Too Much Information? Information Provision and Search Costs *

FERNANDO BRANCO  
(Universidade Católica Portuguesa)

MONIC SUN  
(Boston University)

J. MIGUEL VILLAS-BOAS  
(University of California, Berkeley)

June, 2015

* We would like to thank Su Chen for excellent research assistance, and Heski Bar-Isaac, Drew Fudenberg, Emir Kamenica, and Jidong Zhou for helpful comments. E-mail addresses: fbranco@ucp.pt, monic@bu.edu, and villas@haas.berkeley.edu.
A seller often needs to determine the amount of product information to provide to consumers. We model costly consumer information search in the presence of limited information. We derive the consumer’s optimal stopping rule for the search process. We find that, in general, there is an intermediate amount of information that maximizes the likelihood of purchase. If too much information is provided, some of it is not as useful for the purchase decision, the average informativeness per search occasion is too low, and consumers end up choosing not to purchase the product. If too little information is provided, consumers may end up not having sufficient information to decide to purchase the product. The optimal amount of information increases with the consumer’s ex-ante valuation of the product, as with greater ex-ante valuation by the consumer, the firm wants to offer sufficient information for the consumer to be less likely to run out of information to check. One can also show that there is an intermediate amount of information that maximizes the consumer’s expected utility from the search problem (social welfare under some assumptions). Furthermore, this amount may be smaller than that which maximizes the probability of purchase. That is, the market outcome may lead to information overload with respect to the social welfare optimum. The paper can be seen as providing conditions under which too much information may hurt consumer decision-making. Numerical analysis shows also that if consumers can choose to some extent which attributes to search through (but not perfectly), or if the firm can structure the information searched by consumers, the amount of information that maximizes the probability of purchase increases but is close to the amount of information that maximizes the probability of purchase when the consumer cannot costlessly choose which attributes to search through.
1. Introduction

There are many ways that a seller can disclose product information. She may offer, for example, free trials of the product, informative commercials, detailed product description, or service from knowledgeable sales representatives.\(^1\) To withhold product information, on the other hand, the seller can choose to have a limited product web site, or limit the content in the advertising copy. Sellers consider how to present the information on their products, and how much of that information to present. On one hand one could argue that having more available information may help consumers make better decisions. This argument is especially tempting to managers as the growing penetration of the Internet has tremendously lowered the cost of information dissemination. On the other hand, some authors have argued that too much information may create problems in decision-making, a phenomenon often labeled as “information overload.” As stated by Toffler (1970, pp. 350-351) information overload can occur “when the individual is plunged into a fast and irregularly changing situation, or a novelty-loaded context ... his predictive accuracy plummets. He can no longer make the reasonably correct assessments on which rational behavior is dependent.” Jacoby et al. (1974a) and Jacoby (1977) argue that too much information may lead to poorer consumer decisions.

The concern of information overload in the digital era has been rising among marketers. In a recent survey of more than 7,000 consumers and interviews with hundreds of marketing executives around the world conducted by Corporate Executive Board, Spenner and Freeman (2012) find that the single biggest driver of “consumer stickiness” (measured by indicators such as the likelihood to follow through on an intended purchase) is “decision simplicity,” the ease with which consumers can gather information about a product. The study suggests that while a brand of a digital camera that provides extensive feature information (e.g., megapixels and memory, among hundreds of other features) may instruct the consumer about a given camera’s capabilities, it does little to facilitate an easy decision. If a brand offers instead information on one of the camera’s key features (e.g., a photo-editing feature), consumer stickiness increases dramatically. Information overload can be especially prominent in product categories such as consumer electronics and software as they tend to involve many different and often times features of uncertain importance.\(^2\) Apple, for example, presents information on 10 attributes for its new Apple Watch including Faces, Digital Touch, Activity, Workout.\(^3\) Consumers may not really understand how important each of these attributes is until they research about the attribute further. By the time that they figure out the importance of the attribute, they would already have incurred the search cost for that attribute. It is natural, therefore, that offering information on too many attributes may “overload” the consumer.

\(^1\)Throughout the paper, we refer to the seller as “she” and the consumer as “he.”

\(^2\)Spenner and Freeman also note that it is not only the amount of information that is important to consider, but also how the information is structured and presented. This paper focuses on the question of amount of information, and leaves the important question of the structure of information for future study.

\(^3\)A full list of the attributes is available at https://www.apple.com/watch/guided-tours/, accessed in May 2015.
This paper questions whether there is an optimal “information load” in consumer decision-making, and whether such an optimal load varies with the type of consumer population being considered. These are important questions since as the Internet keeps lowering the cost of information transmission, marketers may want to have a good understanding of whether and when information overload is likely to occur, and what are the factors that determine the optimal information load.

We highlight in this paper the fact that there may be costs in processing information that can make more information availability not necessarily beneficial for decision-making. For information to be used by consumers it needs to be available and processed, which may involve costs. The paper formalizes the existence of these costs of processing information, in the presence of limited information availability. The consumers consider the amount of information available to search through and decide on how much information to costly process until making a decision to either stop searching and purchasing the product, or to stop searching without purchasing the product.

The information that the seller makes available is on product attributes that have different importance on the fit of the product with the consumer. Information available on more attributes means that more information is being provided. Suppose that the seller, given the amount of information provided, wants to provide information on the most important attributes. However, the consumer cannot perfectly control the order in which attributes are checked. This yields that when information on more attributes that is provided by a seller, the average importance of an attribute processed by a consumer becomes lower. If the seller provides too much information, the average importance of each attribute processed may be too low. The consumer may be more likely to decide not to start searching in this case, or, even if the consumer starts searching, he is likely to choose not to purchase the product. This can be seen as a case of too much information hurting consumer choice – an information overload effect.

We describe the search process in which a consumer sequentially searches for product information. In particular, we capture the consumer’s belief updating process with a Brownian motion, which serves as a continuous-time analog to the simple random walk of new information being obtained through search. In the real world consumers are constantly adjusting the beliefs of their expected valuation of products as they process new information. At each step of the search process, the consumer trades off search costs with the likelihood of getting useful product information, which may eventually lead to a purchase or to stop the search process without purchase. We derive an

---

4Jacoby et al. (1974b) perform experiments using instant rice and prepared dinners, varying both the number of brands and the number of attributes per brand, and find that consumer choice accuracy first increases and later decreases as the total amount of available information increases. See Malhotra et al. (1982) for a critical review of early studies of the information overload effect. A related but different issue is whether a greater number of alternatives may lead to fewer and/or worse choices by consumers, choice overload. Jacoby et al. (1975) can also be seen as providing evidence on this effect. See also, for example, Iyengar and Lepper (2000), Kamenica (2008), Kuksov and Villas-Boas (2010).
endogenous stopping rule, which states that the consumer’s expected valuation of the product needs to reach an upper bound for him to purchase the product, and a lower bound for him to exit the market without a purchase. We refer to these bounds as the purchase and exit thresholds.\footnote{See Roberts and Weitzman (1981) and Branco et al. (2012) for similar models of search for information.}

Given the optimal search behavior, we investigate information provision from the seller’s point of view. We focus on the situation where the consumer would not buy the product unless he finds sufficient positive information during the search process – his initial expected valuation is less than the valuation of the no-purchase alternative which we set at zero. The seller tries to maximize the consumer’s purchase likelihood when deciding how much information to provide.

When the consumer knows that the information is provided only for a few attributes, the consumer knows that there is not much positive information to expect and hence would purchase the product as soon as his expected valuation of the product becomes slightly positive. When information is provided about too many attributes, on the other hand, the search process is overall less informative as each step of search yields less information. The consumer in this case is not motivated to search, and would also purchase the product upon a small positive expected valuation. Putting these two cases together, one can obtain under certain conditions that the purchase threshold approaches zero when the amount of product information goes to either zero or infinity. Similarly, the exit threshold approaches zero as well. Considering that the consumer starts with a negative expected valuation of the product, he would not purchase the product if no search occurs. Therefore, it is never optimal to have a zero exit (purchase) threshold. Therefore, in our model the seller never provides zero or an infinite amount of product information. As a result, we show that the optimal strategy for the seller is always to offer an intermediate amount of product information.

Interestingly, the optimal amount of information may increase with the consumer’s initial expected valuation of the product. When the valuation is rather negative, the seller provides information only on a few attributes, such that the average importance stays high. Each of these attributes has a good chance to significantly change the consumer’s valuation of the product. The consumer realizes this fact and is willing to initiate search. If the seller were to offer information on more attributes, the average informativeness of the attributes would decrease, and the negative consumer would simply lose interest. On the other hand, when the consumer’s valuation is only slightly negative, even marginally informative attributes can drive the valuation into the positive domain. The seller in this case provides information on many attributes as the consumer will be more likely to purchase the product.

We also compare the amount of information that maximizes the seller’s expected payoff with the amount that maximizes the consumer’s expected utility. We find that the former may be larger, i.e., the seller may find it optimal to provide more information than what is ideal for the consumer.
The intuition is that the seller only cares about the probability of purchase, while the consumer cares about the certitude with which he makes the right decision, which can be achieved with more informative search. This then also suggests that mandating full disclosure as a policy may potentially not be optimal for consumers that have information processing costs.\footnote{Another related practice is when lawyers are required to pass all relevant information to the opposing side in a court case, and choose to do a data dump, with large quantities of unrequested materials being supplied along with the items actually being sought. This data dump could be beyond any data necessary for the lawyers not to run afoul of the disclosure requirements, and to make search of relevant information more costly to the opposing party (e.g., Nelson and Simek 2010).}

The decision of the amount of information provided by the seller can be seen as the seller structuring the product information in two buckets of attributes: In one bucket of attributes the search cost for information is relatively low (the amount of information that the seller chooses to provide); in the other bucket, search is highly costly (infinite search costs in the limit).

Alternatively, we can also think of the seller structuring the information to be searched through more finely, such that the consumer can choose to some extent which attributes to search through among the ones that can be checked. Numerical analysis of this case illustrates that the amount of information that maximizes the probability of purchase remains relatively stable for a large range of the extent to which the consumer can choose which attributes to search through. The amount of information that maximizes the probability of purchase only starts to be greater when the consumer can choose which attributes to search almost perfectly. However, in that case, choosing a smaller number of attributes to provide information on has only a marginal effect on the probability of purchase.

In addition to the information provided by a firm, consumers in the real world may also have access to other information provided by third parties or by other consumers. If a firm does not have the ability to influence the amount of information that is available for consumers to search through, then the results here can be seen as illustrating how the probability of purchase and the expected payoff for a consumer searching for information is affected by the amount of information that is available to search through (however that amount of information is decided, as long as information is provided on the more important attributes). If a firm has some ability to influence the amount of information that is available to search through, the paper also characterizes how much information is optimal for the firm, and how this amount compares with the amount of information that would maximize the expected payoff for the consumers. If most of the information in the market about a product is provided and structured by third parties the results on the choice of information provided below do not apply, which is a limitation of the paper. To the extent that third parties may provide more information than the seller would choose and provide information on the more important attributes, the paper could be seen illustrating that under some conditions there may be too much information in the market in comparison to what both the seller and consumers would like.
Another dimension of search for information not considered here is the possibility of direct search by consumers in the internet (for example, Google search), on particular attributes. Not considering this direct search for attributes can be seen as a limitation of the paper. In terms of the model presented here, it could be seen that this direct search would be too costly in comparison to the non-direct search of just perusing the information provided by the seller. Another perspective in terms of the model could be that the most important attributes can be considered through direct search, determining the initial expected valuation, and that after those attributes are determined the consumer goes through the information provided by the seller as considered in this paper.

Related Literature

This paper could be seen as providing a formal treatment of the effects of information overload presented in the literature (e.g., Jacoby et al., 1974a, Jacoby 1977). The effects focussed on here regard the possibility of greater information on an alternative (information on a greater number of attributes) potentially leading to less choice and poorer decision quality, while keeping the choice set fixed. Note that Russo (1974) argues that the Jacoby et al. (1974b) data actually lead to the conclusion that consumers wanted and benefitted from more information.\(^7\) This paper can be seen as providing conditions under which we may observe information overload, and conditions under which information overload may not be present.

Branco et al. (2012) also study information gathering within a fixed choice set. That paper focuses, however, on consumer’s optimal search instead of the seller’s information provision strategy. A key difference between that paper and the current one is that all attributes in Branco et al. (2012) carry the same weight in the consumer’s expected utility function. In this paper, we focus on the tradeoff between the quality and quantity of information by allowing the attributes to carry different weights, and endogeneizing the seller’s decision of how much information to provide.

A different, but related issue, is how consumers make their decisions when the choice set is enlarged. It has been argued that if the choice set includes too many alternatives, consumers may prefer not to choose, or make poorer selections – a choice overload effect. Jacoby et al. (1974a) and Jacoby (1977) mentioned above also discuss this effect. Iyengar and Lepper (2000) present results of several experiments documenting this effect.\(^8\) Some work has considered formalizations of this effect, such as Kamenica (2008) and Kuksov and Villas-Boas (2010). Kamenica (2008) considers a firm that is better informed than some consumers about which products are most popular. By offering a smaller set of (the most popular) products, the uninformed consumers are more likely to purchase (at random) a more popular product. Kuksov and Villas-Boas (2010) consider sequential consumer search for alternatives, and show that with a strategic supplier of alternatives a smaller

\(^7\)See also Wilkie (1974) for another critical discussion of the Jacoby et al. (1974b) conclusions.

\(^8\)See also Scheibehenne et al. (2010) for a meta-analytic review of when choice overload may exist.
number of alternatives can benefit consumers by lowering consumer search costs, and allowing consumers to find a product with a reasonably good fit. Other related work is presented in Van Zandt (2004), Norwood (2006), and Anderson and de Palma (2009).\footnote{Van Zandt (2004) considers competition where firms communicate about their products and consumers evaluate a limited and fixed number of alternatives, and finds that there is too much communication in equilibrium, as a firm communicating about its product does not consider the negative externality on consumer information processing that affects the other firms (see also Carlin and Ederer 2014 on the role of search fatigue). Norwood (2006) considers free-entry price competition among fixed products that are vertically differentiated, one product per firm, under the assumption that only the most popular products are offered, and includes an approximation to the consumer sequential evaluation process. Anderson and de Palma (2009) consider costly advertising messages by heterogenous senders, where receivers supply attention according to the average message benefit, and where the marginal sender determines the extent of information congestion. Note also that the information overload on alternatives may be seen as leading to search obfuscation of the benefits of alternative products (e.g., Ellison and Ellison 2009, Ellison and Wolitzky 2012).}

Another related issue not considered here is that some attributes of a product may be easier to evaluate than others. Bar-Isaac et al. (2012b) investigate this effect and find that lower search costs on a product dimension may lead firms to invest more in quality in that dimension, yielding possibly worse products overall.\footnote{Other effects not considered here are that changes in search costs may affect the product design that is available in the market (e.g., Kuksov 2004, Bar-Isaac et al. 2012a), or that the seller may actively initiate which information is revealed (e.g., Bharwaj et al., 2008). See also Bergemann and Wambach (2013) for the issue of sequential information disclosure in auctions.} For seller information provision without costly evaluation by consumers see for example Lewis and Sappington (1994), Anderson and Renault (2006), Johnson and Myatt (2006) and Kamenica and Gentzkow (2011). In that literature, there is a convexity result that the seller wants to disclose either full or no information, which is moderated here by the costly evaluation costs (with the focus on the case that some information is needed for the consumer to be willing to purchase the product). In the example of a seller supplying product information in Kamenica and Gentzkow one can also obtain that the social optimal amount of information to provide may be different than the one that maximizes the expected number of purchases.

One can also think of the seller’s information provision as reducing the search costs of the information provided, with the possibility of consumers searching for additional information with higher search costs. Mayzlin and Shin (2011) can be seen as studying this issue where advertising provides information at zero search costs, and where how much is being advertising can signal about the quality level. Mayzlin and Shin find that high quality and low quality firms can pool together by choosing uninformative advertising whereas the medium quality firm chooses informative advertising. The high quality firms do not provide information in order to encourage the consumers to search on their own and discover additional positive information. This paper focuses on the sequential search costs of information provided, shutting down the signaling effect of the amount of information provided by using horizontal differentiation, in order to focus on the tradeoff between the quantity and the quality of the information that the seller provides.
The remainder of the paper is organized as follows. In the next section we present a basic model of consumer search over product attributes. We characterize the optimal sequential consumer search in Section 3, and discuss the optimal provision of information in Section 4. Section 5 considers the case when consumers or firms can choose in some way the order in which attributes are checked, among the attributes available to check. Section 6 presents concluding remarks.

2. The Model

The Product. Consider the problem of a consumer searching for information on a product to decide whether to purchase it. Suppose that the utility of the product for the consumer is composed of the consumer’s initial expected valuation of the product, $v$, which reflects his prior knowledge of the product and any expectations about the value of the different attributes, plus the changes in utility that come from new possible information on the product attributes, $x_i$, $U = v + \sum_{i=1}^{N} x_i$, where $i$ is the index for product attribute, and $N$ is the number of all possible attributes. The seller may choose not to provide information on all $N$ attributes as discussed below. To focus our analysis on the case where the consumer would not purchase the product without any information search, we assume that $v < 0$ (initially, the expected valuation of the product is lower than the one of the outside option). Before searching attribute $i$ the consumer does not know the value of $x_i$. Suppose $x_i$ can take the value of $+z_i$ or $-z_i$ with equal probability, independent across $i$. That is, the realization of each product attribute can either increase or decrease the consumer’s expected utility of the product. The higher is $z_i$, the more does attribute $i$ change the consumer’s expected valuation, and the more informative is the attribute. By searching an additional attribute, a consumer pays some search cost $c$ and learns the realization of that attribute. The search cost can be seen as the cost of processing information on that attribute. After searching a set $n$ of attributes the expected utility of buying the product is $u = v + \sum_{j \in n} x_j + \sum_{j \notin n} E(x_j) = v + \sum_{j \in n} x_j$.

By checking an extra attribute the expected utility changes according to a binomial process that goes up or down by $z_i$ with equal probability. After checking a set $n$ of attributes, the consumer has to decide among searching for information on more attributes, stopping the search with purchase of the product, or stopping the search without purchase.

The Seller. The seller knows the importance of the different attributes through prior market research and can decide on which attributes to provide information on. The seller chooses the amount of information to provide, $T$, the mass of attributes over which she provides information on, but cannot perfectly order the way in which the consumers search through attributes. This captures the idea that consumers are able to observe the amount of information available as a whole, $T$, but cannot see its structure without incurring evaluation/search costs.

Given that consumers are assumed not to purchase if they cannot find any information (we assume $v < 0$), for a given level of total information $T$, the seller wants to make available information
on the most important attributes, attributes with a greater $z_i$, so that the consumer is more likely to get to the point where $u$ is sufficiently positive.\footnote{See Ottaviani and Pratt (2001) and Johnson and Myatt (2006) for a similar point without consumer search or constraints on information availability.} And the consumers know that information is being provided on the $T$ most important attributes. We then obtain that if the seller provides an amount of information $T$, the average importance of the attributes available can be represented as a function of $T$. Because the performance of any attribute can either be positive or negative for any given consumer the amount of information $T$ does not signal the quality of the product. As the performance of attributes is independent across attributes, having information on some attributes does not also give any information about the other attributes. This allows us to focus on the effects of the amount of information to be processed on choice, without having a signaling effect of the amount of information. By observing the amount information $T$ the only thing that consumers can infer is that they have information that they can check on the $T$ most important attributes.

**The Consumer** Consumers check one attribute at a time, incurring a search cost per attribute checked, and after each evaluation decide whether to continue to evaluate one more attribute, or to stop search and make a decision whether to buy or not buy the product. Consumers know the importance of the different attributes prior to search (made or not made available by the firm), but they cannot order the attributes by importance during search. That means that if during the search process an important attribute is not revealed, a consumer has an incentive to keep looking. An alternative assumption of what the consumer knows prior to search that leads to the same analysis and results is that the consumer do not know the importance of the attributes prior to search and have to learn it during search, which fits product scenarios where the consumer is relatively new to the product category and does not yet have a clear understanding of what are the more important attributes. Under either assumption, the consumer’s belief updates according to the average informativeness of the search process, leading to the same results on the seller’s optimal information provision. For many electronic items, it does appear to be the case that the products go through significant changes over time and that consumers may be learning about the importance of attributes during the search process (potentially with the help of consumer reviews; for example, a consumer learning about new attributes in a new Apple Watch).

In Section 5, we consider numerically the case in which, among the attributes available to check, the consumer is more likely to search first the attributes with greater importance. As long as the order of attributes checked is not guaranteed to be in the order of importance with probability one, the main messages of completely random search continue to follow. The assumption fits the idea that the seller cannot perfectly control the order (of attributes) in which the consumer receives product information and reflects the fact that product information often gets shared in a manner
that cannot be fully controlled by the seller. Even when the consumer concentrates his search effort on information that directly comes from the seller such as the product’s packaging or the product’s official webpage, there is no guarantee that the consumer would read and process information in any particular order. In essence, our results would hold as long as there is a greater search cost of finding a particular attribute the greater the number of attributes that information is provided on: In the example above, it is easier for the consumer to find information about megapixels when megapixels is one of 10 attributes available then when megapixels is one of 100 attributes available.

We capture the essential aspect of the existence of some randomness in the search for information by focusing in this basic case on the extreme case in which the order of attributes checked in the consumer’s information search process is independent of its importance.

**Continuous Approximation.** Imagine now that each attribute $i$ is divided into $k$ sub-attributes, such that if the importance of the attribute $i$ was $z_i$ the importance of each of the sub-attributes is $z_i/\sqrt{k}$. Moreover, if the search cost of evaluating any of the previous attributes was $c$, the search cost of evaluating each sub-attribute is $c/k$. When checking a sub-attribute a consumer pays a search $c/k$, and his expected utility $u$ changes in either $\Delta u = +z_i/\sqrt{k}$ or $\Delta u = -z_i/\sqrt{k}$, with equal probability, which describes the process of the expected utility $u$. In this way the total search costs of evaluating all the $k$ sub-attributes $i$ would be $c$, the same as when checking attribute $i$ and the attributes were not divided in sub-attributes, and the variance of the change in the expected valuation after evaluating all the $k$ sub-attributes $i$ would be $z_i^2$, the same as when checking attribute $i$ and the attributes were not divided in sub-attributes. When checking randomly one attribute from different attributes $i$ in the information available $T$, or checking $k$ sub-attributes from the different sub-attributes in the information available $T$, we would have a variance of the change in the expected valuation of $\sum_{i=1}^{T} z_i^2$. Similarly, when checking randomly a mass $t$ of sub-attributes (that is, $kt$ sub-attributes), the variance of the change in the expected valuation is $t \sum_{i=1}^{T} z_i^2$.

When $k \to \infty$, we have that for any positive $t$, the change in the expected valuation is a sum of an infinitely large number of independent random variables (independent binomials), and we know by the law of large numbers that the change of expected valuation has then a normal distribution. As changes in the expected valuation are independent across attributes checked, we then have that as $k \to \infty$, these small steps become a continuum and the expected valuation process converges to the Brownian motion since it has stationary and independent increments (see Cox et al. 1979, as an example of using this property in the valuation of options with infinitely many binomials). That is, $du = \sigma_T dw$, where $w$ is a standardized Brownian motion, and $\sigma_T$ is the instantaneous

---

12With different consumers caring about different attributes this prioritization becomes also less obvious. For an example of prioritization in search for price in a competitive market see, for example, Armstrong et al. (2009). In order to focus on the role of the amount of information in search, we do not consider the order of attributes to be a choice by the firm. At the end of Section 4 we discuss what happens when price is endogenous and can be observed prior to any search.
standard deviation of the Brownian motion. By construction, we have \( \sigma_{T}^2 = \sum_{i=1}^{T} \frac{z_i^2}{T} \). In summary, as the number of sub-attributes \( k \) goes to infinity, the beliefs of the consumers of the expected utility of the product evolve exactly as a Brownian motion as the consumers search through the different attributes and find out about the fit of each product attribute. While the movement of the expected valuation is purely random from the point of view of the consumer, consumers are strategic in that they can optimize over whether to initiate the process and when to terminate the process.

In this setting, allow now the mass of attributes \( N \) to be infinity, and define the informativeness of the continuous attribute \( i \) as \( \sigma_i \), ordering the attributes in the order of importance such that \( \sigma_i \) is decreasing in \( i \). We also assume that \( \sigma_i \) is continuous in \( i \), that \( \lim_{i \to \infty} \sigma_i = 0 \), and that the total information available is finite, \( \int_{0}^{\infty} \sigma_i^2 \, di < \infty \). Given a mass \( T \) of information made available we can obtain \( \overline{\sigma}_{T}^2 \) as the average of \( \sigma_i^2 \) for \( i \in [0, T] \), \( \overline{\sigma}_{T}^2 = \frac{1}{T} \int_{0}^{T} \sigma_i^2 \, di \). Three properties of the information gathering process are noteworthy. First, the average informativeness, \( \overline{\sigma}_{T}^2 \), decreases with the amount of information made available by the seller, \( T \). Second, the expected total amount of available information, \( T \overline{\sigma}_{T}^2 \), increases with \( T \). Third, the average informativeness, \( \overline{\sigma}_{T}^2 \), converges to zero as \( T \) approaches infinity, and it converges to its maximum, \( \sigma_0^2 \), when \( T \) approaches zero.

**Stationarity.** One approach is to consider the mass of attributes with \( i \in [t_1, t_2] \), for any \( t_1 \) and \( t_2 \), to be \( t_2 - t_1 \). In that case the problem of optimal search of a consumer given the amount of information \( T \) would be non-stationary, as there are fewer attributes to check as a consumer checks more attributes. This case is considered in Section 5 below, where we also allow for the more important attributes to have a greater probability of being checked earlier. For greater tractability and to obtain sharper results, consider the setting in which the mass of attributes \( i \in [t_1, t_2] \), for any \( t_1 \) and \( t_2 \) is \( \varepsilon(t_2 - t_1) \) where \( \varepsilon \), unknown to the firm and consumers, has expected value of one, and is distributed with an exponential cumulative probability distribution, \( 1 - e^{-\varepsilon} \). The firm decides the top \( T \) attributes on which she will provide information on, and this results in a mass of \( \varepsilon T \) total attributes that the consumer can check. That is, the seller cannot choose exactly the amount of information available but can choose the expected total amount of available information, \( T \). This imposes a limitation on the model as the firm does not have full control over the information that it chooses to disseminate (as noted above, the case without this assumption is presented in Section 5). With \( \varepsilon \) having an exponential distribution, the cumulation distribution of the total number of attributes available is also exponential with parameter \( T \), \( 1 - e^{-T\varepsilon} \), which means that there is a constant hazard rate of the consumer running out of attributes to check. That is, a consumer in the search process runs out of attributes to check with a constant hazard rate \( \frac{1}{T} \). During the search process the average informativeness of the attributes checked continues to be \( \overline{\sigma}_{T}^2 = \frac{1}{T} \int_{0}^{T} \sigma_i^2 \, di \). The setup allows for the problem to remain stationary as the consumer searches through attributes,
3. OPTIMAL CONSUMER SEARCH

Let us now characterize the optimal sequential search for information by a consumer. In order to provide such a characterization we have to understand how the consumer’s expected valuation of the option to search changes through the search process. Denote $c$ as the consumer’s search cost per attribute. The consumer’s expected utility of continuing to search the next infinitesimal amount of attributes, $dt$, can be written as

$$ V(u,t) = -c \, dt + \frac{dt}{T} \max[u,0] + (1 - \frac{dt}{T})EV(u + du,t + dt), \quad (1) $$

where $t$ is the number of attributes already searched, $c > 0$ is the search cost per attribute which is incurred after deciding to check the next infinitesimal amount of attributes $dt$, and $u$ is the consumer’s utility if he purchases the product. With probability $\frac{dt}{T}$ the search process runs out exogenously of attributes to check and the consumer has to choose between purchasing the product and getting $u$, or not purchasing and getting zero. This is represented by the second term in (1). With probability $(1 - \frac{dt}{T})$ the consumer does not run out of attributes to check, and after checking $dt$ attributes gets $EV(u + du,t + dt)$. This possibility is represented by the third term in (1).

By a Taylor expansion (see, e.g., Dixit 1993), valid to terms of the first order and less in $dt$, we have $V(u,t) = -c \, dt + \frac{dt}{T} \max[u,0] + (1 - \frac{dt}{T})[V(u,t) + V_u E(du) + V_t dt + \frac{1}{2}V_{uu} E[(du)^2] + V_{ut} E(du) \, dt]$ where $V_u$ is the partial derivative of $V(u,t)$ with respect to $u$, $V_t$ is the partial derivative with respect to $t$, $V_{uu}$ is the second derivative with respect to $u$, and $V_{ut}$ is the cross derivative with respect to $u$ and $t$. By definition, $E(du) = 0$, and $E[(du)^2] = \sigma^2 \, dt$. Also, with a constant hazard rate, the problem of the consumer does not change with the number of attributes already searched: $V_t = 0$. Therefore, $V$ is only a function of $u$. Since $V(u,t)$ is independent of $t$ we then write $V(u,t)$ as $V(u)$. We can hence divide the equation above by $dt$ and get $-cT + \max[u,0] - V + \frac{\sigma^2}{2}V_{uu} = 0$.

We also have a set of conditions for $V(u)$ for when the consumer decides to stop the search process. When $u$ is large enough such that the consumer is indifferent between continuing the search process and stopping the search with a purchase, $V(\overline{U}) = \overline{U}$ and $V_u(\overline{U}) = 1$, where $\overline{U}$ is the purchase threshold such that when $u$ reaches $\overline{U}$ the consumer buys the product. The condition $V(\overline{U}) = \overline{U}$ means that when the expected utility reaches $u = \overline{U}$ the consumer chooses to purchase the product and gets expected utility $\overline{U}$.\textsuperscript{14} When $u$ is low enough such that when $u$ reaches $\underline{U}$ the

\textsuperscript{13}This is also in the spirit of approximating the value of an American put option with random termination times (e.g., Carr 1998).

\textsuperscript{14}The condition $V_u(\overline{U}) = 1$ is known as the “high contact” or “smooth pasting” condition (e.g., Dumas 1991, Dixit 1993). The condition is implied by the fact that the consumer maximizes $V(u,t)$ for all $(u,t)$. Essentially, the condition states that when the consumer’s expected utility, $u$, walks away from $\overline{U}$, in order for him to be indifferent between continuing to search and purchasing right away, the marginal decrease in $V(u)$ when $u$ walks to the left
consumer exists the market without a purchase, we have \( V(\overline{U}) = 0 \) and \( V_u(\overline{U}) = 0 \). These conditions are similar to the conditions above for \( V(U) \). The condition \( V(\overline{U}) = 0 \) indicates that at \( u = \overline{U} \), the consumer expects zero utility from searching. The condition \( V_u(\overline{U}) = 0 \) indicates that if \( u \) increased as the consumer continues to search, the change of \( V(u) \) will be slow. Finally, by the smoothness of \( V \) at \( u = 0 \), we have \( V(0^+ + du) = V(0^- + du) \) and \( V_u(0^+ + du) = V_u(0^- + du) \). The condition \( V(0^+ + du) = V(0^- + du) \) follows directly from (1) at \( u = 0 \) or \( u = 0^- \). The condition \( V_u(0^+ + du) = V_u(0^- + du) \) follows by taking a first order Taylor approximation of (1) at \( u = 0 \) and noting that \( du \) has a symmetric distribution, and that \( EV(0 + du) = \frac{1}{2}[V(0) + E(du|du > 0)V_u(0)] + \frac{1}{2}[V(0) - E(du|du > 0)V_u(0)] \).

Putting all conditions together, we have the following lemma that summarizes how the consumer optimally searches for product information (all proofs are available in the online Appendix).

**Lemma 1:** The consumer purchases the product if either \( u \geq \overline{U} \), or if he runs out of attributes to check and \( u \geq 0 \). The consumer stops search without purchasing the product if either \( u \leq \underline{U} \), or if he runs out of attributes to check and \( u < 0 \). The consumer keeps searching otherwise. The search stopping boundaries are given by \( \overline{U} = \sqrt{T \sigma^2 T} \log \left[ \sqrt{\frac{\sigma^2 T}{8c}} + \sqrt{1 + \frac{\sigma^2 T}{8c}} \right] \) and \( \underline{U} = -\overline{U} \).

Lemma 1 indicates that the stopping boundaries end up being symmetric around zero. The symmetry results from the symmetry of the normal distribution, and goes away if there is discounting of payoffs after checking more attributes. More interestingly, the purchase threshold for the product’s expected valuation is strictly positive while the exit threshold is strictly negative. The intuition is that for each level of the expected product valuation \( u \), the consumer trades off the possibility of purchasing or exiting with the search costs that he expects to spend. When the expected valuation is high enough, the consumer would have to check many attributes and incur a large amount of search costs for the expected utility to walk back to being negative. Therefore, the consumer would simply stop search and purchase the product. Similarly, when the expected utility is low enough, the consumer would also have to check many attributes and incur a large amount of search costs for the expected utility to walk back to being positive. Therefore, he decides to stop the search without buying the product.

It is straightforward to see that the purchase threshold, \( \overline{U} \), decreases with the search cost \( c \). This result is intuitive: when it becomes more costly for the consumer to search for product information, the purchase threshold is lower. Also, the purchase threshold increases with the average informativeness of searchable attributes, \( \overline{\sigma}_T^2 \). Intuitively, when the search process is more informative, the consumer becomes more motivated to search, hence the purchase threshold rises.

The expected number of searchable attributes, \( T \), enters the purchase threshold both directly and indirectly through \( \overline{\sigma}_T^2 \). One can obtain that for a sufficiently low expected number of searchable would have to equal the marginal increase in \( V(u) \) when \( u \) walks to the right.
attributes $T$ the purchase threshold increases with $T$ and then it decreases when the expected number of searchable attributes $T$ is high enough. In fact, one can show that the purchase threshold is small when $T$ is either very big or very small. The purchase threshold converges to zero when the seller chooses an expected number of searchable attributes that is either very small or very large. When $T$ is small, the hazard rate at which the search process terminates exogenously is high. The consumer can barely search any attribute, and therefore needs just a small amount of positive information to make a purchase. When $T$ is large, the average informativeness of searchable attributes, $\sigma^2_T$, goes to zero. Given this, the consumer needs to check many attributes and incur substantial search costs in order for his expected utility to move up or down. As a result, the consumer also requires little positive information to make a purchase. Consider now the probability of purchase given the expected valuation of the product. Lemma 2 presents the result.

**Lemma 2:** Given any purchase threshold $\bar{U} > 0$, the likelihood of purchase is $P(u, T) = 0$ if $u < -\bar{U}$, it is $P(u, T) = \frac{1}{2}e^{2\alpha u} - \frac{1}{2}e^{-\alpha u}$, where $\alpha = \sqrt{\frac{2}{1/T^2}}$, if $u \in [-\bar{U}, 0]$, it is $P(u, T) = 1 - P(-u, T)$ if $u \in (0, \bar{U}]$, and it is $P(u, T) = 1$ if $u > \bar{U}$.

From Lemma 2 one can obtain that, as expected, the likelihood of purchase increases with the initial expected valuation $v$ for $v < 0$. One can also obtain that, other things being equal, the likelihood of purchase increases with the purchase threshold for $v < 0$. To gain intuition on this result, consider two extreme cases. When the purchase threshold is high, the search region is large. The initial negative valuation gets washed out in the search process and the purchase likelihood goes to .5 in the limit. On the other hand, when the purchase threshold is at its minimum, zero, the exit threshold is also zero. The consumer does not do any search, and he would not purchase the product as the initial valuation is negative.\(^{15}\)

Note also that while the purchase likelihood increases with lower search costs for $v < 0$, the purchase likelihood decreases with lower search costs for $v > 0$ (which is assumed away throughout the paper). Intuitively, when facing a consumer who already likes the product, it would in fact help the seller if it is more costly for the consumer to gather additional information about the product.

4. Too Much Information?

Taking into account the optimal search behavior of consumers we can now investigate what is the optimal amount of information to be provided by the seller or how much information should be provided from a social planner’s point of view. Consider first the problem of the seller whose objective is to maximize her demand, which in the model is the same as maximizing the consumer’s purchase probability $P(v, T)$ as a function of the amount of information provided $T$, given the

\(^{15}\)Note that for $u < 0$ the probability of purchase decreases in the search costs $c$.  


consumer’s initial expected valuation $v$. The following proposition states the main result that there is an optimal amount of information to provide.

**Proposition 1:** For any initial valuation $v < 0$, if there exists some amount of information provided $T$ such that $P(v, T) > 0$, then the optimal amount of information $T^*$ that maximizes the purchase likelihood is interior (i.e., $0 < T^* < \infty$).

Proposition 1 provides the important implication that a seller who aims to maximize the consumer’s purchase likelihood would provide neither too much nor too little information. This result is consistent with the information overload effect that too much information can lead a consumer not to choose to purchase a product. The intuition for this effect is as follows. When the amount of information provided $T$ is big, the consumer has too many attributes to check and updates his expected utility slowly, as the average informativeness of these attributes is low. In order to find attributes that are very important the consumer might have to incur search (evaluation) costs of checking attributes that have limited importance. When the amount of information provided $T$ is small, the consumer simply does not have enough attributes to check. In both cases, the purchase threshold approaches zero, and the purchase likelihood becomes zero. Therefore, the optimal strategy for the seller is to trade off the number of available attributes with the average informativeness of these attributes by selecting an intermediate level of $T$.

The condition of $v < 0$, as noted above, is just because for consumers with $v > 0$ the seller is guaranteed selling if no information (or maximum information such that $U = 0$) is provided, hence for such consumers there is no benefit for the seller to provide information. The condition that there is a $T$ such that $P(v, T) > 0$, is just to rule out the cases of $v$ being so low that for any level of the amount of information provided we cannot have $P(v, T) > 0$. This occurs when $v \leq \min_T U$.

Another interesting question is what happens to the optimal $T$ as the initial expected valuation $v$ varies. It is also interesting to see the relationship of the optimal $T$ with the $T$ that maximizes $U$. We formalize these properties of the optimal $T$ in the next proposition.

**Proposition 2:** For any initial valuation $v < 0$, if there exists some amount of information provided $T$ such that $P(v, T) > 0$, then the optimal amount of information $T^*(v)$ that maximizes the purchase likelihood increases with $v$ and is greater than $\arg\max_T U$. In equilibrium, the average informativeness of the search process, $\sigma_{T^*}^2$, decreases with $v$.

The condition in the proposition again imposes the requirement that some search has to take place: the initial expected valuation has to be higher than the lowest possible exit threshold. If this condition is not met, the consumer would exit right away without purchasing the product, regardless of how much product information the seller provides.

Proposition 2 provides an answer to the question of how the seller’s optimal information provision strategy should change with the consumer’s type. It states that when the consumer’s initial expected
valuation of the product becomes more favorable (less negative), the seller should provide more attributes for search, and hence provide a higher amount of total information. As noted in the proposition, $T^*(v)$ is not defined when $v$ is below a certain threshold, which is the opposite of the highest $U$. When $v$ is below this threshold there is no amount of information that can make a consumer engage in the search process.\footnote{Note that the optimal $T^*(v)$ is discontinuous at $v = 0$. For any $v > 0$ the optimal amount of information is infinity or zero, as in either case $U = 0$, and the consumer purchases with probability one.}

When the initial expected valuation is greater, it becomes more important to be sure that the consumer does not run out of attributes to check as he is more likely to purchase the product. Furthermore, a consumer with a greater initial expected valuation is more likely to end up purchasing the product, and therefore can tolerate a lower average informativeness of the attributes checked, lower $\sigma^2_T$. When the consumer is almost indifferent between buying and not buying the product initially (a near-zero initial valuation), the seller’s optimal strategy is to make a large number of attributes available for search as, in this case, the consumer is more motivated to search, and additional information has a high probability of changing his ultimate purchase decision. Since the initial valuation is already close to zero, little positive information is required to trigger a purchase.

This result has important marketing implications. It suggests that a seller should customize the amount of information provided based on her knowledge and assessment of the consumer’s prior attitude toward the product. One application of this could be targeted advertising. Contrary to the intuition that a seller may want to provide less information to consumers with more favorable prior expected valuation toward the product, our results suggest that the opposite could be better to some degree: the seller may want to provide more information to such consumers (if their expected utility is not too high). The key intuition here is that with more product information (higher quantity), the average informativeness decreases (lower quality of information) and the consumer with a prior favorable attitude is less likely to drastically change his expected utility in the course of product research. In other words, if the seller finds it optimal to provide a lot of low-quality product information, it is better to do it with consumers who have had a good impression of the product to begin with, as the other consumers would give up initiating the search process altogether. For those consumers with a less favorable prior attitude toward the product, the seller can increase the quality of information by focusing on only the most important attributes.

Additionally, firms may benefit in investing in raising the initial expected valuation $v$, such that the consumers just need to find some positive information in order to decide to make the purchase. For example, the hype created before the launch of the Apple Watch could be seen in terms of the model as raising the initial expected valuation $v$ such that only some limited positive information on the watch (for example, that the watch taps when a text message arrives) may lead the consumer to make the purchase.
Another important result in the proposition is that the optimal amount of information to provide is greater than the one that maximizes the purchase threshold $U$. This is contrary to the intuitive notion that the two amounts should coincide as the purchase likelihood increases with the purchase threshold. The reason is that, in determining the amount of information that maximizes the purchase threshold, the seller mainly cares about maximizing the possibility that consumers would eventually purchase the product, conditional on the consumers keeping on checking. When the seller maximizes the probability of purchase she has to consider the additional possibility that the consumer may run out of attributes to check. To account for this effect the seller increases the amount of information provided.

It is interesting to investigate the amount of information $T$ that maximizes the expected utility of the consumer, $V(v, T)$. For similar reasons as above one can show that the amount of information that maximizes the expected utility of the consumer is finite. If too little information is provided for search, the consumer runs out of attributes to check. If the seller provides information on too many attributes, the average informativeness on each attribute is low, and the consumer has to incur too many search costs to find out whether the product is a good fit. It is also interesting to compare the amount of information that maximizes the probability of purchase (which can be seen as the amount of information that maximizes the seller’s profit) with the amount of information that maximizes the expected utility of the consumers (which can be seen as the consumer surplus, which in this case is equivalent to social welfare under some conditions). The following proposition presents a result on this comparison.

**Proposition 3:** Suppose that $v, c \to 0$. Then, the amount of information that maximizes consumer surplus is less than the amount of information that maximizes the probability of purchase. That is, 
$$\arg\max_T V(v, T) < \arg\max_T P(v, T).$$

To maximize the probability of purchase, the seller does not care about the degree of certitude by the consumer as to whether purchasing is the right decision. That is, the seller only cares that either $u$ reaches the purchase threshold (even if that purchase threshold is low), or that $u$ is slightly above zero, if the consumer runs out of attributes to search over. On the other hand, the consumer wants to make sure that, when making the purchase decision, it is the right choice, which means that the consumer prefers to purchase when $u$ is high. This happens when the purchase threshold is relatively high, which can be obtained by reducing the amount of information provided. Therefore, the amount of information that maximizes consumer surplus ends up being lower than the amount of information that maximizes the probability of purchase.$^{17}$

If we interpret the case of maximization of the probability of purchase as the market outcome (seller maximizing her expected profit), then the proposition can be viewed as saying that in the

---

$^{17}$The result in the proposition is for $v, c \to 0$ but we could not find examples where the result does not hold for different levels of $v$ and $c$. 
Combined with the other results, Proposition 3 suggests that when the seller starts to reduce information from a large amount to a small amount, three phases occur. At the beginning ($T \geq \arg\max_T P(v, T)$), the information reduction benefits both the consumer and the seller. In the second phase ($\arg\max_T V(v, T) \leq T < \arg\max_T P(v, T)$), it benefits the consumer but not the seller. Only in the third phase when information gets really limited ($0 \leq T < \arg\max_T V(v, T)$) would further reduction hurt both the consumer and the seller.

The proposition can also be interpreted with probability of purchase being a measure of short-term benefits for the firm, and the expected customer surplus being a measure of long-term benefits for the firm of providing customer value. The proposition would then mean that the firm would provide a smaller amount of information when the long-term benefits become relatively more important in comparison to the short-term benefits.

5. Non-Perfectly Random Search

In this section we consider the case of the search for information not being perfectly random among the attributes available for search. That is, consumers may be able to search, to some extent, the more important attributes first. In order to study this case we consider first the case in which the number of attributes available to check is deterministic with perfectly random search among the available attributes, and then consider the non-perfectly random search of attributes.

**Deterministic Termination of Search.** Consider a deterministic environment in which the seller chooses the exact maximum number of attributes that the consumer could check, rather than a hazard rate of no more attributes being available for search. Suppose the seller chooses to offer $T$ attributes for the consumer to check. The average informativeness of the available attributes is represented by $\sigma_T^2$. Since the consumer can not search the attributes in a particular order, he learns about the product according to the average informativeness.

Note that Anderson and de Palma (2009), in a sender-receiver framework, also find that information may be congested when the cost of information provision is low and social welfare could be improved by taxing such provision. While our results are similar, we highlight the inherent tradeoff between the amount and quality of the information provided and abstract away from the cost of information provision.

The analysis above does not consider the impact of price in order to focus on the effects of the amount of information to search through. Including price in the analysis could be done by assuming that price is observed freely prior to any search, such that the initial expected net valuation of a consumer is $v = v' - p$, where $v'$ is the expected gross valuation, and $p$ is the price. The search problem of the consumer would then be similar to the analysis presented above, and the problem of the firm for zero production costs could then be seen as $\max_{p,T} P(v' - p, T)p$.

Per Proposition 2 note that given the same initial valuation $v'$, when the price is higher (lower $v$) then the optimal amount of information to provide is lower. We can also obtain the same results as in Propositions 1 and 2. The corresponding variation of Proposition 3 would now involve comparing optimal amounts of information with two different prices, one that maximizes the consumer surplus ($p = 0$) and the other that maximizes the expected profit. This change makes it hard to prove a general relationship. In an example with $\sigma_T^2 = e^{-i}$ and $c = .01$, however, the amount of information that maximizes consumer surplus is indeed lower than the profit-maximizing amount of information for $v < 0$. We provide further discussion for this case of an optimal price in the online Appendix.
The setup here is similar to the previous section except that the consumer knows exactly when he will run out of attributes to check. That is, the number of attributes already searched now enters into the optimal decision of the consumer. The value function of the consumer, $V(u, t)$, can now be represented as

$$-c + V_t + \frac{\sigma^2_t}{2} V_{uu} = 0,$$

with boundary conditions now also being a function of $t$. With the same intuition as above, the boundary conditions now become, for all $t \leq T$,

$$V(u(t), t) = 0, V_u(u(t), t) = 1, V_t(u(t), t) = 0, \text{ and } V(u, T) = \max[0, u].$$

It is not possible to analytically solve for the stopping boundaries in this case, but we can still show that the optimal number of product attributes for the seller to offer is interior, as stated in the next proposition.20

**Proposition 4:** Consider the deterministic termination of search case. For any initial valuation $v < 0$, if there exists some amount of information provided $T$ such at $P(v, T) > 0$, then the optimal amount of information $T^*$ that maximizes $P(v, T)$ is interior.

**Non-Perfectly Random Search of Attributes.** Consider now the case in which the consumer is able to search the more important attributes earlier with greater probability. This problem cannot be considered analytically, as the problem is non-stationary, and the boundary conditions are convex in one region and concave in another region. In this setting, when checking the first attributes, the purchase threshold can become convex as the consumer is far from running out of attributes to check, and the attributes checked decrease in importance on average. However, when checking the last attributes available to check, the purchase threshold is concave, as the purchase threshold has to go to zero relatively fast. This is further discussed below.

Let $\tilde{\sigma}^2_{tT}$ be the expected informativeness when checking the $t$-th attribute and there are $T$ attributes available to check. We assume $\tilde{\sigma}^2_{tT} = \gamma \sigma^2_t + (1 - \gamma) \tilde{\sigma}_T^2$, such that $\gamma \in [0, 1]$ is an index of how perfectly ordered the search over attributes is. When $\gamma = 0$, we are back in the situation above where $\tilde{\sigma}^2_{tT}$ is independent of $t$ and it is the average attribute informativeness of the attributes available to be checked; that is, any of the attributes available to be checked is checked at random. When $\gamma = 1$, the consumer is perfectly able to choose which attributes to check when, and chooses to check attributes in decreasing order of informativeness. When $\gamma \in (0, 1)$, the consumer has some ability to try to search first on the attributes which are most important, but is not able to do so perfectly. The greater the $\gamma$ the greater the ability of the consumer to search first information on the most important attributes. The conditions for the optimum are similar to the ones of the previous subsection except that now the informativeness of the attributes checked changes during the search process. The value function of the consumer, $V(u, t)$, can now be represented as

$$-c + V_t + \frac{\tilde{\sigma}^2_{tT}}{2} V_{uu} = 0,$$

with boundary conditions $V(U(t), t) = U(t), V(U(t), t) = 0, V_u(U(t), t) = 1, V_u(U(t), t) = 0, \text{ and } V(u, T) = \max[0, u].$

---

20 Simulations allow us also to obtain in this case that, for $v$ and $c$ close to zero, the amount of information $T$ that maximizes the probability of purchase is greater than the amount of information that maximizes the consumer expected utility (this result is obtained analytically for the random termination case in Section 4).
For the case of \( c = .01 \), and \( \sigma_i^2 = e^{-2i} \), Figure 1 presents the numerical simulation of the purchase and exit thresholds for different \( \gamma \) as a function of the number of attributes checked, when the number of attributes available to check is \( T = 20 \). Note that for each \( \gamma \in (0,1) \) each threshold has a region where it is convex and another region where it is concave. Consider, for example, the purchase threshold. For the first few attributes checked the threshold is convex while the threshold becomes concave after some point. In order to gain some intuition on the shape of this curve consider first the initial attributes being checked. When checking these initial attributes, the consumer is far away from running out of attributes to check, and therefore the shape of the threshold depends on how the relative importance of attributes evolves. With \( \gamma > 0 \), the importance of the attributes decreases at a decreasing rate, which then leads the purchase threshold to be convex (and the exit threshold to be concave) for these initial attributes checked. In other words, the thresholds get closer to zero as the importance of attributes checked decreases, and as the importance of attributes decreases faster for earlier than later attributes, the thresholds get closer to zero for earlier than later attributes, which means that the purchase threshold is convex, and the exit threshold is concave.

Figure 1: Purchase and exit thresholds with non-perfectly random search of attributes for \( \gamma = .9, .5, .1 \) for the case \( \sigma_i^2 = e^{-2i} \) and \( c = .01, T = 20 \). The two thresholds are symmetric around zero.

After checking some attributes, the consumer starts realizing that he may run out of attributes to check. That is, the consumer starts being less demanding on the expected utility to decide to purchase the product (or to exit the market without the purchase). As this effect gets stronger and stronger as the consumer approaches the number of attributes available to check, \( T \), the purchase threshold decreases at an increasing rate (and the exit threshold increases at an increasing rate), as
the number of attributes checked approaches its limit $T$. That is, for these later attributes checked the purchase threshold is concave, and the exit threshold is convex. Note also that as $\gamma$ increases the thresholds are more demanding for the initial attributes checked, and are less demanding for the later attributes checked. This reflects the fact that as $\gamma$ increases, the initial attributes checked are more informative, while the later attributes checked are less informative.

It is also interesting to check how the $T$ that maximizes the probability of purchase and the consumer’s expected payoff evolves as $\gamma$ changes. Figure 2 presents the numerical computation of this evolution for the same parametrization as above, and for the initial expected valuation $v$ close to zero. As in the results of the previous section we can obtain that the amount of information $T$ that maximizes the probability of purchase is finite for $\gamma \in [0, 1)$ and greater than the amount of information that maximizes the expected payoff of the consumer. More interestingly, the amount of information that maximizes the probability of purchase (or the one that maximizes the consumer’s expected payoff) is relatively stable and increasing over $\gamma$, only increasing at a fast rate when $\gamma \to 1$. That is, for this example, the optimal amount of information does not vary too much with the ability to check first the more important attributes, $\gamma$. Furthermore, we also find that for $\gamma = 1$, the probability of purchase and the consumer’s expected payoff evolves at a very slow rate beyond a limited amount of information (as the informativeness of attributes beyond that amount of information is lower and lower).

Figure 2: *Amount of information $T$ that maximizes the probability of purchase and the expected consumer payoff with non-perfectly random search of attributes as a function of $\gamma$ with $\sigma_i^2 = e^{-2i}$, and $c = .01$.*

6. Concluding Remarks

In this paper we study a seller who needs to determine the amount of product information to provide to her consumers. We show that the consumer’s optimal search rule is characterized by two
symmetric boundaries between which the consumer would keep searching. More importantly, by highlighting the tradeoff between the quantity and the quality of the information, we find that it is never optimal for the seller to provide the maximum amount of information. Instead, she finds it optimal to provide an intermediate level of information. Besides providing a theoretical explanation for the classic information overload effect, our results also suggest to the managers that too much product information may deter their consumers from initiating the product research process and hence lower their profit.

We also find that the optimal amount of product information should increase with the consumer’s initial valuation of the product. In other words, managers should provide more information to consumers who are almost ready to buy the product prior to search, and less (but high quality) information to those who have less favorable prior attitude. Moreover, the amount of information that a seller wants to provide is larger than what maximizes consumer surplus. That is, the market outcome may lead to a second layer of information overload with respect to what would be desirable from the consumer welfare point of view.

With the increasing volume of consumer generated content such as product reviews and the growing accessibility of popular press, expert opinions and retailer web sites, firms often do not have complete control of the amount of product information and how it is transmitted among consumers. The results presented here can be seen as capturing the effects of the ability of firms to influence the balance between the quantity and quality of the available information. In some cases these additional sources of information may also require higher search costs, than what is communicated by a firm. This greater amount of information available to search through can also be seen as generating information overload with respect to what both firms and consumers would prefer.

In future research, it would be interesting to investigate what happens when a seller can provide information on multiple products, or when competing sellers provide attribute information on their products. In a competitive market, two forces would come into play. First, when the consumer has many options to choose from, the outside option increases and his prior valuation of the focal product may decrease, which motivates the seller to decrease the total amount of information. Second, one may obtain in a competitive market that information overload would persist as firms do not internalize the evaluation costs to consumers (as in Van Zandt 2004).

REFERENCES


See Bhardwaj et al. (2008) for an interesting discussion on the importance of empowering the customers with choices on which information to receive from the firm.


Carlin, B.I., and F. Ederer (2014) “Search Fatigue,” *working paper*, University of California, Los Angeles, and Yale University.


ONLINE APPENDIX

A Two-Attribute Model

We present the following two-attribute model to present further intuition on the nature of the problem. Consider a seller whose product has only two attributes, attribute $A$ and attribute $B$. A consumer is interested in this product, with a commonly known prior expectation of its value $v < 0$, and can incur search cost $c > 0$ to check out each additional product attribute. The consumer would purchase the product only if he expects a positive utility from it.\textsuperscript{22}

The seller can choose to offer no information, offer information on the most important attribute, call it attribute $A$, or offer information on both attributes. Information on the most important attribute makes the consumer’s valuation of the product go either up or down by $z_A$ with equal probability; the other attribute, call it attribute $B$, makes the valuation go either up or down, with equal probability, by $z_B$, and $z_B < z_A$. Both the seller and the consumer know the values of $z_A$ and $z_B$, the amounts by which the expected valuation can go up or down when checking that attribute. As an example, for a consumer who is shopping for a printer online, the two most important attributes could be the size of the printer (attribute $A$) and the speed of printing (attribute $B$).

When the seller offers no information, the consumer exits without buying the product as his initial valuation is negative. When the seller offers information on only one attribute, the seller offers information on the more important attribute, and the consumer would search for this information as long as the expected benefit from search exceeds the cost: $\frac{1}{2}(v + z_A) > c$. Upon search, the probability that he buys the product is $1/2$.

When the seller offers information on both attributes, the consumer faces a more complicated decision. Suppose that both attributes can affect the purchase decision: (1) $v + z_A + z_B > 0$ (if both attributes are positive, the consumer buys the product), and (2) $v + z_A - z_B < 0$ (if only the less important attribute is negative, the consumer does not buy the product). Consider also the case in which $v + z_A > 0 > v + z_B$, such that if information only on the more important attribute is obtained, and that is positive, then the consumer would prefer buying to not buying the product, but that is not the case for the less important attribute.

Suppose that prior to search the consumer knows which attributes are the most important but cannot perfectly order the search of the attributes by their importance.\textsuperscript{23} In other words, when information on both attributes is provided on the product page, there is no guarantee that the consumer would see attribute $A$ first. If the consumer first sees attribute $A$ and obtains a negative value (e.g., the printer is larger than what is ideal for him), the consumer chooses not to buy and stops searching. If he obtains a positive value then he chooses to buy and not to check the other attribute for more information if $v + z_A > \frac{v + z_A + z_B}{2} - c$, which reduces to $c > \frac{z_B - z_A - v}{2}$. Otherwise, the consumer checks the next attribute and buys only if he obtains a positive value on that attribute.

If the consumer sees attribute $B$ first and obtains a negative value (e.g., the printing speed is slower than what is ideal for him), the consumer chooses not to buy and stops searching. If a positive value is obtained the consumer searches again if $\frac{1}{2}(v + z_B + z_A) > \frac{1}{2}(v + z_A) > c$, buying the product if a positive value is obtained also in attribute $A$.

We can therefore obtain that if $\frac{z_B - z_A - v}{2} < c < \frac{v + z_A}{2}$ then the expected value of the first search when two attributes are available is $-c + \frac{1}{2}(v + z_A) + \frac{1}{2}(-c + \frac{1}{2}(v + z_A + z_B)) = -\frac{v}{2}c + \frac{3}{2}(v + z_A) + \frac{3}{8}z_B$. Under

\textsuperscript{22}In the presentation of this illustrative example we restrict attention to the cases that are important in the more general model.

\textsuperscript{23}Our analysis remains the same with the alternative assumptions that (1) the consumer knows the distribution of weights across all the attributes but not the weight of each specific attribute, and (2) in each step of the search he is presented with a random attribute and learns both the importance ($z_i$) and the realization (+/-) of that attribute after checking it.
this condition for the search cost \( c \), we can then derive that the expected utility of the consumer is greater when information is available on only one attribute than when information is available on both attributes. Similarly, we can obtain that the probability of purchase is greater when information is offered on only one attribute, \( 1/2 \), than when information is offered on both attributes, \( 3/8 \).

This illustrates how too much information may hurt the expected utility of a consumer, or the probability of purchase. Less information (with a strategic supplier of information) allows the consumer to concentrate his search on the more important attributes, which may not be possible to guarantee when information on more attributes is available. In this example, note also that if search costs are sufficiently low, the consumer may always prefer to search both attributes.

This simple two-attribute example can also illustrate how the incentives to provide information are different if one wants to maximize the probability of purchase (for example, the firm wants to maximize demand) or if one wants to maximize the expected payoff for the consumer (maximize consumer surplus). For this consider that \( \frac{z_A-z_B}{2} < c < \frac{v+z_B}{2} \). Then, in this case if only one attribute is available for search, the firm presents attribute \( A \), and the consumer searches and purchases if this attribute is positive. Then, the probability of purchase is \( \frac{1}{2} \) and the expected payoff for the consumer is \( \frac{1}{2}(v+z_A) - c \).

Consider now the case of the firm offering two attributes for search. If the first attribute searched is attribute \( A \) the consumer stops searching purchasing the product if attribute \( A \) has a good fit. If the first attribute searched is attribute \( B \) the consumer also stops searching, and purchases if attribute \( B \) has a good fit. In this case the probability of purchase is still \( \frac{1}{2} \), but the expected payoff for the consumer is now \( \frac{2v+z_A+z_B}{4} - c \). That is, although a firm is indifferent between offering information on one or two attributes (the same probability of purchase), a consumer would prefer that the firm offered information on only one attribute.\(^{24}\)

For more than two discrete attributes the analysis can get analytically more complicated. With a small number of attributes we can numerically evaluate the optimal search behavior, and also find that the optimal number of attributes to disclose information on may not be all the possible attributes. As an example, consider the case where there are 20 product attributes, the importance of attribute \( k \in \{1, 2, \ldots, 20\} \) is \( z_k = e^{-k/2} \), the consumer can check one attribute at a time if he decides to continue searching, and the search costs are \( c = .01 \). If the initial expected valuation is \( v = -1.0 \), for example, the optimal number of attributes is 3 and the corresponding probability of purchase is .125. Similar to the intuition above, the seller trades off the number of attributes to present information on and the average informativeness of these attributes in order to maximize the consumer’s purchase probability.

Proof of Lemma 1: Solving the differential equation \(-cT + \max[u, 0] - V + \frac{T\sigma^2}{2}V_{uu} = 0\) for \( u > 0 \) one obtains

\[
V(u) = A_1e^{\sqrt{\frac{2}{T\sigma^2}}u} + A_2e^{-\sqrt{\frac{2}{T\sigma^2}}u} + u - cT,
\]

where \( A_1 \) and \( A_2 \) are constants to be determined with the boundary conditions.

Solving the differential equation for \( u < 0 \) one obtains

\[
V(u) = A_3e^{\sqrt{\frac{2}{T\sigma^2}}u} + A_4e^{-\sqrt{\frac{2}{T\sigma^2}}u} - cT,
\]

where \( A_3 \) and \( A_4 \) are constants to be determined also with the boundary conditions.

Consider now the constraints imposed on the constants \( A_1, A_2, A_3, A_4, \bar{U}, \) and \( U \) by \( V(\bar{U}) = \bar{U} \) and \( V_u(\bar{U}) = 1 \), \( V(U) = 0 \) and \( V_u(U) = 0 \), and \( V(0^+) = V(0^-) \), and \( V_u(0^+) = V_u(0^-) \). The condition \( V(0^+) = V(0^-) \) leads to \( A_1 + A_2 = A_3 + A_4 \). The condition \( V'(0^+) = V'(0^-) \) leads to \( A_1 - A_2 + \sqrt{\frac{T\sigma^2}{2}} = A_3 - A_4 \). The

\(^{24}\)A similar result can be obtained also, for example, for \( v + z_A - z_B > 0 \) and \( \frac{1}{2}(v + z_A - z_B) > c \).
condition \( V(U) = U \) leads to
\[
A_1 e^{\frac{2}{T^2} U} + A_2 e^{-\frac{2}{T^2} U} = cT.
\]
The condition \( V(U) = 0 \) leads to
\[
A_3 e^{\frac{2}{T^2} U} + A_4 e^{-\frac{2}{T^2} U} = cT.
\]
The condition \( V'(U) = 1 \) leads to \( A_2 = A_1 e^{\frac{2}{T^2} U} \). The condition \( V'(U) = 0 \) leads to \( A_4 = A_3 e^{\frac{2}{T^2} U} \).

From this, one can obtain \( A_1 = A_4 = \frac{cT}{2} e^{-\frac{2}{T^2} U}, A_2 = A_3 = \frac{cT}{2} e^{\frac{2}{T^2} U} \), and \( U = -\bar{U} \).

Putting the equations above together, we have
\[
\bar{U} = \sqrt{\frac{T^2}{2}} \log \left[ \frac{\sigma^T}{2} \sqrt{\frac{\sigma^T}{8c^2T}} + \sqrt{1 + \frac{\sigma^T}{8c^2T}} \right].
\]

**Proof that** \( \lim_{T \to 0} \bar{U} = 0 \), \( \lim_{T \to \infty} \bar{U} = 0 \), \( \frac{\partial \bar{U}}{\partial T} > 0 \) for \( T \) small enough and \( \frac{\partial \bar{U}}{\partial T} < 0 \) for \( T \) large enough: Using L’Hôpital’s rule, we have that
\[
\lim_{T \to 0} \bar{U} = \sqrt{\frac{\sigma^T}{2}} \lim_{x \to 0} \sqrt{x} \log(2\sqrt{\frac{\sigma^T}{8c^2x}}) = 0.
\]
We also have
\[
\lim_{T \to \infty} \bar{U} = \frac{\sigma^T}{2} \lim_{T \to \infty} \left[ \frac{\sqrt{\frac{\sigma^T}{8c^2T}}^{T^{-3/2}} + (1 + \frac{\sigma^T}{8c^2T})^{-1/2} \frac{\sigma^T}{8c^2T}T^{-2}}{T^{-3/2} \left[ \sqrt{\frac{\sigma^T}{8c^2T}} + (1 + \frac{\sigma^T}{8c^2T}) \right]} \right] = 0
\]
since by the Lebesgue’s dominated convergence theorem \( \lim_{T \to \infty} \sigma^T_T = \int_0^1 \lim_{T \to \infty} \sigma^T_T dy = 0 \).

Then, given that \( \bar{U} > 0 \) for \( T > 0 \) and \( \bar{\sigma}^T > 0 \), we have that \( \frac{\partial \bar{U}}{\partial T} > 0 \) for \( T \) small enough and \( \frac{\partial \bar{U}}{\partial T} < 0 \) for \( T \) large enough.

**Proof of Lemma 3:** Consider the time interval \([0, dt], \forall u \in (-\bar{U}, 0),\)

\[
P(u, T) = 0 \cdot (\frac{\partial}{\partial t}) + E \left[ P(u + du, T) \cdot (1 - \frac{\partial}{\partial T}) \right]
= (1 - \frac{dt}{T}) \left[ P(u, T) + P_u(u, T) \cdot du + \frac{1}{2} \cdot P_{uu}(u, T) \cdot du^2 \right]
= (1 - \frac{dt}{T}) [P(u, T) + \frac{1}{2} \sigma^2_T \cdot P_{uu}(u, T) \cdot du]
\Rightarrow P(u, T) = \frac{1}{2} \cdot P_{uu}(u, T) \cdot T \sigma^2_T
\]  

(2)

The solution to this differential equation has solution \( P(u, T) = A_1 e^{\alpha u} + A_2 e^{-\alpha u} \), where \( \alpha = \sqrt{\frac{2}{T^2}} \) and \( A_1 \) and \( A_2 \) are constants to be determined. Given that \( P(-\bar{U}, T) = 0 \), and that by symmetry \( P(0, T) = \frac{1}{2} \), we obtain
\[
A_1 = \frac{1}{2} e^{2\alpha \bar{U}} \quad \text{and} \quad A_2 = -\frac{1}{2} \frac{1}{e^{2\alpha \bar{U}} - 1}.
\]

**Proof of Proposition 1:** By Lemma 2, \( \bar{U} \to 0 \) when \( T \to 0 \) or \( T \to \infty \). With \( v < 0 \), we know that the
Figure 3: **Purchase Threshold $\bar{U}(T)$**

Evolution of $\bar{U}$ as a function of $T$ for $\sigma_i^2 = e^{-i}$ and $c = .01$.

Purchase likelihood is zero if $U = -\bar{U}$ is close to zero. Since $P(v, T)$ is continuous in $T$, the proposition holds.

**Proof of Proposition 2:** Pick any $v_0 \in (-\bar{U}^*, 0)$, where $\bar{U}^*$ is the $\bar{U}$ given in Lemma 1 for the case in which $T$ is optimal for $v_0$. We show first the result that $T^*(v)$ in increasing in $v$ by showing that there exists $\delta > 0$ such that $\forall v \in (v_0 - \delta, v_0) \Rightarrow T^*(v) \leq T^*(v_0)$. Denote $T_0 = T^*(v_0)$. If $v_0 \leq -\bar{U}(T_0)$, where $\bar{U}(T_0)$ is the $\bar{U}$ defined in Lemma 1 for $T = T_0$, then $P(v_0, T_0) = 0$, which contradicts the fact that there exists $T'$ such that $P(v_0, T') > 0$. Therefore, $v_0 > -\bar{U}(T_0)$, and we can define $\delta = v_0 + \bar{U}(T_0)$. Suppose we have $v_1 \in (v_0 - \delta, v_0)$ such that $T_1 = T^*(v_1) > T_0$, we will show that $P(v_0, T_1) \geq P(v_0, T_0)$, which contradicts the optimality of $T_0$.

Let $u_s(v, \sigma^2)$ represent the value of a Brownian motion at time $s$ starting from $v$ with variance $\sigma^2$. By the Markovian Property of the Brownian Motion and the memory–less property of the constant hazard rate model,

$$P(v_0, T_0) = P(0, T_0) \times \Pr(u_s(v_0, \sigma^2) \text{ hits 0 before being terminated or hitting } v_1) + P(v_1, T_0) \times \Pr(u_s(v_0, \sigma^2) \text{ hits } v_1 \text{ before being terminated or hitting 0})$$

where $P(0, T) = \frac{1}{2}, \forall T$ and $P(v_1, T_0) < P(v_1, T_1)$ by the optimality of $T_1$.

Let $\tau'$ be distributed as an exponential distribution with parameter equal to one. For any $T > 0$, $\tau \overset{d}{=} T \tau'$ is then distributed as an exponential distribution with parameter $T$. We then have

$$\Pr(u_s(v_0, \sigma^2) \text{ hits 0 before hitting } v_1 \text{ or being terminated}) = \Pr(u_s(0, \sigma^2) \text{ hits } -v_0 \text{ before hitting } v_1 - v_0 \text{ and before } T_0 \tau')$$
where the last line is implied by the important property of Brownian motion that \( \theta u_{s/\theta^2}(0,1) = u_s(0,1) \) in distribution. As we have shown that \( T \sigma_T^2 \) is increasing as \( T \) increases, we get

\[
Pr(u_s(0,1) \text{ hits } v_0 \text{ before hitting } v_1 - v_0 \text{ and before } T_0 r' \times \sigma_{T_0}^2),
\]

Similarly we can show that

\[
Pr(u_s(0,1) \text{ hits } v_0 \text{ before hitting } v_1 - v_0 \text{ and before } T_0 r' \times \sigma_{T_0}^2).
\]

Combining all inequalities, we have \( P(v_0, T_0) < P(v_0, T_1) \), which is a contradiction.

Consider now the proof that \( T^* > \arg \max_T U \). For this let \( z = \alpha U \) where \( \alpha \) is defined in Lemma 3. Then the \( T \) that maximizes \( U \) satisfies \( \frac{\partial}{\partial T}(\frac{z}{\alpha}) = 0 \), which yields \( \alpha \frac{\partial z}{\partial T} = z \frac{\partial \alpha}{\partial T} \). Differentiating \( P(v, T) \) with respect to \( T \) one obtains:

\[
2(e^{2z} - 1)^2 \frac{\partial P}{\partial T} = 2 \frac{\partial z}{\partial T} (e^{2z-\alpha v} - e^{2z+\alpha v}) + v \frac{\partial \alpha}{\partial T} (e^{2z} - 1)(e^{-\alpha v} + e^{2z+\alpha v}).
\]

Evaluating this at the \( T \) that maximizes \( U \) one obtains \( \frac{\partial P}{\partial T} > 0 \), as \( \frac{\partial \alpha}{\partial T} < 0 \) and \( v < 0 \) by assumption. That means that the optimal \( T^* \) is then strictly greater than the \( T \) maximizing \( U \).

**Proof of Proposition 3:** Let us first re-write \( P \) and \( V \) with a simpler notation. Define \( Y \equiv \frac{\sigma_T^2}{\sqrt{2} \alpha}, \; X \equiv [\sqrt{Y} + \sqrt{1+Y}]^2, \; \) and \( W \equiv e^{\alpha v}, \) and note that \( X = e^{2\alpha \sigma} \).

Defining \( \tilde{P} = 2P \), we can then obtain from Lemma 3 (for \( v < 0 \), the case considered),

\[
\tilde{P} = \frac{1}{W(X-1)} (XW^2 - 1).
\]

Defining, \( \tilde{V} \equiv 2V/c \), one can obtain from the Proof of Lemma 1 and the derivations above that

\[
\tilde{V} = T \frac{(W \sqrt{X} - 1)^2}{W \sqrt{X}}.
\]

Note that \( W \) is monotonic (increasing) in \( T \). Differentiating \( P \) with respect to \( W \) and equalizing to zero one obtains

\[
W(1 - W^2)X' + (X - 1)(1 + W^2)X = 0,
\]

where \( X' = \frac{dX}{dW} \).

For a given \( T \), when \( v \to 0 \), we have \( W \to 1 \). At the limit we then have that (3) converges to

\[
X'(1 - W) = \frac{1 - X^2}{2}.
\]
Differentiating $V$ with respect to $W$ one obtains:

$$\frac{dV}{dW} = \frac{dT}{dW} \left( \frac{W\sqrt{X} - 1}{W\sqrt{X}} \right) + \frac{T}{2W^2X^{3/2}} (W^2 - 1)(2X + WX') \tag{4}$$

Using $\frac{dW}{dT} = vW\frac{d\alpha}{dT}$, and factoring out $\frac{T}{2W^2X^{3/2}}$, one obtains

$$\text{sign} \left\{ \frac{dV}{dW} \right\} = \text{sign} \left\{ \frac{2X}{\alpha \log W} \left( W\sqrt{X} - 1 \right) + (W^2 - 1)(2X + WX') \right\}, \tag{5}$$

where $\tilde{\alpha} = \frac{T}{\alpha} \frac{d\alpha}{dT}$.

Multiplying by $(1 - W)$ (note $W < 1$), and evaluating it when $\frac{dP}{dW} = 0$ for $v \to 0$ (which, remember, yields $X' (1 - W) = \frac{1 - X^2}{2}$) we have

$$\text{sign} \left\{ \frac{dV}{dW} \right\} = \text{sign} \left\{ \frac{-4X}{\alpha} \left( \sqrt{X} - 1 \right) + (X - 1) \frac{(1 - X^2)}{2} \right\},$$

as

$$\lim_{v \to 0} -\log \frac{W}{1 - W} = 1$$
as $W \to 1$ as $v \to 0$.

Consider now making $c \to 0$ and its effect on $\frac{dP}{dW} = 0$. Note first that $X' = \frac{X}{\sqrt{Y(1 + Y)}} Y'$ and we can obtain

$$4c^2 Y' = -\frac{2v}{T \frac{d\alpha}{dT}} \left[ \frac{1}{T \frac{d\alpha}{dT}} + \log W \right].$$

The condition $\frac{dP}{dW} = 0$ can then be written as

$$\frac{2X(1 - W)v}{4c^2 T^2 (\log W)^2 \sqrt{Y(1 + Y)}} \left[ \frac{1}{T} \frac{d\alpha}{dT} + \frac{1}{\alpha} \right] = \frac{X^2 - 1}{2}$$

which can be simplified to

$$\frac{2X}{4c^2 T^2 \alpha \sigma^2 T \sqrt{Y(1 + Y)}} \left[ \frac{1}{T} \frac{d\alpha}{dT} - \frac{1}{\alpha} \right] = \frac{X^2 - 1}{2}. \tag{6}$$

From this condition, when $c \to 0$, we have then that $X,Y,$ and $T$ cannot be finite, and therefore $X \to \infty, Y \to \infty,$ and $T \to \infty$ when $c \to 0$. As $T \to \infty$ we then have $\tilde{\alpha} \to 0$, as $\lim_{T \to \infty} T \sigma^2 T$ is finite and non-zero.

From (6) one can still obtain

$$\frac{2XY}{\sqrt{Y(1 + Y)}} \left( -\frac{1}{\tilde{\alpha}} - 1 \right) = \frac{X^2 - 1}{2}$$

from which for $c \to 0$, one obtains

$$-\frac{4X}{\tilde{\alpha}} = X^2.$$

Substituting in (5), the sign of $\frac{dV}{dW}$ can be obtained to be

$$\text{sign} \left\{ \frac{dV}{dW} \right\} = \text{sign} \left\{ X^2(\sqrt{X} - 1) + (X - 1)(1 - X^2) \right\} < 0$$

when $c \to 0$. 

6
As the sign of \( \frac{dV}{dW} \) is strictly negative when evaluated at \( \frac{dP}{dW} = 0 \), one can conclude that the maximum of \( V \) is with a lower \( W \) if \( V \) is quasi-concave, which means, a lower \( T \).

Figure 4: **Comparison of Expected Utility** \( V(v,T) \) and **Purchase Likelihood** \( P(v,T) \) as a function of \( T \).

In this figure, \( v = -0.1 \), and \( c = 0.01 \). Expected utility \( V \) is maximized at \( T_v \approx 3.0 \), and purchase likelihood \( P \) is maximized at \( T_p \approx 4.8 \).

**Proof of Proposition 4:** Since \( \sigma_i^2 \) is continuous in \( i \), we know that \( P(v,T) \) is continuous in \( T \). Obviously, \( P(v,T) \geq 0 \). So we only need to show that

\[
P(v,T) \to 0, \text{ as } T \to 0 \text{ or } T \to \infty.
\]

We first show that \( P(v,T) \to 0 \) when \( T \to \infty \). For a given \( T \), let \( U_T = -\frac{\sigma_i^2}{c} \) be the exit threshold for a search process where the average informativeness of the searchable attributes is \( \bar{\sigma}_T^2 \) and there is an infinite mass of attributes available to check. With deterministic termination, the consumer stops search more easily since searching does not give him as much information. That is, the exit threshold with deterministic termination is \( U(t) \geq U_T \), for \( 0 \leq t \leq T \). Therefore it is sufficient to show that \( U_T \to 0 \) as \( T \to \infty \), which is implied by the fact that \( \lim_{T \to \infty} \sigma_i^2 = 0 \).

Next we show that when \( T \to 0 \), \( P(v,T) \to 0 \).

\[
P(v,T) \leq P(v + \bar{\sigma}_T w_t \text{ hits 0 before } T)
= P(w_t \text{ hits } \frac{-v}{\bar{\sigma}_T} \text{ before } T)
\]
Figure 5: Optimal $T$ for the consumer and the seller as a function of $v$ for different search costs.

In this figure, there are three levels of search cost $c$: .005, .01, and 0.5. $T_v = \text{argmax}_T V(v, T)$ and $T_p = \text{argmax}_T P(v, T)$.

where $w_t$ is the standardized Brownian motion. By monotonicity of $\bar{\sigma}_T$, we have

$$P(v, T) \leq P(w_t \text{ hits } \frac{v}{\bar{\sigma}_0} \text{ before } T) = 0 \quad \text{as} \quad T \to 0. \quad \text{(7)}$$

The Optimal Price:

The analysis of Section 4 does not consider the impact of price in order to focus on the effects of the amount of information to search through. Including price in the analysis could be done by assuming that price can be observed freely prior to any search, such that the initial expected net valuation of a consumer would be equal to $v = v' - p$, where $v'$ is the expected gross valuation, and $p$ is the price charged by the firm.\(^{25}\)

The search problem of the consumer would then be similar to the analysis presented above, and the problem of the firm for zero production costs could then be seen as $\max_{p,T} P(v' - p, T)p$. Per Proposition

\(^{25}\)Branco et al. (2012) considers this problem with no limits on the amount of information that can be searched.
2 note that given the same initial valuation \( v' \), when the price is higher (lower \( v \)) then the optimal amount of information to provide is lower.

The optimal price for \( v' \) not too high would then be characterized by \( P(v' - p, T) - p \frac{\partial P(v' - p, T)}{\partial v} = 0 \). Suppose that \( v' - p < 0 \), using the purchase likelihood obtained earlier, we can reduce this first order condition to \( e^{2\alpha(U + v' - p)} = \frac{1 + \alpha p}{1 - \alpha p} \). Although we cannot derive the closed-form solution of the equation above, it gives us several properties of the optimal price. First, \( p^* < \min\{v' + U, \frac{1}{\alpha} \} \). That is, the optimal price is bounded above by a function that is increasing in the initial valuation, the purchase threshold, and the total amount of available information. Second, using the implicit function theorem, one can obtain that the optimal price increases with the consumer’s initial valuation \( v' \). Further, the net prior evaluation, \( v' - p^* \) also increases with \( v' \). That is, improvement in the consumer’s prior evaluation does not get completely extracted by a higher price, as the seller uses the more favorable starting point to increase purchase likelihood.

Consider now the optimal \( T \). A modified version of Proposition 1 now holds: For any initial valuation \( v < 0 \), if there exists some amount of information \( T \) such that \( P(v, T) > 0 \), then the optimal amount of information \( T^* \) that maximizes the expected profit is interior (i.e., \( 0 < T^* < \infty \)). To see this, note that the seller’s profit goes to 0 as \( U \) approaches 0, which happens when \( T \to 0 \) or \( T \to \infty \).

As \( v' - p^* \) increases with \( v' \), Proposition 2 still holds. In particular, note that the \( T \) that maximizes the purchase likelihood at the optimal price also maximizes the profit, pointing to a positive correlation between the initial valuation and the optimal amount of information. In sum, as the consumer’s prior attitude toward the product becomes more favorable (conditional on that he would still need to search for more information prior to purchasing the product), the seller should increase the price of the product, and at the same time increase the total amount of information available for search.

The corresponding variation of Proposition 3 would now involve comparing optimal amounts of information with two different prices, one that maximizes the consumer surplus (\( p = 0 \)) and the other that maximizes the expected profit. This change makes it hard to prove a general relationship. In the example above with \( \sigma_i^2 = e^{-i} \) and \( c = .01 \), however, the amount of information that maximizes consumer surplus is indeed lower than the profit-maximizing amount of information for \( v < 0 \).