second period (this inequality is discussed in greater detail in §4.3). If $\alpha = 0$ then no firm would enter in the first period (contrary to what happens in the previous section). Considering α small but positive leads to several firms entering the market

[18] For example, Xiao and Orazem (2009) show that the fourth entrant in a local U.S. broadband market has little effect on the competitive behavior.

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in the first period, prior to a steep demand growth. The inequality $\alpha < 1$ captures the effect that demand grows from the first to the second period.[19]

In a symmetric equilibrium in a given period, each firm will enter with some probability p. Given that N firms independently follow this strategy, the actual number of firms that enter the industry is a random variable. With N large, the probability distribution of the number of firms that enter the market can be approximated by a normal distribution with mean pN and variance $p(1-p)N$, which we will use in what follows. Let $g(x;\mu,\sigma^2)$ and $G(x;\mu,\sigma^2)$ be, respectively, the density and cumulative distribution function of a normally distributed random variable x with mean μ and variance σ^2.

Finally, define m as the number of firms such that the profit per firm in the second period is equal to the fixed cost of entry, $\pi(m) = F$.

4.2. Second Period

Consider the equilibrium strategies in the second period of the market assuming that n_1 firms entered in the first period. If $n_1 + 1 \geq m$, no firm will enter the market in the second period, as even if only one firm entered the market, the profit obtained, $\pi(n_1+1)$, would be less than the fixed cost of entry F.

Suppose now that $n_1 + 1 < m$. Then the probability of entry in the market should be such that the expected profit of entering is equal to the expected profit of staying out, zero. The probability of entry in the second period p_2 would be then defined by

$$-F + \int e^{-\beta(n_1+n_2)} g\big(n_2; p_2(N-n_1), p_2(1-p_2)(N-n_1)\big)\, dn_2 = 0. \quad (3)$$

Given that $g()$ is the normal density, this expression can be reduced to

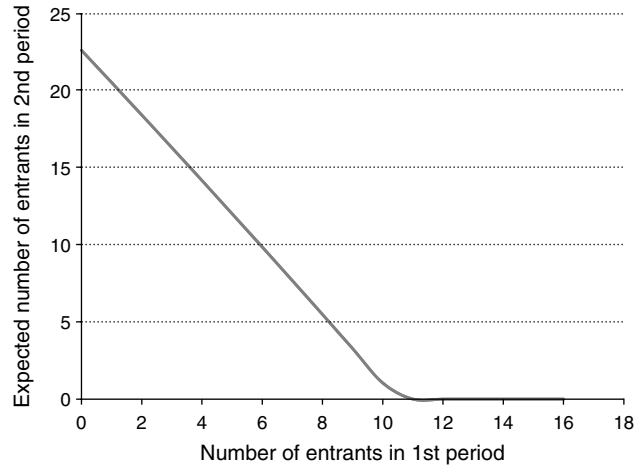
$$-F + e^{-\beta(n_1+p_2(N-n_1))+(\beta^2/2)p_2(1-p_2)(N-n_1)} = 0, \quad (4)$$

which defines the equilibrium probability of entry p_2 as

$$p_2 - \frac{\beta}{2} p_2(1-p_2) = \frac{-\log F - \beta n_1}{\beta(N-n_1)}. \quad (5)$$

From this, one can obtain directly that the probability of entry is decreasing in the number of potential entrants, N; the number of incumbents, n_1; and in the entry costs, F.[20] A greater number of potential entrants makes each firm be more conservative with its probability of entry, for fear of too many firms entering the market. However, one can obtain that the expected number of firms in the market $n_1 + p_2(N-n_1)$ is increasing in the number of potential entrants, N. A greater number of incumbents, n_1, or a greater entry cost F makes the market less attractive to entry, and the probability of entry is lower.

Figure 2 Expected Number of Entrants in the Second Period as a Function of the Number of Entrants in the First Period for Example in the Text



Interestingly, note that the expected number of firms in the market, $n_1 + p_2(N-n_1)$, is greater than the number of firms m whose entry would lead to a profit per firm equal to the entry cost. That is, with probability greater than $\frac{1}{2}$, there are too many firms in the market. The intuition for this result follows from the convexity of the profit function $\pi(n)$. With a convex profit function, firms like uncertainty and are more likely to enter, enticed by the possibility of the market having very few firms and generating high profits per firm.[21]

To illustrate these effects, we consider the following example with $\alpha = 0.1$, $\beta = 1.1$, $N = 1{,}000$, $F = 0.00001$, and $\delta = 0.95$. From this one can obtain $m = 10.47$ so that if 10 or more firms enter in the first period, no firm enters in the second period. Figure 2 shows the expected number of firms entering in the second period as a function of the number of firms that entered in the first period. Figure 2 also illustrates how the expected number of firms in the market at the end of the second period is greater than the number of firms m whose entry would lead to a profit per firm equal to the entry cost. For example, if 4 firms entered in the first period, the expected number of firms that would enter in the second period would be around 15, with an expected number of firms in

[19] If $\alpha > 1$ then demand is decreasing through time and there are even greater incentives to firms entering in the first period.

[20] Note that $F < 1$ and $\log F < 0$ for some number of firms to want to enter the market.

[21] See also Dixit and Shapiro (1986). Mankiw and Whinston (1986) and Amir and Lambson (2003) present also a force for excessive entry based on the private incentives for entry possibly being greater than the social incentives for entry.

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the market in the second period of around 19, greater than $m = 10.47$. The probability of ending up in the second period with more than m firms in the market given that four firms entered in the first period is slightly greater than 98%.

4.3. First Period

Consider now the decisions by the firms in the first period. In a symmetric equilibrium, firms will enter with some probability p_1 in the first period. If the actual number of firms that enter the market in the first period is greater than m, we have from above that no other firm will enter in the second period. If the actual number of firms that enter the market in the first period is less than m, then firms enter in the second period with a probability of entry $p_2(n_1)$, which is determined by (5). Making the expected present value of profits equal to the entry costs F, one obtains

$$\begin{aligned}
&-F + \int \alpha\pi(n_1) g(n_1; p_1 N; p_1(1-p_1)N)\, dn_1 \\
&\quad + \delta \int_0^m \Big[\int \pi(n_1+n_2) g\big(n_2; p_2(n_1)(N-n_1), \\
&\qquad\qquad p_2(n_1)(1-p_2(n_1))(N-n_1)\big)\, dn_2 \Big] \\
&\quad \cdot g(n_1; p_1 N, p_1(1-p_1)N)\, dn_1 \\
&\quad + \delta \int_m^N \pi(n_1) g(n_1; p_1 N, p_1(1-p_1)N)\, dn_1 = 0, \qquad (6)
\end{aligned}$$

where the second term is the expected profit in the first period; the third term is the part of the second period expected profit when few firms enter in the first period, such that there is more entry in the second period; and the fourth term is the part of the second period expected profit when too many firms enter in the first period, such that there is no more entry in the second period.

Using (5) and $g()$ being a normal density, one can then obtain the equilibrium condition for p_1 as

$$\begin{aligned}
&-F + e^{-\beta p_1 N + (\beta^2/2) p_1(1-p_1)N} \big\{ \alpha + \delta[1 - G(m; p_1 N - \beta p_1 \\
&\qquad\qquad \cdot (1-p_1)N, p_1(1-p_1)N)] \big\} \\
&\quad + \delta F G(m; p_1 N, p_1(1-p_1)N) = 0. \qquad (7)
\end{aligned}$$

This expression yields several observations. First, note that if $\alpha = 0$ (as in §2), then no firm would enter in the first period. To see this, note that the left-hand side is negative when evaluated at $\alpha = 0$ and $p_1 = 0$. That is, if no firm enters, the expected profit of a firm entering the market would be negative, and this would be even more negative if more firms entered. Now consider what has to be a condition on α such that at least one firm would like to enter the market in the first period. In that case, if that firm is the only firm entering the market, its expected payoff (the left-hand side of (7) with certainty of only one firm entering in the first period) would be $-F + \alpha e^{-\beta} + \delta F$. Making this expression strictly greater than zero leads to the condition $\alpha > (1-\delta) F e^{\beta}$ noted above. To gain intuition on this result, note that in the second period the expected profit of a firm entering in the first period is below or equal to F. From the first period perspective, this is discounted to be worth at most δF. For a firm entering in the first period, and if $\alpha = 0$, that firm would get, at most, δF minus the entry costs F, which is strictly negative. If $\alpha = 0$, it is then better not to enter in the first period.

Second, as in the second period analysis, given the convexity of the profit function, there is a force for more firms to enter in the first period than the number of firms that would lead to zero ex ante profits. To focus on this effect, note that the number of firms entering in the first period that would lead to zero ex ante profits, m', is defined by $\alpha\pi(m') = \alpha e^{-\beta m'} = F - \delta\pi_2^*$, where π_2^* is the expected second-period profit, which can potentially be a function of m'.[22] If the expected number of firms entering in the first period is m', that is, $p_1 N = m'$, and the expected second period profits remained unchanged, then the expected discounted profits for a firm entering in the first period would be $\alpha e^{-\beta m'} e^{\beta^2 m'(1-p_1)/2} - F + \delta\pi_2^*$, which is strictly positive as $e^{\beta^2 m'(1-p_1)/m'} > 1$. That is, for Equation (7) to be satisfied, the expected number of firms entering the market has to be greater than m'. In the example above, one can compute $m' = 10.51$ and $p_1 = 0.022$, which leads to an expected number of firms entering in the first period of around 23 firms, greater than m'.

Third, if the discount factor δ is close to one, we have that the number of firms that enter in the first period that would lead to zero ex ante profits is greater than the number of firms that would lead to zero ex ante profits for firms entering in the second period. This is because firms that enter in the first period also benefit from the first period profits. If all the firms entered in the first period, the number of firms that would lead to zero discounted profits for δ close to one would be determined by $\alpha\pi(m') + \pi(m') = F$, which means that m' is greater than the number of firms that would lead to zero ex ante profits for firms entering in the second period, m, which as noted above was defined by $\pi(m) = F$. This is a force for the number of firms to increase prior to demand increasing. In the example above, this holds as $m' = 10.51 > m = 10.47$. One can also obtain that the probability of entry in the first period such that

[22] Note that for α low, we have the expected second period profit, π_2^*, equal to F, because firms enter in the second period such that the expected profits are equal to the entry costs.

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there is no entry in the second period is greater than 99%; almost all entry is in the first period.

Fourth, as the number of potential entrants N increases, following up on the analysis of the second period, there is a force for the expected number of firms in the market to increase. To see this, consider the effect of the number of potential firms N on the expression $e^{-\beta p_1 N+(\beta^2/2)p_1(1-p_1)N}$ in (7). If N increases and $p_1 N$ remains constant, this expression would increase (greater variance of the number of firms in the market). This means that for (7) to hold, we would need more firms to enter the market, that is, a greater p_1, leading to a greater $p_1 N$.

Fifth, and interestingly, with a large number of firms, the possibility of later entry in the second period is a force toward less entry in the first period. To see this, note that if too few firms enter in the first period, then they know that firms will come in the second period such that the expected second period profits is F. On the other hand, if too many firms enter in the first period, then potential entrants in the second period stay out. That is, first period entrants will have a limit on how much they can gain if the entry uncertainty results in too few firms but have no limit on their losses if the entry uncertainty results in too many firms. To gain further intuition, suppose that firms could only enter in the market in the first period. Then the equilibrium condition would be $-F + e^{-\beta p_1 N+(\beta^2/2)p_1(1-p_1)N}\{\alpha+\delta\} = 0$ instead of (7). In this case the expected second period profits would be

$$e^{-\beta p_1 N+(\beta^2/2)p_1(1-p_1)N} = \int_0^N e^{-\beta n_1} g\big(n_1; p_1 N, p_1(1-p_1)N\big)\, dn_1,$$

whereas in the case in which firms can enter in the second period the expected second period profits would be the lower amount

$$\int_0^m \Big[\int e^{-\beta(n_1+n_2)} g\big(n_2; p_2(n_1)(N-n_1), p_2(n_1)(1-p_2(n_1))(N-n_1)\big)\, dn_2\Big] \cdot g\big(n_1; p_1 N, p_1(1-p_1)N\big)\, dn_1 + \int_m^N e^{-\beta n_1} g\big(n_1; p_1 N, p_1(1-p_1)N\big)\, dn_1.$$

Therefore, if firms were not allowed to enter in the second period, more firms would actually enter in the first period.

Note that in the context of the model, all firms that enter in the first period decide to stay on in the second period independent of the number of firms that entered in the first period because $\pi(n) > 0$ for any n. That is, in the context of the model, whether firms have the option to exit in the second period does not affect the result that the possibility of entry in the second period limits entry in the first period. In a variation of the model where $\pi(n)$ can be negative for a sufficiently large n, then if too many firms entered in the first period, some would exit in the second period. In this case, the effect that the possibility of entry in the second period limits entry in the first period would still be present (because firms pay a sunk entry cost in the first period) but would be diminished.[23]

Note also that if the market has not many firms (integer problems), then the effects of the previous sections start coming into play, with entering firms knowing that by being in the market they will deter entry by the competitors and have positive profits.

5. Private Information, Pure Strategies, and Firm Capabilities

5.1. Private Information Independent Through Time

The analysis above considered the case in which firms were symmetric and there was no private information, with the equilibrium being in mixed strategies in every period. We now consider the existence of private information by each potential entrant and discuss how this possibility leads to pure strategy equilibria, as noted by Harsanyi (1973) for the static case. Here we consider the case when the private information of each firm is only relevant in the period under consideration. In the next subsection we allow the private information to have effect throughout the lifetime of the potential entrant.

In the model of §3, consider now that in each period t each potential entrant i draws a fixed cost F_{it} from a uniform distribution with support $[\underline{F}, \overline{F}]$, with $\pi(2)/(1-\delta) < \underline{F} < \overline{F} < \pi(1)/(1-\delta)$ and with F_{it} being independent across i and t.[24]

Then denoting as $V(t)$ the expected net present value of payoffs for a firm if no firm has entered the market at the beginning of period t, before the realization of F_{it}, we have that the threshold $\hat{F}_t$, such that a firm enters in period t if and only if it draws a fixed cost less than or equal to $\hat{F}_t$, is determined by

$$-\hat{F}_t + p_t\delta^{\tau-t}\frac{\pi(2)}{1-\delta} + (1-p_t)\delta^{\tau-t}\frac{\pi(1)}{1-\delta} = (1-p_t)\delta V(t+1), \tag{8}$$

[23] Another possibility not considered here is that firms in a market may decide to merge, which could lead to further effects on the dynamics of entry. See, for example, Gowrisankaran (1999) for a model investigating these effects with numerical simulations.

[24] Alternatively, the private information of the firms could be a signal of the market demand level. This possibility is discussed in §6.

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where the probability of entry by a firm in period t is $p_t = (\hat{F}_t - \underline{F})/(\bar{F} - \underline{F})$.

By the definition of $V(t)$, we also have that with probability p_t the firm enters the market in period t with an expected entry cost of $(\hat{F}_t + \underline{F})/2$ and with probability $(1-p_t)^2$, no firm will enter the market, and the firm can get $V(t+1)$ in the next period. That is,

$$V(t) = p_t\left[-\frac{\hat{F}_t + \underline{F}}{2} + p_t\delta^{\tau-t}\frac{\pi(2)}{1-\delta} + (1-p_t)\delta^{\tau-t}\frac{\pi(1)}{1-\delta}\right] + (1-p_t)^2\delta V(t+1), \tag{9}$$

which can be rewritten, using (8), as

$$V(t) = (1-p_t)\delta V(t+1) + p_t^2\frac{\bar{F} - \underline{F}}{2}. \tag{10}$$

This representation generates several points of difference with respect to the analysis above. Now firms have a strictly positive expected payoff, in comparison to zero. This is because firms can draw a lower fixed cost of entry that is smaller than the threshold fixed cost of entry, which yields entry being optimal. Furthermore, now firms can choose to wait because of the option value of drawing a low fixed cost of entry in future periods. This option value is included in $V(t)$, the expected net present value of profits at the start of period t if both firms have not yet entered and before the entry cost is observed.

To proceed, we consider $\delta \to 1$ with $\bar{F} = \tilde{\bar{F}}/(1-\delta)$, and $\underline{F} = \tilde{\underline{F}}/(1-\delta)$ for fixed $\tilde{\bar{F}}$ and $\tilde{\underline{F}}$, such that the condition above for $\bar{F}$ and $\underline{F}$ remains satisfied. Note also that for $\delta \to 1$, we have $V(t) - V(t') \to 0$ and $p_t - p_{t'} \to 0$ for any t and t'. Define $\lim_{\delta\to 1}(1-\delta)V(t) = \tilde{V}$, the average per period expected payoff, and $\lim_{\delta\to 1} p_t = p$. Substituting this into (10), one obtains

$$\tilde{V} = p\frac{\tilde{\bar{F}} - \tilde{\underline{F}}}{2}, \tag{11}$$

which can be substituted into (8) to obtain

$$p^2\frac{\tilde{\bar{F}} - \tilde{\underline{F}}}{2} - p\left[\pi(1) - \pi(2) + \frac{3}{2}(\tilde{\bar{F}} - \tilde{\underline{F}})\right] + \pi(1) - \tilde{\underline{F}} = 0, \tag{12}$$

from which one can obtain the equilibrium p, and then, substituting in (11), $\tilde{V}$. From this, one can then obtain the following proposition.

Proposition 3. *Suppose that the draws of entry costs are independent over time and the discount factor $\delta \to 1$. Then the probability of entry p_t converges to being constant over time, increasing in the firms' profits, $\pi(1)$ and $\pi(2)$, and decreasing in the average entry cost, $(\tilde{\bar{F}} + \tilde{\underline{F}})/2$, and in the spread of possible average per period entry costs, $\tilde{\bar{F}} - \tilde{\underline{F}}$.*

When the discount factor converges to one, and entry costs are drawn independently over time, firms count the future as much as the present; at each moment in time where no entry has occurred, the market conditions are the same, whether or not demand is realized. We obtain then that the probability of entry converges to being constant through time, whether the firms are or are not in a period in which demand has started. As expected, the greater the profits that firms can earn, whatever the market structure, the more likely firms are to enter the market. Also as expected, as the average entry cost increases, the less likely firms are to enter the market, because they try to avoid entering at the same time as the competitor.

More interestingly, the greater the spread of possible entry costs (variance of entry costs), the less likely firms are to enter the market and the more likely they are to wait for a future period. The intuition is that the greater the variance of entry costs, the more it pays off to wait for the option value of drawing a very low entry cost in the next period. Note also that this implies that the probability of entry is lower in this case than in the case where there is no private information about entry costs.

One can also obtain that the probability of entry is greater than the case of no private information and entry costs equal to $\bar{F}/(1-\delta)$ and lower than the case of no private information and entry costs equal to $\underline{F}/(1-\delta)$.

In terms of expected present value of profits, we can obtain that it is increasing in the profits that firms can obtain with either one or two firms in the market and decreasing in the average entry cost. Again, more interestingly, the expected present value of profits is increasing in the variance of possible entry costs. The reason is that with a greater variance of entry costs, firms tend to wait for the option of a low value of entry costs and only then enter the market.

One can also compute the expected costs of entry conditional on firms entering the market. These can obtain to be equal to $\tilde{\underline{F}} + p(\tilde{\bar{F}} - \tilde{\underline{F}})/2$. One can then obtain that the expected costs of entry conditional on firms entering the market are increasing in the firms' profits, $\pi(1)$ and $\pi(2)$; increasing in the average unconditional entry cost, $(\tilde{\bar{F}} + \tilde{\underline{F}})/2$; and decreasing in the spread of possible average per period entry costs, $\tilde{\bar{F}} - \tilde{\underline{F}}$, as expected, given the arguments above.

Compared with monopoly (only one potential entrant), note that competition leads again to a positive probability of firms entering the market before demand is realized and that this is more likely to occur the greater the discount factor and the greater the number of periods in which entry is possible and demand is still not in place. Note also that with competition there is a positive probability of two firms being in the market, which does not happen under monopoly.

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Note that from a social welfare point of view, firms may enter too fast in equilibrium. This could be the case if the benefits in terms of consumer surplus of competition are not too large. In competition, a second firm considering entering compares π_2 with the entry costs, whereas in terms of social welfare the comparison is of $2\pi_2 - \pi_1$ with the entry costs (and note that $\pi_2 > 2\pi_2 - \pi_1$). Note also that if the social welfare optimum is for just one firm to enter the market, and if a regulator can set a threshold entry cost for entry, then it may be optimal for the regulator to set a lower threshold entry cost than the equilibrium one considered above, so that the likelihood of too many firms entering the market is lower. A regulator could change the firms' entry costs, for example, by affecting the cost of licenses to enter a market (e.g., licenses in mobile telecommunications markets).

5.2. Private Information Fixed Through Time

Consider now the case in which firms draw independently an entry cost that is fixed throughout the lifetime of the market. That is, the drawn entry cost could be seen as the firm capability for that market. However, a firm might not know the entry cost drawn by the competitor, the firm capability of the competitor. But the decision of a firm not to enter provides some information to the competitor about the firm's entry cost.

The equilibrium then involves firms waiting to enter in a certain period if the other firm has not entered previously. In each period t there will then be a threshold $\hat{F}_t$ of the last firm's type entering in that period. Firms with $F \in (\hat{F}_{t-1}, \hat{F}_t]$ enter in period t if the other firm has not entered before, with firms with a lower F entering earlier. The probability of a firm entering in period t given no prior entry is $p_t = (\hat{F}_t - \hat{F}_{t-1})/(\bar{F} - \hat{F}_{t-1})$. Denoting $V(t+1)$ as the expected present value of a firm with type $\hat{F}_t$, delaying entry until period $t+1$, we have that the condition for the threshold $\hat{F}_t$ entering in period t, prior to the period τ when demand starts, is

$$-\hat{F}_t + p_t\delta^{\tau-t}\frac{\pi(2)}{1-\delta} + (1-p_t)\delta^{\tau-t}\frac{\pi(1)}{1-\delta} = (1-p_t)\delta V(t+1), \tag{13}$$

where

$$V(t+1) = -\hat{F}_t + p_{t+1}\delta^{\tau-t-1}\frac{\pi(2)}{1-\delta} + (1-p_{t+1})\delta^{\tau-t-1}\frac{\pi(1)}{1-\delta}. \tag{14}$$

Consider now $\delta \to 1$ with the notation, similar to above, that $F = \tilde{F}/(1-\delta)$. The expressions above then become

$$-\tilde{F}_t + p_t\pi(2) + (1-p_t)\pi(1) = (1-p_t)[-\tilde{F}_t + p_{t+1}\pi(2) + (1-p_{t+1})\pi(1)], \tag{15}$$

which yields

$$\frac{p_{t+1}}{p_t} = \frac{\tilde{F}_t - \pi(2)}{(1-p_t)(\pi(1)-\pi(2))}. \tag{16}$$

From this, given that the threshold $\tilde{F}_t$ increases in t, we have that the probability of entry p_t changes through time, contrarily to the case in which the entry costs are drawn independently over time. Furthermore, as stated in the next proposition, one can show that the probability of entry is decreasing through time.

Proposition 4. *Suppose $\delta \to 1$ and the entry costs drawn by each firm are fixed through time. Then, the probability of entry of each firm, conditional on no prior entry, is decreasing through time.*

The proof is presented in the appendix. In equilibrium, firms can never enter with probability zero or one, which means that $\tilde{F}_t$ is strictly increasing in t without ever reaching $\bar{\tilde{F}}$. This means that at least at some point, $\tilde{F}_t$ increases at a decreasing rate, which then implies that the probability of entry is decreasing through time.

As above, note that with several potential entrants, firms come into the market before demand is realized with positive probability. Note also that if one calibrates the support of entry costs such that the average entry probability conditional on no prior entry is the same with independent draws as with fixed capabilities, entry occurs faster under fixed capabilities.

Note also that all types of firms have to have a chance of entering the market because otherwise the probability of entry would become at some point so low that it would pay off for a firm with a high entry cost to enter the market. This means that $\tilde{F}_t$ converges to $\bar{\tilde{F}}$ when t goes to infinity. At the limit one can obtain the probability of entry as $(\pi(1) - \bar{\tilde{F}})/(\pi(1) - \pi(2))$, which is the probability of entry in the case without private information and entry costs $\bar{\tilde{F}}/(1-\delta)$.

Given that the probability of entry is decreasing in t, $p_{t+1}/p_t < 1$, we have, from (16), that $p_t < (\pi(1) - \tilde{F}_t)/(\pi(1) - \pi(2))$. That is, for each threshold $\tilde{F}_t$, the probability of entry with private information and fixed capabilities is lower than the probability of entry without private information at that entry cost. Given that the firms with entry costs such that they enter in period t have lower entry costs than $\tilde{F}_t$, then the probability of entry in period t is also lower than if there were no private information at any entry cost of the firms that enter in period t under private information and fixed capabilities. This also then implies that if there is no private information with fixed capabilities at the average entry costs, the probability of entry is greater than it is with private information and fixed capabilities.

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With private information, firms are strategic in delaying entry because there is an expected positive payoff of delaying entry.

Comparing the case of entry costs independently drawn over time with the case of fixed capabilities, one can obtain that after some time with no entry, the probability of entry when entry costs are independently drawn through time is greater than it is in the case of fixed capabilities.

6. Conclusion

This paper considers strategic entry in a market where firms may be able to enter the market prior to demand growth. We show that competition leads to entry prior to demand growth, as presented in empirical studies, and discuss how the probability of entry increases through time prior to demand realization.

For the case of potentially many firms in the market, the paper shows two additional important effects. First, given the convexity of the profit function, firms like uncertainty, and the expected number of firms is greater than the one that would lead to zero ex ante profits. Second, forward-looking firms understand that if too few firms enter in the beginning of the market, other firms will come into the market later to correct any potential rents, and this reduces the benefit of entering the market early.

The paper also shows that with private information one may replicate the complete information mixed strategy equilibria with pure strategies, but there are additional important effects. Private information can be considered as either independent draws of entry costs through time or as fixed draws at the beginning of the market. In both cases, the expected profits are greater than they are in models of complete information. In the independent draws case, the greater expected profits result from the option value of waiting for a lower draw of entry costs. In the fixed draws case, the greater expected profits result from the potential of strategic delay in entering by firms that do not have a very low entry cost.

The paper considers private information by firms on their entry costs. Alternatively, one could consider that firms receive a private signal on the demand conditions. This possibility could be important in several markets, and it would be interesting to investigate. The analysis in such a case would be similar to the one considered here, with the difference that a positive signal would likely mean that the competitor also received a positive signal, and therefore the news of a positive signal would have to be tempered by the increased likelihood of entry by competitors. Another issue that could be potentially interesting to investigate in future research is that in several market situations, entry costs are invested gradually rather than the discrete entry decision that is considered here.

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Appendix

Time Between Periods. In the model of §3, consider that the time between periods is reduced while maintaining the same time value of money. That is, denoting as Δ the time interval between periods, we have that the discount factor as a function of Δ is $\delta(\Delta) = e^{-r\Delta}$, where r is the continuous time constant interest rate. If demand increases at moment in time τ, then there are τ/Δ periods from time zero until time τ. Defining $\tilde{\pi}(n)$ as the profit rate when there are n firms in the market, we have that the present value of the profit for a period as a function of the interval of time between periods is $\pi(n;\Delta) = \int_0^\Delta \tilde{\pi}(n)e^{-rt}\,dt = \tilde{\pi}(n)/r(1-e^{-r\Delta})$.

Denoting $\underline{\tau}$ as the moment in time at which it becomes potentially profitable to enter the market, $e^{-r(\tau-\underline{\tau})}\pi(1)/r - F = 0$, the number of periods in which firms have a probability of entry prior to time τ is $(\tau-\underline{\tau})/\Delta$ up to an integer approximation. The probability of entry for a period i between time $\underline{\tau}$ and time τ can be written as

$$p_i = \frac{e^{-r(\tau-i\Delta)}\pi(1) - rF}{e^{-r(\tau-i\Delta)}(\pi(1)-\pi(2))}.$$

The probability of at least one firm entering prior to time $\tau' < \tau$ can then be written as

$$1 - \Pi_{i=\underline{\tau}/\Delta}^{\tau'/\Delta}(1-p_i)^2. \tag{17}$$

For any $\tau' \in (\underline{\tau}, \tau]$, one can then obtain that when the time interval between periods converges to zero, $\Delta \to 0$, the probability of entry of at least one firm prior to time τ' converges to one. This can be obtained by noting that when $\Delta \to 0$, there is an infinite number of periods between $\underline{\tau}$ and τ' with $p_i > 0$, which leads the expression in (17) to converge to one. To see what happens in the market when the time interval between periods converges to zero, $\Delta \to 0$, consider a time interval $[\tau'', \tau']$ with $\underline{\tau} < \tau'' < \tau' < \tau$, under the condition that no firm entered the market prior to τ''. By the argument above when $\Delta \to 0$, and for any (τ'', τ'), at least one firm enters in the interval $[\tau'', \tau']$ with probability converging to one. Furthermore, we have that if $\tau' \to \tau''$, the probability of two firms entering in the interval $[\tau'', \tau']$ converges to

$$q_{\tau''}^2 + q_{\tau''}^2\left(1-q_{\tau''}\right)^2 + q_{\tau''}^2\left(1-q_{\tau''}\right)^4 + \cdots = \frac{q_{\tau''}}{2-q_{\tau''}},$$

where

$$q_{\tau''} = \frac{e^{-r(\tau-\tau'')}\pi(1) - rF}{e^{-r(\tau-\tau'')}}(\pi(1)-\pi(2)).$$

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Then we have that if in addition $\tau'' \to \underline{\tau}$, the probability of two firms entering the market converges to zero. That is, if $\Delta \to 0$, the equilibrium converges to only one firm entering the market at time $\underline{\tau}$ with probability approaching one. This is the result discussed in Fudenberg and Tirole (1985) in the context of adoption of a new technology, with a continuous time specification that yields the same result.

Proof of Proposition 4. Given that $\tilde{F}_t$ is strictly increasing in t, we have that $p_{t+1}(1-p_t)/p_t$ is strictly increasing in t, and therefore p_t cannot be constant in t. For any t this means $p_{t+2}(1-p_{t+1})/p_{t+1} > p_{t+1}(1-p_t)/p_t$ for all t. The proof is by contradiction. Suppose $p_{t+1} > p_t$. Then we have $p_{t+2} > p_{t+1}$, and iterating we have $p_{t'+1} > p_{t'}$, for all $t' > t$. Furthermore, for any $t' > t$ we have $p_{t'+2}/p_{t'+1} > p_{t'+1}/p_{t'}$, which means that p_t goes to its maximum, one, at an accelerating rate. But then in the period in which p_t goes to one, entering is not optimal for any of the firms because the other firm is entering with probability one, a contradiction.

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