Competitive Product Lines with Quality Constraints*

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1. Introduction

In many markets, competing firms offer a product line (or a menu of contracts) in order to screen their potential customers. For example, in general markets such as automobiles, airline travel, fax machines, irons, microwaves, coffee-makers, etc., firms target different products to different customer types according to purchasing ability or preference for quality (also known as second degree price discrimination). In other arenas, such as credit markets, firms screen customers according to the risk of the project using the level of collateral required, and in insurance markets, they screen customers according to the risk of accident, with the amount of coverage provided. Similarly, in labor markets, firms may use wage schedules as a function of education levels in order to screen the potential employees' ability. In some of these markets there can be some natural quality constraints: a maximum available quality in general markets, no negative collateral in credit markets; coverage not above 100% in insurance markets; minimum education level in labor markets.

At a general level, pricing and designing a product line without competition (the monopolist case) has been studied since Mussa and Rosen [1978]. Here we look at the competition case, when differentiated firms compete in product lines and there are quality constraints. We address the following questions: What product line should be offered by a firm under competition? Under which conditions should a firm decide not to screen its customers? How should the product line design change with more intense competition?

The answers to these questions are not straightforward. Competition may dramatically complicate the optimal screening of the different types of customers. Firms, while trying to screen customer types, are also now interested in "stealing" customers from the competitors, and in particular, customers from the most profitable types. At the same time, because screening involves the costs of "separating" the different types, competing firms may consider offering a single product (without distortions) targeted at different customer types, in order to profitably "steal" customers from each other.

Examples of market situations where firms compete in product lines are everywhere. Consider the following examples as a general illustration of the type of problem being studied. Consider first the three main American automobile manufacturers. General Motors offers the brands Cadillac, Oldsmobile, Buick, Saab, Chevrolet, Pontiac, and Saturn, as a way to screen customer types.[1] Ford offers the brands Lincoln, Jaguar, Mercury, and Ford. Chrysler offers the brands Chrysler, Dodge,

[1] Obviously, for each brand there is a further screening of types which is represented by a variety of models and optional equipment.

Eagle, and Plymouth. These three manufacturers compete among each other, offering different products targeted at different customer types.

Another interesting example of product line competition is the case of firms competing through couponing, in the sense that using coupons changes the product being bought: handling and search costs but lower prices. Customer types are then screened according to their handling and search costs of using coupons. Customers with lower costs search more, find the best coupons and pay a lower price. Customers with higher costs do not search and pay a higher price. Note also that the different customer types with costs of handling (and search for) coupons above a certain threshold buy all the same product, the product that does not involve using coupons.[2]

Having illustrated the problem under study and some stylized facts, let us now briefly summarize the results of the paper and their intuition. We present conditions for the existence of a pure strategies Nash equilibrium in the case in which there is a sufficient amount of horizontal differentiation across firms.[3] We also present the conditions that the equilibrium product line has to satisfy.

The equilibrium product line results, under certain general conditions, in each firm offering a single product (contract) for the top portion of the customer types (pooling at the top), a single product (contract) for the bottom portion of the customer types (pooling at the bottom), and a different product (contract) for each customer type in the middle portion of the customer types (complete screening in the middle). This little variation in the product offerings at the high and low ends, and the substantial variation in the middle, could be seen as consistent with the couponing example presented above.

The conditions for these product line features have to do with each firm having some constraints on the optimal level of the screening device being used to screen customer types (this is made more clear below). For example, when a firm screens different customer types through the use of coupons the firm cannot make customers incur negative search costs; i.e., the search costs are constrained to be positive. The same type of condition arises when firms screening customers according to their preferences for quality are limited by an exogenous (binding) maximum quality level. In the credit markets example, firms screening according to the collateral required in the loan are also constrained by the fact that the collateral level has to be positive. In the same way, in insurance markets, firms

[2]See Nevo and Wolfram [2002] for an empirical investigation of coupon offerings and price levels. There, it is found that couponing is more likely to happen when prices are lower. This can be seen as consistent with the results in this paper that with more competition (lower prices) firms are more likely to screen consumers (offer coupons).

[3]This is in contrast to the case with no differentiation which has been shown by Rothschild and Stiglitz [1976] to result, under some general conditions, in the non-existence of an equilibrium in pure strategies.

screening customers according to the level of coverage may be constrained by not being able to offer more than full coverage. In labor markets, firms screening potential employees according to their education level may be constrained by not being able to require negative education levels.

These constraints on the level of the screening device result in firms offering different customer types a single product characterized by the constrained level of the screening device (in the examples above this represents, respectively, "no coupons", "maximum level of quality", "zero collateral", "full coverage", "zero education"). Moreover, at the other end of the product line, firms also choose to offer one single product to different customer types because it becomes too expensive to screen customers; i.e., there is too much distortion in the screening device (in the examples above the distortions are, respectively, "positive search costs", "less than the maximum quality level", "costly collateral", "less than full coverage", "high education").

These results are in contrast with the non-existence results when customer types are continuously distributed in the literature on non-differentiated oligopoly with asymmetric information (Rothschild and Stiglitz [1976], Wilson [1977], Riley [1979], Stockey [1980]).[4] The results also contrast with those obtained under the equilibrium concepts used in part of that literature, non-anticipatory (Wilson) and reactive (Riley) equilibria. Under these equilibrium concepts the market outcome has complete screening of all customer types, while we show that, with differentiation, firms will always offer a same product to a set of different customer types.

An important component of our analysis is the presence of horizontal differentiation among firms. In fact, in all the examples presented above there seems to be some degree of horizontal differentiation among firms; i.e., some customers have a relative preference for one firm while other customers have a relative preference for another firm. In the example of screening according to purchasing ability or preference for quality, these horizontal preferences could depend on customer perceptions of each firm. In the example of credit or insurance markets, these horizontal preferences could result from past relationships (see, e.g., Baltensberg and Devinney [1985]). In the labor market these preferences could simply depend on geographical factors.

Other related papers include the work on competition in credit markets by Bester [1985], and Besanko and Thakor [1987], who look at the non-differentiated oligopoly case with a discrete distribution of customer types. There has also been some research on screening according to preference for quality – Locay and Rodriguez [1992], Stole [1995], Epstein and Peters

[4]This literature considers similar constraints to the ones stated above. Note, however, that Riley [1985] shows that in the non-differentiated oligopoly case, an equilibrium in pure strategies exists if there is a sufficient mass of customers for whom firms serving them is socially inefficient and the rate of decline of the marginal cost of signaling across types is sufficiently large. The results presented here do not require the existence of such a mass of customers.

[1999], PETERS [2001], ROCHET AND STOLE [2002], and MARTIMORT AND STOLE [2002], without considering quality constraints. LOCAY AND RODRIGUEZ restrict attention to competition using two-part tariffs. STOLE considers screening (without perfect discrimination) in only one dimension at a time (either vertical or horizontal). ROCHET AND STOLE (also looking at screening according to preference for quality) is closer to the results presented here by restricting attention to the case in which firms are unable to screen customers according to their horizontal preferences. In relation to that paper this paper shows that under with quality constraints there is pooling both at the top and at the bottom, and shows that product differentiation may lead to the existence of pure-strategy equilibria under quality constraints. EPSTEIN AND PETERS, PETERS, and MARTIMORT AND STOLE consider the important possibility of the allocation provided to consumers being a function of what they report about what the other firms are offering. This possibility is not considered here, with allocations being only a function of the reports about the consumer preferences. See MARTIMORT AND STOLE for further discussion on this general issue.[5]

The paper is organized as follows: We start by presenting in the next section a general model of competition in product lines that includes as particular cases the examples above. Section 3 derives the main results of the paper, giving their intuition, and linking them to the markets discussed above. Section 4 concludes, discussing empirical implications.

2. The Model

Consider the following general model of product line competition. The demand side of the market is composed of a continuum of customers identified by a pair (θ, d), where θ is an index of vertical preferences and d is an index of horizontal preferences. θ is assumed to be unidimensional and is in $[\underline{\theta}, \overline{\theta}]$ in $\Re$. d can be multidimensional and is in $\mathcal{D}$, a compact and convex set. Each customer knows his pair (θ, d), and each firm knows only the distribution of (θ, d). This is in the spirit of, for example, MUSSA AND ROSEN [1978]. The total mass of the market is standardized at 1.

When choosing the product, customers care about the pair (R, C), the product design. R is a variable that represents a cost to the customers and a benefit to the firm. C is a variable that represents the screening device, and is defined here as costly to the customer. Often, but not necessarily, R is as costly to the customer as it is beneficial to the firm, while C is more costly to

[5] Other related work on competition on product lines is presented in KATZ [1984] who looks at a discrete distribution of types, in MOORTHY [1987] who considers simultaneous choice of design and output (with a fixed number of products), in CHAMPSAUR AND ROCHET [1989] who look at the important case in which firms first commit on the quality interval (under non-differentiated oligopoly), in DESAI [2001], and in ARMSTRONG AND VICKERS [2001]. For dynamic product line design in competition please see ZHANG [2006].

the customer than it is beneficial to the firm. R and C allow firms to screen customers according to θ, but not according to d. The role of d is simply to create differentiation among firms without interfering on the screening strategies based on θ. C is restricted to belonging to $[\underline{C}, \overline{C}]$ in $\Re$.

In the example of screening according to the preference for quality (automobiles, airline travel, fax machines, irons, microwaves, coffee-makers, etc.), R could be the price paid by the customer (the revenue received by a firm), $-C$ could be the quality of a certain product, and $-\theta$ would be an index of the customer's preference for quality. In the credit markets example R could be the interest rate paid by the entrepreneur/"customer", C could be the collateral associated with the loan (or minus the amount of the project that is financed by the bank), and θ would be the risk of the entrepreneur's project. In the insurance markets example R could be the premium of the insurance policy, $-C$ could be the amount covered by the policy, and θ would be the probability of damages. In the labor market example $-R$ could be the wage received by the employee, C could be the education level of the employee, and θ would be the employee's ability.

Given these assumptions, the utility derived by a customer identified by the pair (θ, d) from buying a product with characteristics (R, C) from the firm i could be defined by

$$\widehat{U}^i(\theta, d, R, C, t) = \widetilde{U}(\theta, R, C) - h(i, d, t) \tag{1}$$

where $\widetilde{U}_R < 0, \widetilde{U}_C < 0$, and the function $h(.)$ represents the horizontal preferences term and is increasing in some measure of distance between d and i, with $h(.) \geq 0$. The parameter t is an index of "transportation costs" between d and i, and can also be interpreted as an index of differentiation; i.e., the higher t the less intense the competition is, and therefore $h_t > 0$.[6] If t is small, the distance between d and i is less important for the consumers to choose among firms, i.e., firms are not very differentiated. If t is large, the distance between d and i is more important for the consumers to choose among firms, i.e., firms are very differentiated. Finally, each customer chooses the product (within a product line and across firms) that yields the greatest utility.

For firms to be able to screen customer types according to θ, one has to assume that the sorting condition holds on θ; i.e., that $\frac{\partial}{\partial \theta}(-\frac{\widehat{U}^i_C}{\widehat{U}^i_R}) > 0$, which is equivalent to $\frac{\partial}{\partial \theta}(-\frac{\widetilde{U}_C}{\widetilde{U}_R}) > 0$. That is, customer types with higher θ are more willing to accept a higher C, the screening device, for any given reduction in R. For example, customers that appreciate quality more are more willing to accept a higher price for a given increase in the quality of the product. Customers that have a higher

[6]Throughout the paper the notation for the partial derivative of a function F with respect to its argument x will be denoted by F_x. For some analysis on the case in which the vertical and horizontal parameters are not additively separable see STOLE [1995].

tolerance for coupon search costs are more likely to use coupons.

On the other hand, firms are not able to screen customers according to d because the sorting condition does not hold for d; i.e., $\frac{\partial}{\partial d}(-\frac{\widehat{U}^i_C}{\widehat{U}^i_R}) = 0$, where $\frac{\partial}{\partial d}$ means the derivative on any direction in the space of d. As stated above, the role of d is simply to introduce horizontal differentiation in the model without interfering in the screening strategies based on θ.

In order to be able to clearly separate the impact of θ and d, we assume that these two variables are stochastically independent. This allows the demand for each firm given θ to be independent of θ, which can be seen as the natural extension to the case with no horizontal differentiation. Several of the results presented here go through when θ and d are not independent.[7] The marginal distribution of θ is characterized by the smooth density function $f(\theta)$.

On the supply side of the market there are n firms. The profit of a firm selling to a customer with type (θ, d) a product with characteristics (R, C) is $\widehat{\pi}(\theta, R, C)$, with $\widehat{\pi}_R > 0$, i.e., firms like higher R.[8]

For simplicity we will consider the symmetric case among firms, the case in which the firms are in an equal position in relation to the customers. One simple example of this with $n = 4$ is the case in which $\mathcal{D}$ is a square, each firm is positioned at a different corner of the square, and the marginal distribution of d on $\mathcal{D}$ is uniform.

Finally, the market game is played as follows: firms offer an incentive–compatible product line to the customers, taking offers by the competitors as given. Customers either buy one of the products (in one of the product lines) or reject all of them. The incentive–compatibility feature of the product line has to do with the firms realizing in advance which customer types will buy which products.

An important assumption in this set-up is that firms decide both characteristics of the product simultaneously (or, at least, without the competitors knowing anything about the firm's decisions before any of their own decisions). Alternatively, this can be seen as the firms deciding on which product characteristics to activate simultaneously. This assumption is acceptable in important market situations. However, there are situations where firms commit first to the possible physical designs of the product (C), and then compete on prices (R). For an analysis of this case with no horizontal differentiation see Champsaur and Rochet [1989].

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[7] In that case the function $D^i(.)$ below would also be a direct function of θ. Without quality constraints, the assumption of independence of d and θ allows for further characterization of the equilibrium contracts (see Rochet and Stole [2002]).

[8] Note that for the example in which firms screen customers according to their preference for quality, $\widehat{\pi}_\theta = 0$; i.e., the index that represents the preference for quality does not enter directly into the profits of the firms.

3. The Main Results

Since screening customers on the basis of different d, the horizontal differentiation parameter, is infeasible, the problem of each firm comes down to designing a product line $\{R(\theta), C(\theta)\}_{\theta\in[\underline{\theta},\overline{\theta}]}$ that maximizes profits subject to incentive compatibility (firms understand which product each type of customer is going to choose), individual rationality (the participation constraints – a customer only buys a product if he gets a greater surplus from buying a product offered by that firm than not buying any product or buying a product from another firm), and feasibility constraints (the product designs are feasible).

By the revelation principle (see, e.g., Fudenberg and Tirole [1991, p.253]), incentive compatibility is equivalent to the requirement that truth telling is a dominant strategy for all customers in a direct revelation game. So, the incentive compatibility requirement is given by

$$U(\theta) \equiv \widetilde{U}(\theta, R(\theta), C(\theta)) = \max_{\widetilde{\theta}} \widetilde{U}(\theta, R(\widetilde{\theta}), C(\widetilde{\theta})). \qquad (IC)$$

To make (IC) operational, observe that (IC) and the envelope theorem imply

$$\frac{dU(\theta)}{d\theta} = \widetilde{U}_{\theta}(\theta, R(\theta), C(\theta)). \qquad (IC')$$

However, (IC') is necessary but in general not sufficient for incentive compatibility. It is well known by now that (IC') and the monotonicity requirement

$$\frac{dC(\theta)}{d\theta} \geq 0 \qquad \text{(monotonicity)}$$

are necessary and sufficient conditions for global incentive compatibility (see, e.g., Fudenberg and Tirole [1991, pp. 258 – 262]).

Let us turn to the participation constraints next. There are two types of participation constraints: a customer not buying from any firm, and a customer buying from another firm. The former is assumed not to be binding for all customers; i.e., in equilibrium all customers buy a product. This assumption can be derived from conditions on $\underline{\theta}, \overline{\theta}$, the distribution of d and its support, $\mathcal{D}$, and the functions $\widehat{\pi}(.)$ and $\widehat{U}(.)$. In the example of screening according to the preference for quality, this would require that all customers value the product very highly. In the credit markets example, this assumption would require that all the entrepreneurs' projects be highly profitable. This assumption is not crucial in the analysis and is stated to make the presentation simpler. An analysis without this assumption can be seen in Villas-Boas and Schmidt-Mohr [1999] for the two-type case and in Stole [1995].[9]

The latter type of participation constraints, the possibility of a potential customer buying from another firm, is considered through the existence of an allocation of the total demand across firms according to the utilities ($\widehat{U}$) customers derive from the competing product lines being offered. Given the product lines being offered, each customer of type θ will derive utility $U^i(\theta)$ (as defined above) from buying a product from firm i (for each firm i). The allocation across firms of the total demand of all the customers of type θ can then be defined as a demand for firm i, $D^i(U^i(\theta), U^{ci}(\theta), t)$, where $U^{ci}(\theta)$ is a vector of the utilities derived by a customer with type θ from all the firms except firm i, and $D^i_{U^i} \geq 0, D^i_{U^{ci}} \leq 0$. The functions $D^i(.)$ are derived from the $h(.)$ function presented above. Note that, because of the additive separability and independence of θ and d, the functions $D^i(.)$ are directly independent of θ. Note also that given the assumption that the participation constraints of not buying any product are not binding and the standardization of the size of the market to one, we have $\sum_{i=1}^n D^i(.) = 1$. Finally, note that, because of the symmetry assumption, if all the firms offer the same product line we have $D^i(.) = \frac{1}{n}, \forall i$. The functions $D^i(.)$ are assumed to be continuous, and at least twice differentiable.[10] With t being an index of differentiation, we have $D^i_{U^it} \leq 0$, and when t tends to zero differentiation, $D^i_{U^i}$ evaluated at a point in which all firms offer the same product line, tends to infinity.

Now, using $U = \widetilde{U}(\theta, R, C)$, and because the function $\widetilde{U}(.)$ was assumed strictly monotone in R, one can obtain $R = \widetilde{U}^{-1}(\theta, U, C)$ satisfying $U \equiv \widetilde{U}(\theta, \widetilde{U}^{-1}(\theta, U, C), C)$. This allows us to eliminate R in the conditions of the problem by defining

$$\frac{dU}{d\theta} = \widetilde{U}_\theta(\theta, R, C) = \widetilde{U}_\theta(\theta, \widetilde{U}^{-1}(\theta, U, C), C) \equiv g(\theta, U, C) \tag{2}$$

$$\widehat{\pi}(\theta, R, C) = \widehat{\pi}(\theta, \widetilde{U}^{-1}(\theta, U, C), C) \equiv \pi(\theta, U, C). \tag{3}$$

[9] The case of this assumption has been called the "pure competition case". Without this assumption, there are several other cases to be considered: (i) there is at least one d for which any customer with type (θ, d) does not buy from any firm ("pure local monopoly"); (ii) there is at least one θ for which any customer with type (θ, d) buys from a firm, and there is at least one customer that does not buy from any firm ("the mixed case"). Additionally, in cases (i) and (ii) it might happen (or not) that there is a θ such that for all d a customer with type (θ, d) does not buy from any firm (this is the differentiation extension of the case considered in Riley [1985]).

[10] This assumption is obviously related to properties on the function $h(.)$ and on the distribution probability on d. For example, this assumption would hold if there are only two firms in the market located at the extremes of the segment $[0, 1]$, with d uniformly distributed on this segment and the function $h()$ representing linear transportation costs on the distance between a consumer's location and each firm.

Given the assumptions above ($\widetilde{U}_R < 0$ and $\widehat{\pi}_R > 0$), we now have $\pi_U < 0$, the more "utility" a firm gives a customer, the less profitable that customer is for the firm.

Finally, in order to state each firm's problem let us define the auxiliary control variable $\beta(\theta)$ by $\beta(\theta) \equiv \frac{dC(\theta)}{d\theta}$, the rate of change of the product variable C with the type of customers. The firm's problem of setting the product line $\{R(\theta), C(\theta)\}_{\theta \in [\underline{\theta}, \overline{\theta}]}$ can now be stated as an optimal control problem with control β and states U and C. This problem can be stated for firm i as

$$\max_{\beta^i(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} \pi^i(\theta, U^i(\theta), C^i(\theta)) D^i(U^i(\theta), U^{ci}(\theta), t) f(\theta) \mathrm{d}\theta \tag{$MAX1$}$$

subject to:

$$\frac{dU^i(\theta)}{d\theta} = g(\theta, U^i(\theta), C^i(\theta)) \quad (\lambda^i(\theta)); \quad \frac{dC^i(\theta)}{d\theta} = \beta^i(\theta) \quad (\mu^i(\theta)); \quad \beta^i(\theta) \geq 0 \quad (\gamma^i(\theta)) \tag{IC''}$$

$$C(\theta) \geq \underline{C} \quad (\underline{\psi}^i(\theta)); \quad C(\theta) \leq \overline{C} \quad (\overline{\psi}^i(\theta)) \tag{feasibility}$$

where $\lambda^i(\theta)$ is an adjoint function associated with the equation of motion of U, $\mu^i(\theta)$ is an adjoint function associated with the equation of motion of C, $\gamma^i(\theta)$ is an adjoint function associated with the monotonicity constraint on C, and $\underline{\psi}^i\theta$ and $\overline{\psi}^i(\theta)$ are adjoint functions associated with the feasibility conditions on C.

This is a standard optimal control problem, with each firm facing this type of problem. A Nash equilibrium is characterized by the solution to this problem by each firm, and substituting the equilibrium competitors' strategies for U^{ci}. Concentrating on symmetric equilibria, and following Mangasarian [1966], we present in the next proposition sufficient conditions for the existence of a Nash equilibrium in pure strategies.

Proposition 1: *A symmetric Nash equilibrium in pure strategies with* $U^i(\theta) = U(\theta), C^i(\theta) = C(\theta), \lambda^i(\theta) = \lambda(\theta), \mu^i(\theta) = \mu(\theta), \gamma^i(\theta) = \gamma(\theta), \underline{\psi}^i(\theta) = \underline{\psi}(\theta), \overline{\psi}^i(\theta) = \overline{\psi}(\theta), \forall i, \theta$ *exists if the function* $g(.)$ *and the product* $\pi(.)D^i(.)f(.)$ *are concave in* (U, C) *for all* i*, and the equilibrium strategies satisfy the conditions* (IC'') *and (feasibility) and the following conditions:*

$$\frac{d\lambda}{d\theta} = -\frac{\pi_U f}{n} - \pi D_U f - \lambda g_U \tag{4}$$

$$\frac{d\mu}{d\theta} = -\frac{\pi_C f}{n} - \lambda g_C - \underline{\psi} - \overline{\psi} \tag{5}$$

$$\mu = \gamma \tag{6}$$

$$\lambda(\overline{\theta}) = \lambda(\underline{\theta}) = 0 \tag{7}$$

$$\mu(\overline{\theta}) = \mu(\underline{\theta}) = 0 \tag{8}$$

$$\beta\gamma = 0; \quad \gamma \leq 0 \tag{9}$$

$$\underline{\psi}(\underline{C} - C) = 0; \quad \overline{\psi}(\overline{C} - C) = 0; \quad \underline{\psi} \geq 0; \quad \overline{\psi} \leq 0 \tag{10}$$

$$\lambda \geq 0. \tag{11}$$

$U(\theta)$ and $C(\theta)$ satisfying these conditions constitute the Nash equilibrium among firms. Moreover, condition (11) is not required if $g(.)$ is linear in (U, C).[11]

The proof follows directly from Theorem 2 in MANGASARIAN [1966]. Note that conditions (4)-(11) are also necessary conditions. They define the equilibrium product lines being offered by the firms in a market with horizontal differentiation.

Consider an example of the credit market where θ represents the probability of a project having return X (having zero return otherwise), investments require one unit, and where collateral can be liquidated by the lender at a cost of $C/2$ per unit. In this case we would have $\widetilde{U}(\theta, R, C) = \theta(X - R) - (1 - \theta)C$ and $\widehat{\pi}(\theta, R, C) = \theta R + (1 - \theta)C(1 - \frac{C}{2}) - 1$. From this one can then obtain $g(\theta, U, C) = \frac{U+C}{\theta}$ and $\pi(\theta, U, C) = \theta X - U - (1 - \theta)\frac{C^2}{2} - 1$. If there are two lenders in the market located at the extremes of the interval [0,1], with d uniformly distributed on [0,1] with $h(i, d, t)$ being t times the distance between a borrower located at d and lender i, we would then have $D^i(U, U^c, t) = \frac{t+U-U^c}{2t}$. One can then obtain that both $g(.)$ and the product $\pi(.)D^i(.)f(.)$ are concave in (U, C) under some conditions on t and the support of θ.

[11] D_U denotes the partial derivative of D^i with respect to U^i.

Proposition 1 has to be related to the non-existence results in the literature on oligopoly under asymmetric information and quality constraints and with no differentiation (ROTHSCHILD AND STIGLITZ [1976], RILEY [1979]). This can be seen in the next proposition.

PROPOSITION 2: *With a sufficiently high level of differentiation, a symmetric Nash equilibrium in pure strategies may exist among competing firms. With a sufficiently small level of differentiation, a symmetric Nash equilibrium in pure strategies cannot be guaranteed to exist under the conditions of Proposition 1.*

PROOF: One of the conditions in Proposition 1 is that the product $\pi(.)D(.)f(.)$ is concave in (U, C). This implies that the Hessian of $\pi(.)D(.)$ on (U, C) is negative definite, which implies $\pi_{UU}D+2D_U\pi_U+D_{UU}\pi < 0$, $\pi_{CC}D < 0$, and $(\pi_{UU}D+2D_U\pi_U+D_{UU}\pi)\pi_{CC}D-(\pi_{UC}D+\pi_C D_U)^2 \geq 0$. Consider this last condition. Because $D_{Ut} < 0$, with a sufficiently high t the sign of the last term can be dominated by the size of π_{CC} and π_{UU} which, if large enough, can allow the last condition to hold; i.e., under a sufficiently high level of differentiation, a symmetric Nash equilibrium in pure strategies can exist. Finally, when t tends to zero differentiation, D_U tends to infinity, and in that case the sign of the last term is negative, i.e., the product $\pi(.)D(.)f(.)$ is not concave in (U, C). That is, when there is a sufficiently low level of differentiation a symmetric Nash equilibrium in pure strategies cannot be guaranteed to exist under the conditions of Proposition 1. QED

This result shows that Proposition 1 is consistent with the non-existence results in the literature on oligopoly under asymmetric information with quality constraints and with no differentiation: even with a small level of differentiation a pure strategies equilibrium cannot be guaranteed to exist. Note that the Proposition 2's statement is only about non-existence under the conditions of Proposition 1, so that an equilibrium may exist for a small level of differentiation under other conditions.

However, with a sufficiently high level of differentiation an equilibrium in pure strategies may exist.[12] Note that the result is only about the possibility of existence, as there may be other conditions in the problem that do not satisfy the conditions in Proposition 1, or there may be that the level of differentiation is high enough such that there is not full coverage, as assumed above. If $\overline{\theta} - \underline{\theta}$ is small enough and the density $f(.)$ is sufficiently close to the uniform distribution there exist levels of differentiation where the pure strategy equilibrium exists under full coverage. If there is no full coverage then one can also show that a pure strategy equilibrium exists, but the necessary conditions of Proposition 1 change to include this possibility.

[12] An example of this, with an application to credit markets is available from the authors upon request.

Pure-strategy equilibria may exist with more differentiation because differentiation may not allow a firm offering one single product to all the customer types to attract all the market demand profitably.

Consider a small amount of differentiation. If all the firms offer product lines that screen some customer types, a deviating firm offering one same product to all the customer types can profitably attract all the market demand, and attract the high customer types in particular (as in Rothschild and Stiglitz [1976]). If, on the other hand, all the firms offer product lines with only one single product for all the customer types, a deviating firm can profitably attract only the high customer types (with a certain distortion in the screening device). Then, there is no equilibria in pure strategies, and the only market equilibrium involves mixed strategies.

With a sufficiently high degree of differentiation among firms, the first form of deviation does not work because offering a single product to all the customer types does not attract enough demand to be profitable. In this case, one can then obtain an equilibrium in pure strategies.

Given the conditions in Proposition 1, one can also obtain some properties of a possibly existing symmetric equilibrium. The following result shows that under certain general conditions the equilibrium, if it exists, has firms offering a single product to the top portion of the customer types (pooling at the top), and another single product to the bottom portion of the customer types (pooling at the bottom).

Proposition 3: *Consider π such that $\pi(\overline{\theta}, U, C)$ and $\pi(\underline{\theta}, U, C)$ are strictly monotone in C. Then, any pure strategies symmetric Nash equilibrium has firms offering one single product to the top portion of the customer types, and another single product to the bottom portion of the customer types.*

Proof: If the firms are screening the customer types in a certain interval we have $\beta > 0$ in that interval. Then, because $\beta\gamma = 0$, $\gamma = 0$, which implies $\mu = 0$, and therefore $\frac{d\mu}{d\theta} = 0$. From equation (5) and because in such an interval $\underline{\psi} = \overline{\psi} = 0$, this yields $\lambda = -\frac{\pi_C f}{n g_C}$ in that interval. Then, if the interval has one of the limits as $\overline{\theta}$ or $\underline{\theta}$, $\lambda \neq 0$ which contradicts the condition $\lambda(\overline{\theta}) = \lambda(\underline{\theta}) = 0$. This implies that any existing equilibrium has firms offering a same single product to the top portion of the customer types, and another single product to the bottom portion of the customer types. QED

This result is in some contrast with the results in the literature on oligopoly under asymmetric information with quality constraints and no differentiation. There, it is shown that the non-anticipatory or reactive equilibrium have complete screening of customer types. Here, we show that with some degree of differentiation a Nash equilibrium in pure strategies may actually exist, and it involves the firms offering a same product to a set of different customer types. Note that the pooling product is not necessarily at one of the quality constraints. For example, in the credit markets example the pooling contract for the good borrowers is not at the maximum possible collateral level. Note also that without differentiation, even though there are quality constraints, the suggested equilibria concepts would lead to no pooling.

This proposition is also consistent with VILLAS-BOAS AND SCHMIDT-MOHR [1999] who indicate, in a specific model with discrete types, that with less differentiation (i.e., more intense competition) firms should do more screening.

The conditions for these product line features have to do with the strict monotonicity of π on the product characteristics C, and with each firm having some constraints on the optimal level of the screening device. Note that these assumptions are not very strong in some of the examples presented above. For example, when a firm screens customer types through the use of coupons, the firm can not make customers incur negative search costs, i.e., the search costs are constrained to be positive. The same type of condition arises when a firm screening customers according to their preferences for quality is limited by an exogenous (binding) maximum quality level. In the credit markets example, firms screening borrowers according to the level of collateral required in the loan are also constrained by the fact that the collateral level has to be positive. In the same way, in insurance markets, firms screening customer types according to the level of coverage may be constrained by not being able to offer more than full coverage. In the labor market, firms screening potential employees through their education levels may be constrained by not being able to require negative education levels.

These constraints in the level of the screening device result in firms offering a single product characterized by the constrained level of the screening device (in the examples above, this represents, respectively, "no coupons", "maximum level of quality", "zero collateral", "full coverage", "zero education") to a set of different customer types. Moreover, at the other end of the product line, firms also choose to offer a single product to different customer types because it becomes too expensive to screen customers, i.e., there is too much distortion in the screening device (in the examples above the distortions are, respectively, "positive search costs", "less than the maximum quality level", "costly collateral", "less than full coverage", "inefficiently high education level").

This result of little variation in the product lines at the low and high ends and substantial variation in the middle is consistent with the couponing example that was briefly discussed in the introduction. Note, however, that alternative explanations for the observed little variation at the high and low ends could be a smaller proportion of customers at the high and low ends (jointly with the existence of a fixed cost per product being offered) or a greater homogeneity in tastes at both the high and low ends.

The results of Proposition 3 can also be shown to occur in the the local monopoly case where D^i is only a function of U^i and is independent of U^{ci}. In comparison to the traditional models under monopoly (e.g., Mussa and Rosen [1978]), here the number of customers that buy from the firm is determined by the product line for all customer types (i.e., there are random participation constraints).

An interesting and important counter-example to Proposition 3 is that of screening customer types according to their preference for quality, without a maximum binding quality level (see Rochet and Stole [2002]). C is minus one times quality, and $\widetilde{U} = \theta V(-C) - R, \widehat{\pi} = R + \alpha C$, with $C \in [\underline{C}, 0]$ with $\underline{C}$ very small, α is a constant (the marginal cost of quality), and the function $V(.)$ satisfies $V' > 0, V'' < 0, \underline{\theta} V'(0) > \alpha$, and $\overline{\theta} V'(-\underline{C}) < \alpha$. Then, one can get $\pi = \theta V(-C) - U + \alpha C$, and $\frac{dU}{d\theta} = V(-C) = g(\theta, U, C)$. This results in the functions $\pi(.)$ and $g(.)$ being concave (quasi-concave) in (U, C), as required in Proposition 1, and the condition (5) can be satisfied with $\frac{d\mu}{d\theta} = 0$ at $\overline{\theta}$ and $\underline{\theta}$, i.e., the equilibrium can have complete screening at both the top and bottom portion of the customer types, by making C such that $\overline{\theta} V'(-C(\overline{\theta})) = \alpha$, and $\underline{\theta} V'(-C(\underline{\theta})) = \alpha$. This means that in equilibrium the qualities received by the lowest and the highest customer types can be the efficient ones.

However, in this case, if, for example, there is a binding limit on the highest quality level ($\underline{C}$ is not too small, such that $\overline{\theta} V'(-\underline{C}) > \alpha$) any equilibrium may have firms offering a same product to a set of the highest customer types. Another interesting example is the case of $\alpha = 0$, in which firms screen customers by making them purchase products of less than efficient quality (increase customer search costs, increase purchasing costs through the use of coupons, etc.). See, for example, Deneckere and McAfee [1996] for some analysis on this case.

4. Conclusion

In this paper we present sufficient conditions for the existence of a pure strategies Nash equilibrium in product line competition for the case in which there is horizontal differentiation across firms and there are quality constraints. We show that a sufficient amount of differentiation yields this result. Furthermore, under certain general conditions, the equilibrium has firms offering a single product to the high customer types, and offering another single product to the low customer types. These conditions have to do with the non-existence of an optimal interior full-information level of the screening device.

These results are in contrast with the results under no–differentiation with quality constraints in the sense that in that case the equilibria were always in mixed strategies. Furthermore, the existing literature focused on the definition of equilibria which involved complete screening (e.g., Riley [1979]). One can then interpret our results as showing that under less competition (more differentiation), firms should offer shorter product lines and only partially screen customers (as there are pooling regions for both the high and low types). This is consistent with the developments in the airline industry in the eighties, where deregulation (more competition) made airline companies offer more variability in the product lines (Borenstein and Rose [1994]). Note also that this finding of shorter product lines with more differentiation can be empirically tested. Finally, the above results also predict that a high degree of differentiation yields a market where firms offer similar product lines which get stable demands, while a lesser degree of differentiation can result in different product lines across firms (mixed strategies) with some demand unstability (products being offered which do not attain their expected demand).[13]

[13]In the context of uncertain demand and product offerings, it would also be interesting to look at the effects of search costs on product line design in competition (see related work for the monopoly case in Kuksov [2004], and Villas-Boas [2004]).

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