Price Promotions and Trade Deals with Multiproduct Retailers

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In this paper we study retail price promotions and manufacturer trade deals in markets with multiproduct retailers. We find that in situations where retailers carry more than one competing brand, the promotions across brands can be positively or negatively correlated depending on the structure of the market: the relative sizes of the various market segments (in terms of loyalty to manufacturer, retailer, or the pair manufacturer-retailer). We show that sometimes retailers offer the same discount on different products, but at other times they offer a smaller discount on a brand supported by a bigger trade deal. We also present results on the effects of changes in the sizes of the different market segments on the depth of price promotions and trade deals and on pass through.

(Price Promotions; Trade Deals; Multiproduct Retailers; Category Management; Competition)

1. Introduction

One major factor in recent times in the management of frequently purchased consumer products has been the increased price sensitivity of shoppers. Recent surveys conducted by Progressive Grocer (see also Farris and Ailawadi 1993 and Messinger and Narasimhan 1995) have concluded that the state of economy in general, and consumers' personal economic situation in particular, have affected consumer lifestyles and shopping patterns in a major way. "Low prices" seem to be the most important attribute, behind cleanliness, in choosing stores—a major change, since price was only the fifth most important attribute in the last such survey. This change also seems to have been well understood in the supermarket industry, given the prevailing practices over the last ten years. According to Progressive Grocer, both manufacturers and retailers have been putting a continuous emphasis on prices and have moved away from advertising attributes like quality and value.

Given these developments, understanding the pricing interactions—and in particular, competitive price promotions—in a market where manufacturers compete with other manufacturers, retailers compete with other retailers, and retailers carry the products of several manufacturers, has become more and more important.

Price promotions are short-term price discounts offered by retailers. These price cuts are often supported by manufacturers in the form of trade deals. Most of the existing literature on price promotions1 (e.g., Varian 1980, Narasimhan 1988, Raju et al. 1992, Agrawal 1995, Simester 1996) interprets a mixed strategy to offer an explanation for price promotions. These analyses typically are conducted in the context of competing firms selling one product to satisfy the demand of a homogeneous consumer base (in terms of loyalty towards the different brands or in terms of the degree of information about the prices being charged). Since there is no equilibrium in pure strategies, these authors employ a mixed strategies equilibrium framework.2 The

1 See also Conlisk et al. (1984), Green and Porter (1984), Sobel (1984), Rotemberg and Saloner (1986), and Rao (1991) for other perspectives on price promotions.

2 Note, however, that, following Harsanyi (1973), the mixed strategies equilibria can be obtained as the limit of pure strategies equilibria in a sequence of games that are perturbed by a slight amount of private
characterization of the equilibrium probability distribution over a range of prices is viewed as promotions and is assumed to be implemented over a large number of periods where the demand over subsequent periods is independent of others. Agrawal (1995) extends the analysis of Raju et al. (1992) to include a retailer that carries both brands but does not incorporate retail competition. Simester (1997) studies the complementary products case under competition, with individual continuous demand equations (similarly to Rosenthal 1980), and without considering the manufacturers' strategic behavior. All these papers recognize, however, the need to integrate the retailer carrying substitutable brands in a competitive environment (both at the manufacturer and at the retail level) in order to provide a better understanding of price promotions.

In this paper we take a first step in this direction, building on this literature by extending the model to include multiple retailers (retail competition), more than one brand in the category (category management), a more extensive specification of consumer heterogeneity and the explicit role of manufacturers and retailers, i.e., we consider the existence of both trade deals and price promotions. Consumer heterogeneity is specified in terms of nine different segments, including those who are completely price-sensitive, those who may be loyal to either of the two manufacturer brands or the two competing retailers, and those who are extremely loyal in the sense that they buy only the preferred brand at the preferred retail outlet.

We see one of the contributions of this work to be the analysis of price promotions in a more complete institutional environment. Our approach characterizes the equilibrium pricing strategies at both the retail and the wholesale level. By conducting this analysis in a more complete/complex institutional environment, we are able to identify the conditions under which the results from previous analyses by Narasimhan and Varian continue to be valid, and also to identify conditions when these analyses may not hold.

We show that when there is no retailer loyalty, the insights offered by the above-mentioned analyses can be easily applied to these circumstances. In contrast, in situations when there are consumers who are loyal to a retailer in some form, the equilibrium may be qualitatively different from that characterized in the above described analyses. For example, with multiproduct retailer competition our analysis has implications for category management and the relationship among prices in a certain retailer. Moreover, we show that retailers might sometimes charge a higher price for the brand with the lowest wholesale price. We also show that these features of the equilibrium critically depend on the relationship between the sizes of the various segments. In particular, they depend on the comparison between the retailer's incentives to lower the prices of its higher and lower priced brands, which is one of the major results.

A second important contribution relates to the study of retail passsthrough. Manufacturers offer discounts to retailers with the assumption that some of these would be passed on to the consumers. Furthermore, it is not

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3 Information by each player (the purification theorem). In this case the private information could be on costs, size of loyalty segments, etc.

4 In this setup price promotions (or trade deals) should not be interpreted as any price below the consumers reservation price, but rather, any price "far away" from the reservation price. The analysis is performed on the continuum for simplicity, while in reality firms price on a grid. Villas-Boas (1995) tests some of the empirical implications of both the Varian and the Narasimhan models.

5 The mixed strategies type of equilibrium obtained in most of this literature (and for that matter, the equilibria obtained in this paper) depends on the existence of a mass of consumers that always buy the brand that has the lowest price, which can be seen as a likely property in frequently purchased product categories. If, on the other hand, the distribution of preferences in the population was "smooth" (and by this we mean that there is not a concentration of very price-sensitive consumers), the equilibria would be in pure strategies.

6 Trade deals often have two components: price cuts and contractual agreements from the retailer to perform some other function. We concentrate here exclusively on the first component. In the same vein, we do not consider here any of the dynamic effects of trade deals and price promotions so that the analysis be as simple as possible, given the objectives of this work.

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6 As noted above, a modeling alternative would have been to consider a smooth distribution of consumer preferences and obtain pure strategies equilibria. However, this modeling approach would not allow us to address most of the objectives of this paper: understanding price promotions and trade deals in an environment with competing multiproduct retailers (probability distributions of price promotions and trade deals, correlations of price promotions in a certain retailer, are price promotions always smaller for brands that receive smaller trade deals?, etc.).
unreasonable to expect that retailers would charge a lower retail price for the brand with the lower wholesale price. However, our analysis shows how this heuristic is dependent on the relative sizes of the various segments in a heterogeneous consumer population. Under certain conditions the equilibrium retail pricing strategies are such that the brand with the highest wholesale price may have the lowest retail price.

Finally, our analysis allows us to investigate the relationship between retail prices of competing brands at a retail outlet. In particular, one might wonder whether the retail prices of competing brands should move together or in opposite directions. We again show that this relationship depends on the relative magnitudes of the various consumer segments.

The rest of the paper is organized as follows. We present the basic framework in the next section. Section 3 offers an analysis of restricted cases that helps illustrate the impact of the different types of consumer segments, and §4 presents the main intuition for the results and some managerial insights. The more technical §5 presents the general case. We close with a summary of our results and offer directions for future research.

2. The Model

We consider the simplest framework in which one could have competing manufacturers and competing retailers carrying several products. There are two manufacturers, A and B, and two retailers, 1 and 2. The manufacturers set the prices they sell at to the retailers, $w_A$ and $w_B$, and the retailers set the retail prices, $P_{A1}$ and $P_{B1}$ (for retailer 1), and $P_{A2}$ and $P_{B2}$ (for retailer 2). The manufacturers do not price discriminate between retailers. This could be justified by the existing nonprice discrimination antitrust laws. Figure 1 illustrates the market situation.

In order to characterize the demand conditions we consider the simplest and most interpretable possible case, although stylized, in which we have the complete spectrum of types of loyalty in this market: no loyalty, loyalty to a manufacturer, loyalty to a retailer, or loyalty to a specific manufacturer brand at a specific retail outlet. This yields the existence of nine segments of consumers:

- One segment, of size $S$, consists of the price-sensitive consumers. They buy the product that has the lowest retail price ($P_{A1}, P_{B1}, P_{A2},$ or $P_{B2}$).
- Two segments, each of size $R$, consist of the retailer-loyal consumers (one segment per retailer). They go always to the same retailer, and once there, they buy the product at the lowest price.
- Two segments, each of size $M$, consist of the manufacturer-loyal consumers (one segment per manufacturer). They always buy the brand of the same manufacturer, but they go to the retailer that has it at the lowest price.
- Four segments, each of size $I$, consist of the most price-insensitive consumers, loyal to a specific manufacturer brand at a specific retail outlet (one segment per manufacturer-retailer pair).

Empirically, using scanner data, numerous studies have looked at the distribution of the relative preference of consumers for several manufacturers (see, for example, Chintagunta et al. 1991, Gönen and Srinivasan 1993), i.e., the relation between $4I + 2M$ and $2R + S$. Gönen and Srinivasan, for example, suggest that $(4I + 2M)/(2R + S)$ is close to $1/2$ for the product category investigated by them. Other studies have also looked at the distribution of the relative preference of consumers.
for retailers (see, for example, Bell 1995), i.e., the relation between $4I + 2R$ and $2M + S$. However, we do not know of any study that investigated the distribution of the relative preference of consumer over both manufacturers and retailers.

This heterogeneity among consumers can be generalized without changing the main message of the results of the model to the case in which the distribution of loyalty across consumers is smoother. As noted above, the only crucial aspect of the heterogeneity being assumed is that there is a positive mass of consumers who always buy the brand that has the lowest price. Note also that the size of the manufacturer-loyal and the price-insensitive segments can be seen as a measure of the manufacturer power. Similarly, the size of the retailer-loyal and the price-insensitive segments can be seen as a measure of the retailer power.

Note also that while conceptually the manufacturer and retailer loyalties are similar, the source of each type of loyalty might be different. While the manufacturer loyalty might result from preferences of consumers across physical attributes of the products, the retailer loyalty could depend on physical search costs of consumers.

All consumers have reservation price $r$. For simplicity, all retailers and manufacturers have zero costs of production. We look for the subgame perfect equilibrium where, first, manufacturers set wholesale prices simultaneously, and then the retailers also simultaneously set the retail prices. We restrict our attention to symmetric equilibria. The strategy to compute the equilibrium is similar to that in McGuire and Staelin (1983). First we compute the equilibrium in the retail market for all the pairs $(w_A, w_B)$. Then, we find the equilibrium in $(w_A, w_B)$ given the retail market equilibrium that follows. Before getting started, let us state a lemma (similar to one in Varian 1980, but here for the multiproduct seller) that will be useful in the subsequent analysis.

**Lemma 1.** Given $w_A < r$, and $w_B < r$, $I > 0$, $M > 0$, $R > 0$, and $S > 0$, there is no symmetric retail market equilibrium where a price is set with positive mass, i.e., all the prices being charged are drawn from a probability distribution with smooth density.

**Proof.** See appendix.

We therefore know that in our model of the retail market, the only symmetric equilibria are in mixed strategies, with no price of any product being set with positive probability.

## 3. The Restricted Cases

In order to get the intuition for the impact of the different consumer segments on the market equilibrium, we consider some of the restricted cases of the model presented above. These cases give the general intuition for the main results and will also help us understand when we should expect a positive or negative correlation in the price promotions in a certain retailer. They are particular cases of the generalized model presented in the more technical §5.

Note first that when there is no retailer loyalty (i.e., $R = I = 0$), the retailers do not have any market power (they price at cost, i.e., $P_A = w_A$ and $P_B = w_B$), and the manufacturers price exactly as in the case of the two-sellers model considered in Varian (1980) or Narasimhan (1988) (where the sellers are the manufacturers).

The results of the two-sellers model are as follows (this term is also used below). There are two sellers in the market, each selling one product (and setting the price for that product). The marginal cost is $c$. The consumers reservation price is $r$. Each seller is guaranteed a demand of $X$. Additionally, if its price is the lowest one, its demand increases by $Y$. Then the equilibrium is in mixed strategies with cumulative distribution

$$F(P) = 1 - \frac{X(r - P)}{Y(P - c)},$$

with support between

$$\frac{Xr + Yc}{X + Y} \text{ and } r.$$

If the sellers are the retailers, $P$ is $P_{iA}$ or $P_{iB}$ ($i = 1, 2$), $c$ is $w_A$ or $w_B$, $X$ is $I$ and $R$, and $Y$ is $S$ and $M$. If the sellers are the manufacturers, $P$ is $w_A$ or $w_B$, $c$ is 0, $X$ is $I$ and $M$, and $Y$ is $S$ and $R$.

In the same way as above, when there is no manufacturer loyalty (i.e., $M = I = 0$) the manufacturers do

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8 Similarly, one could also allow consumers to buy more than one unit of the product.
not have any market power (they price at cost, i.e., \( w_A = w_B = 0 \)), and the retailers price exactly as in the case of the two-sellers model where the sellers are the retailers (and we look only at the lowest price of each retailer). On the other hand, if there is no type of manufacturer switchers (i.e., \( R = S = 0 \)), all prices are equal to the reservation price \( r \); and if there is no type of retailer switchers (\( M = S = 0 \)), the retailers price at the reservation price, and the manufacturers price as in the two-sellers model.

Consider now more complicated cases in order to better understand the forces at work in the general case.

3.1. No Consumers who are Loyal to a Manufacturer but not to a Retailer \((M = 0)\)
Consider first the case when there are no manufacturer-loyal consumers, i.e., there are no consumers that are willing to shop around for the best deal available (across retailers) on a certain brand. Let us consider the incentives for the pricing decision of a retailer.

For the brand that has the lowest retail price (the lowest priced brand) the incentives are as follows: it is guaranteed a demand of \( R + I \); if the price is sufficiently low this demand can be increased by as much as \( M + S = S \) (because \( M = 0 \)).

For the brand that has the highest retail price (the highest priced brand) the incentives are as follows: it is guaranteed a demand of \( I \); if the price is sufficiently low this demand can be increased by as much as \( M = 0 \).

Then, there is no incentive of lowering the price of the highest priced brand, i.e., this will always be equal to \( r \). The equilibrium has then the following characteristics: the brand that receives the smallest trade deal is priced at \( r \) (no promotions); the brand that receives the greatest trade deal is priced as in the two-sellers model; the equilibrium in the wholesale market is also exactly the same as in the two-sellers model.

3.2. No Consumers Loyal to the Manufacturer-Retailer Combinations \((I = 0)\)
Consider now the case when there are no consumers that are loyal to the pairs manufacturer-retailer, i.e., there are no consumers that are willing to buy a certain brand in a certain retailer, whatever the prices of other brands in that retailer or in the other retailer.

Let us consider the incentives for the pricing decision of a certain retailer for its highest priced brand. The incentives are as follows: it is guaranteed a demand of \( I = 0 \); if the price is sufficiently low this demand can be increased by as much as \( M \). Then, there is no incentive to increase the price of the highest priced brand, i.e., this will always be equal to the price of the lowest priced brand.

The equilibrium has then the following characteristics: both brands are always equally promoted;\(^9\) the brand that received the greatest trade deal gets the retailer-loyal consumers and the “switchers” in the case that retailer has the lowest price in the market.\(^10\) There is then a positive correlation across brands of the promoted prices of a certain retailer.

Given this retail market equilibrium, the manufacturer market behaves in exactly the same way as the one in the two sellers model (the manufacturer with lowest price gets a demand of \( R + M + 2I \), and the manufacturer with the highest price gets a demand of \( M + 2I \)), and the equilibrium has the same form.

3.3. No Consumers Who Are Loyal to a Retailer but not to a Manufacturer \((R = 0)\)
Consider now the case when there are no retailer-loyal consumers, i.e., there are no consumers that are loyal to a certain store and once there look for the best in-store deal. Examples of product categories where \( R \) might be smaller, as compared to \( I, M, \) and \( S \), are electric appliances categories or, as discussed below, categories used as “loss-leaders.”

Consider the retail market equilibrium. Note first that the brand with the greatest trade deal is not always the most promoted one. Suppose that the brand \( A \) is the one with the greatest trade deal, i.e., \( w_A < w_B \), but the trade deals are not very different.\(^11\) Then consider that the equilibrium is such that for each retailer we have always \( P_A = P_B \), i.e., the brand most promoted is always the one that received the greatest trade deal. Then, the lowest \( P_A \) charged, \( P_A \), has to satisfy \( (P_A - w_A)(I + M + S) \)

\(^9\) Except when the retailer is supposed to promote the most promoted brand, \( A \), at a price below the price paid for the other brand by the retailer to the manufacturer \( (P_A < w_B) \). In this case we have \( P_A = w_B \).

\(^10\) The probability distribution of the equilibrium mixed strategies of the retailers is characterized in §5 for the general case.

\(^11\) This is with loss of generality. As it is made clear below, the manufacturer equilibrium results in the manufacturers charging different wholesale prices with probability one.
which results to the charge \( w_b \), which is \((P_b - w_b)\)I, the expected payoff for each retailer is \((r - w_A)I + (r - w_b)I\), and

\[
F(P_A, r) = 1 - \frac{(r - P_A)I}{(P_A - w_A)(M + S)}. \tag{12}
\]

Consider now a deviating strategy where \( P_b < P_A \), which is \((P_b = P_B, P_A = r)\). Then, this retailer gets for brand A a demand of only segment I, and for brand B a demand of segments I and M with probability one, and the segment S with the probability \(1 - F(P_b, r)\). Its expected payoff is

\[
(r - w_A)I + (P_b - w_b)(I + M) + (P_b - w_b)[1 - F(P_b, r)]
\]

\[= (r - w_A)I + (r - w_b)I + (r - P_b)\frac{I}{M + S}
\]

\[> (r - w_A)I + (r - w_b)I.
\]

Then, this deviating strategy gets a payoff greater than the equilibrium payoff, i.e., the equilibrium can not have the retailers charging only prices such that \( P_A \leq P_b \).

The reason for this result has to do with the incentives to lower prices for the most- and the least-promoted brand. For the most-promoted brand, it is guaranteed a demand of I and can increase its demand by as much as \( S + M \) if it has a sufficiently low price. For the least-promoted brand, it is guaranteed, in the same way, a demand of I, but it can increase its demand by only M.

Then the least-promoted brand has a smaller incentive to lower prices. Then, if \( P_A \leq P_B \) for all the prices being charged, this implies relatively high prices for the B brand, which creates an opportunity for a retailer to charge a very low \( P_B \) such that \( P_B < P_A \).

Furthermore, when \((w_b - w_A)\) converges to zero, the probability with which each retailer sets \( P_B < P_A \) converges to 50 percent. \(^{13}\) It can also be shown that there is an \( x_2 \) (to use the same notation as in Table 4) for which each retailer either sets \( P_A \leq x_2 \leq P_B \) or \( P_B \leq x_2 \leq x_A \) (see Table 4 when \( R = 0 \)), i.e., retailers promote heavily one or the other product. This can result in a negative correlation across brands of the price promotions of a retailer, given the wholesale prices.

Another important aspect of this case is that the manufacturers equilibrium is not of the same type of the one in the two sellers model, in contrast with the previous cases. Even when a manufacturer offers the smallest trade deal, it can get the “switchers” segment (segment S) with positive probability. The equilibrium expected payoff for each manufacturer is still equal to the payoff resulting only from the segments that are loyal in some form to the manufacturer (i.e., equal to \( r(2I + M) \)). \(^{14}\)

4. The Main Intuition

At an intuitive level and as suggested in the previous section, the main results have to do with the relative incentives a retailer has to lower the price of the highest or the lowest priced brand.

Consider the case in which a retailer has greater incentives to lower the price of the highest priced brand. This is the case if the ratio of switchers to loyal consumers for the highest priced brand is greater than the ratio of switchers to loyal consumers for the lowest priced brand. Note that the “switchers for the highest priced brand” include only the manufacturer-loyal consumers (i.e., \( M \)), while “the switchers for the lowest priced brand” include both the manufacturer-loyal consumers and the price sensitive consumers (i.e., \( S \)). Similarly, the “loyal consumers for the highest priced brand” include only the price insensitive consumers (i.e., \( I \)) while the “loyal consumers for the lowest priced brand” include both the price insensitive consumers and the retailer-loyal consumers (i.e., \( R \)). Then the retailer having greater incentives to lower the price of the highest priced brand can be represented by

\[+ I(r - w_A) = (r - w_A)I + (r - w_b)I. \]

Then, when \((w_b - w_A)\) goes to 0, we have \[F(r, x) = F(x, r)\] also going to 0. As this is true for all \( x \), and there is no mass point or mass on a line, we have that the probability of each retailer setting \( P_b < P_A \) goes to 1/2 when \((w_b - w_A)\).

\(^{13}\) The characterization of this equilibrium is presented in §5 for the general case.

\(^{14}\) The characterization of this equilibrium is presented in §5 for the general case.
\[
\frac{M}{I} > \frac{M + S}{I + R}, \quad \text{where } \frac{M}{I}
\]
is the incentive to lower the price of the highest priced brand, and \((M + S)/(I + R)\) is the incentive to lower the price of the lowest priced brand.

Then, in this case we find intuitively that the price promotions across brands for each retailer are close and move together: either deep price promotions for all the brands in the category, or light price promotions for all the brands in the category (this is presented in Proposition 1, and one extreme case is discussed in §3.2). Given this positive correlation on price promotions, the retailer always promotes more or equally the product for which it receives a greater trade deal: this is because the retailer might as well have a greater demand for the product that is yielding a greater margin, where the greater demand consists of the retailer-loyal segment for sure and the price-sensitive segment with positive probability. Also intuitively, the expected value of the trade deals offered by the manufacturers increases with increases in the size of their "switcher" segments, i.e., the retailer-loyal and price sensitive segments, and decreases with the size of their "loyal" segments, i.e., the manufacturer-loyal and the price insensitive segments.

Consider now the case in which a retailer has greater incentives to lower the price of the lowest priced brand, and this can be seen as the case of a "loss-leader" product category. This is the case if the ratio of switchers to loyal consumers for the lowest priced brand is greater than the ratio of switchers to loyal consumers for the highest priced brand, i.e.,

\[
\frac{M + S}{I + R} > \frac{M}{I}.
\]

We then find that the retailer might promote more heavily a product that did not receive the greatest trade deal. In this case, we should observe the price promotions across brands in a product category, and for each retailer, to move in different directions: one of the brands is always heavily promoted while the other brands are lightly promoted. The brand that is most heavily promoted varies from period to period. This negative correlation results in the equilibrium retail pricing strategies being such that the brand with the highest wholesale price may have the lowest retail price (this is presented in Proposition 3, and one extreme case is discussed in §3.3).

The results above can also be used to derive important managerial implications. Suppose that the incentive to lower the price of lowest priced brand is higher (this can also be seen as the case of a "loss-leader" product category) than the one to lower the price of highest priced brand. Then, in this case a retailer should promote only one brand very heavily, and keep the other brands in the category relatively unpromoted. Manufacturers also selling products in "loss-leader" type categories should also offer more medium sized trade deals rather than heavy trade deals, given that even if a manufacturer's trade deal offer is not the largest one, the retailers might still promote its product more heavily. Manufacturers should also offer heavier trade deals the greater the sizes of the price sensitive and retailer-loyal consumer segments. At the same time, retailers should price promote more heavily the greater the sizes of the price sensitive and manufacturer-loyal segments.

5. The General Case

We now consider in this more technical section the general case where all the segments have positive mass. All the intuition for the results is presented above and this section is presented for completeness. It has a combination of the results presented in §3.

In order to generate the general results we have to consider the expected payoffs for each retailer of each pair of prices being set. Without loss of generality let us consider the case in which \(w_A < w_B\). In order to compute the manufacturer equilibrium we have to first learn what happens in the retail market equilibrium for any pair of wholesale prices \((w_A, w_B)\). As seen below, the manufacturer equilibrium results, in fact, in the two manufacturers charging different wholesale prices with probability one.

Being \(F(P_A, P_B)\) the cumulative distribution function of the mixed strategy of each player \((F(r, r) = 1)\), the expected payoff if \(P_A = P_B = r\) is

\[
(I + R)(P_A - w_A) + I(P_B - w_B) + S(P_A - w_A)
\]

\[
\times [1 - F(r, P_A) - F(P_A, r) + F(P_A, P_A)]
\]

\[
+ M(P_A - w_A)[1 - F(P_A, r)]
\]

\[
+ M(P_B - w_B)[1 - F(r, P_B)].
\]

(1)
The first and second components are the retailer profits from the market segments that are loyal to that retailer for, respectively, brands A and B. Brand A gets an additional demand of \( R \) because \( P_A \leq P_B \). The third component has to do with the possibility of the brand the retailer is promoting more intensely, brand A, having the lowest price in the market, and attracting the “switchers” segment (S). This happens with probability \( 1 - F(r, P_A) - F(P_A, r) + F(P_A, P_A) \). Finally, each brand may attract the manufacturer-loyal segments (M consumers that are loyal to a certain brand and shop around for the retailer that has that brand at the lowest price). This is represented by the fourth and fifth components for, respectively, brands A and B. The probability of attracting the manufacturer-loyal segment for brand A is \( 1 - F(P_A, r) \); the probability of attracting the manufacturer-loyal segment for brand B is \( 1 - F(r, P_B) \).

If \( P_B < P_A \leq r \) the expected payoff is now

\[
(I + R)(P_B - w_B) + I(P_A - w_A) + S(P_B - w_B) \\
\times [1 - F(r, P_B) - F(P_B, r) + F(P_B, P_B)] \\
+ M(P_A - w_A)[1 - F(r, P_A)] \\
+ M(P_B - w_B)[1 - F(r, P_B)].
\]  

Proposition 1. If the incentives for a retailer to lower the price of the lowest priced brand are smaller than the incentives to lower the price of the highest priced brand (i.e., \((M + S)/(1 + R) < M/I\)) in the retail market equilibrium is in mixed strategies with each retailer never promoting less the brand which receives the greatest trade deal: (1) if the trade deals are sufficiently different, the retailers promote more heavily the brand with the greatest trade deal; (2) if the trade deals are close enough, the retailers promote both brands equally; and (3) if the difference between trade deals is in an intermediate range, the retailers, when promoting heavily, promote more heavily the brand with the greatest trade deal, and, when promoting lightly, promote both brands equally.

For \( w_A \leq w_B \), the expected payoff for each retailer given \( (w_A, w_B) \) is \((I + R)(r - w_A) + I(r - w_B)\). \(^{15}\)

A few comments and remarks are warranted here. First of all, as stated above, in this case of greater incentives for a retailer to lower the price of the highest priced brand than to lower the price of the lowest priced brand, there is a positive correlation in the price promotions across brands in each retailer, given the wholesale prices.

Second, the likelihood with which each retailer promotes strictly more the brand for which it receives a greater trade deal is greater (1) the greater the difference in trade deals, (2) the greater the incentives to lower the price of the lowest priced brand (incentives to get the price sensitive segment), (3) the smaller the incentives to lower the price of the highest priced brand (incentives to get the manufacturer-loyal segments), and (4) the smaller the level of trade deals being offered. Point (2) is a very important one: if the retailer gains a lot from getting the price sensitive segment, it will compete more for it, offering a greater depth in promotion\(^{17}\) for the

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\(^{15}\) The proof of this proposition, as well as the proofs of Propositions 2 through 4, is available upon request from the authors.

\(^{16}\) The strategies played by each retailer and each manufacturer (and obviously the conditions for cases (1), (2), and (3) in the proposition) are completely characterized in Table 1. The equilibrium strategies for the next three propositions are also completely characterized in Tables 2 through 5.

\(^{17}\) In terms of this paper, and except where noted, we will use the terms “depth of promotion” and “greater promotional intensity” interchangeably to mean first order stochastic dominance. This is, brand A is promoted more intensely than product B if \( F_A(P) \geq F_B(P) \) \( \forall P \), where \( F_\cdot(\cdot) \) represents the cumulative distribution function of the prices of brand \( i \).
brand for which it received a greater trade deal. Results (1)–(4) can be obtained directly from differentiation of the likelihood with which each retailer promotes strictly more the brand for which it receives a greater trade deal (defined in Table 1) with respect to the variables of interest.

Third, if both brands are equally promoted by the retailer, then, as expected, the depth of retail promotions is greater (greater passthrough), the greater the price-sensitive and the manufacturer-loyal segments, and the smaller the price-insensitive and retailer-loyal segments. If the brands are not supported by the same depth of retail promotions, then the depth of retail promotions of the brand that is promoted most increases (greater passthrough) with increasing size of the price-sensitive and the manufacturer-loyal segments, and decreasing size of the price-insensitive and retailer-loyal segments. However, the depth of retail promotions (passthrough) of the brand that is promoted least increases only with the size of the manufacturer-loyal segment and decreases with the size of the price-insensitive segment. These results can be obtained directly from differentiation of $F(x, y)$ (defined in Table 1) with respect to $S, M, I,$ and $R$.

Now that we have characterized the retail market equilibrium for the case in which the incentives for a

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Retail Market Equilibrium if $M/I &gt; (M + S)(I + R)$ (Proposition 1) ($w_A = w_B$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Condition</td>
<td>Equilibrium</td>
</tr>
<tr>
<td>$I(S + M(r - w_A) = M(I + R)(r - w_B)$</td>
<td>$P_A = P_B$</td>
</tr>
<tr>
<td>$R(x, r) = 1 - \frac{(I + R)(r - x)}{(M + S)x} P_A = \frac{(I + R) + (M + S)w_A}{I + R + M + S}$</td>
<td>$R(x, x) = 1 - \frac{(r - x)}{P_A}$</td>
</tr>
<tr>
<td>[Case (i)]</td>
<td></td>
</tr>
<tr>
<td>$I(S + M)(r - w_A) &lt; M(I + R)(r - w_B) &lt; I(S + M)(r - w_A) + \frac{4I + 2R + 2M + S}{2I + R}$</td>
<td>Define $\hat{x}$ as $-I(M + S)^2(\hat{x} - w_A)^2 + M(M + S)(\hat{x} - w_B)^2 + 2I + R(M + S)(\hat{x} - w_B)^2 - (2I + R)(M + S)M(r - \hat{x})(w_B - w_A) = 0$.</td>
</tr>
<tr>
<td>$\times (S + M)(w_B - w_A)$</td>
<td>If $P_A, P_B &gt; \hat{x}$, then $P_A = P_B$ and $R(x, x) = 1 - \frac{(I + R)(r - x)}{(S + M)(x - w_B) + M(x - w_A)}$</td>
</tr>
<tr>
<td>[Case (ii)]</td>
<td></td>
</tr>
<tr>
<td>$M(I + R)(r - w_B) = I(S + M(r - w_B) + \frac{4I + 2R + 2M + S}{2I + R}$</td>
<td>$P_A = P_B$</td>
</tr>
<tr>
<td>$M(S + M)(w_B - w_A)$</td>
<td>$R(x, x) = 1 - \frac{(I + R)(r - x)}{(S + M)(x - w_B) + M(x - w_A)}$</td>
</tr>
<tr>
<td>[Case (ii)]</td>
<td></td>
</tr>
<tr>
<td>$P = \frac{(2I + R + Mw_B + S + Mw_A)}{2I + R + 2M + S}$</td>
<td>$PB$</td>
</tr>
</tbody>
</table>

Note: $P_A$ and $P_B$ are the minimum prices charged for brands $A$ and $B$ in Case (i). $P$ is the minimum price being charged in Case (ii). $P_A$ and $P_B$ are the minimum prices charged for brands $A$ and $B$ in Case (iii).
retailer to lower the price of the highest priced brand are greater than the incentives to lower the price of the lowest priced brand, we can compute the manufacturers' equilibrium. Note that if both manufacturers set the same price, each of them gets expected demand of $2I + M + R + S/2$ (assuming that both retailers would treat both retailers equally). If one manufacturer sets a price lower than the other, the lowest price manufacturer gets demand of $2I + M + 2R + S$, and the highest price manufacturer gets demand of $2I + M$. Then, following Varian (1980), we know that both manufacturers play a mixed strategies equilibrium. Proposition 2 gives the result.

**Proposition 2.** If the incentives for a retailer to lower the price of the lowest priced brand are smaller than the incentives to lower the price of the highest priced brand (i.e., $(M + S)/(I + R) < M/I$), the manufacturers equilibrium is in mixed strategies and as in the two sellers model. The expected payoff for each manufacturer is $r(M + 2I)$.

Note that the depth of trade deals offered by the manufacturers is greater the greater the retailer-loyal and the price-sensitive segments, and the smaller the manufacturer-loyal and the price-insensitive segments (this results directly from the differentiation of the equilibrium mixed strategy with respect to $I, M, R$, and $S$).

One example for the price and trade promotions equilibrium strategies for the case of Propositions 1 and 2 (numerical example 1) is presented in Figures 2 through 4. The example has $R = 20, M = 20, I = 10, S = 10, r = 8$.

Now, let us consider the case in which each retailer has greater incentives to lower the price of the lowest priced brand than to lower the price of the highest priced brand. As in the previous case, we start by analyzing the retail market equilibrium. The following proposition states the result.

**Proposition 3.** If the incentives for a retailer to lower the price of the lowest priced brand are greater than the incentives to lower the price of the highest priced brand (i.e., $(M + S)/(I + R) > M/I$), the retail market equilibrium is in mixed strategies: (i) if the trade deals are close enough, the retailers promote the brand that received the smallest trade deal with positive probability; (ii) if the trade deals are sufficiently different the retailers promote always more the brand that received the greatest trade deal. For $w_A = w_B$, the expected payoff for each retailer given $(w_A, w_B)$ is $(I + R)(r - w_A) + I(r - w_B)$.

18 It is possible to show (not shown here) that in case (i) there is no equilibrium where $\min_{p_A, p_B} P_B < \max_{p_A, p_B} P_B$ and $P_B \approx P_A P_B < P_A$, i.e., there are no holes in any of the marginal distributions (which is a familiar result from Varian and Narasimhan).
Note first that, in contrast with Proposition 1, the retailers can promote more the brand for which they received the smallest trade deal (greatest wholesale price). When the incentives to lower the price of the lowest priced brand are greater than the incentives to lower the price of the highest priced brand (i.e., \((M + S)/(I + R) > M/I\)), we get that if the brand that receives the greatest trade deal, \(A\), was always the most promoted \((P_A = P_B)\), this would result in the brand that receives the smallest trade deal, \(B\), being promoted with very little intensity. There could then be a profitable opportunity (if the trade deals are not too different) for a retailer to promote more the brand that received the smallest trade deal, i.e., \(P_B < P_A\), and get a demand for this brand of \(I + M\) for sure and \(S\) with some probability. This results in the retail market equilibrium having each retailer promoting more the brand for which they received the smallest trade deal with positive probability (one extreme case is discussed in §3.3). The probability with which a retailer promotes more the brand for which it received the smallest trade is, however, always smaller than 50 percent. This can also lead, as discussed above, to a negative correlation between the price promotions across brands for each retailer, given the wholesale prices.\(^{19}\)

Second, the likelihood with which the retailers can promote more the brand with the smallest trade deal (case (i) above) is greater (1) the greater the incentives the retailer has to lower the price of the lowest priced brand in comparison with lowering the price of the highest priced brand, (2) the smaller the difference \(w_B - w_A\) (as expected), and (3) the greater the level of trade deals offered by the manufacturers. (1) is discussed above, and (2) and (3) can be obtained from a direct evaluation of the condition for case (i); see Table 3. That condition is satisfied only if \(w_B\) is close to \(w_A\), and it is more likely to be satisfied if \(w_A\) and \(w_B\) are small (if \(w_A\) and \(w_B\) are high, then the relative importance of \(w_B - w_A\) to the level of trade deals is bigger).\(^{20}\)

It is interesting to try to understand the implications of Propositions 1 and 3 for empirical correlations between pairs of competing brands in a retailer. Let us interpret the conditions of Proposition 3 (the most important retailer incentives are the ones to lower the price of the lowest priced brand) as conditions identifying potential “loss-leaders” (product categories for which a large proportion of consumers switch retailers for a sufficiently good deal in any major brand in that category). We can then see that Proposition 3 suggests that for products that are “loss-leaders,” we may obtain a negative correlation between pairs of retail prices.\(^{21}\)

Now that we have analyzed the retail market equilibrium for the case in which each retailer has greater incentives to lower the price of the lowest price brand (than the price of the highest priced brand), we can characterize the equilibrium in the manufacturer market. Note that \(w_A = w_B\) cannot be an equilibrium because a small cut in price of one of the manufacturers yields a discontinuous gain in demand, though smaller than

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\(^{19}\) Note the relation of this with Proposition 1. In Proposition 1 we have \((M + S)/(I + R) < M/I\) and the bigger \((M + S)/(I + R)\) the more likely we have \(P_B\) to be strictly smaller than \(P_A\). However, when \((M + S)/(I + R)\) becomes greater than \(M/I\), there is the possibility of \(P_B\) being promoted with “very little” intensity (when we restrict \(P_B \geq P_A\)). Then, there is an opportunity to charge \(P_B < P_A\) and gain the segments \(I + M\) of this brand for sure and the segment \(S\) with some probability. This opportunity is larger the bigger is \((M + S)/(I + R)\). Note also that, given \(w_A < w_B\), \((M + S)/(I + R)\) represents the incentives to lower the price of the lowest priced brand, be it brand \(A\) or brand \(B\).

\(^{20}\) Notice that when \(w_B - w_A\) tends to zero, the probability with which the brand with the greater trade deal is promoted more tends to \(SI/(2SI - MR) \approx 1/2\). (To see this one just has to see the value of \(1 - B(x, r)\) in Table 5 when \(w_B - w_A\) tends to zero.) This probability is strictly greater than \(1/2\) if \(MR \neq 0\).

\(^{21}\) Preliminary evidence obtained by the authors seems to support this hypothesis.
Table 3  Retail Market Equilibrium if $M/l = (M + S)/(l + R)$ (Proposition 3) ($w = w_b$)

<table>
<thead>
<tr>
<th>Condition</th>
<th>Equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_a = w_b$</td>
<td>$F(x, r) = F(r, x) = 1 - \frac{l(r - x)}{M(x - w)} \forall x = x^*$</td>
</tr>
<tr>
<td></td>
<td>$F(x, r) = F(r, x) = \frac{S + M}{2S + M} - \frac{(l + R)(r - x)}{(2S + M)(x - w)} \forall \ P_a = x = x^*$</td>
</tr>
<tr>
<td></td>
<td>$F(x, x) = 1 - \frac{2l(r - x)}{M(x - w)} + \frac{R(r - x)}{S(x - w)} \forall x = x^<em>$, with $x^</em> = \frac{(2l - R)R + SMw}{2lS + SM - RM}$</td>
</tr>
</tbody>
</table>

[Casex (i.1)]

$w_a < w_b$ and $\max_{p \in \{p_a\}} (p_a - w_b) \left[ R + \frac{(l + R)(r - P_a)}{(S + M)(P_a - w_a)} \right] = R(r - w_a)$ See Table 4

[Casex (i.2)]

$P_a = p_b$ $F(x, r) = 1 - \frac{(l + R)(r - x)}{(M + S)(x - w_a)}$

$w_a < w_b$ and $\max_{p \in \{p_a\}} (p_a - w_b) \left[ R + \frac{(l + R)(r - P_a)}{(S + M)(P_a - w_a)} \right] < R(r - w_a)$ $F(x, x) = 1 - \frac{l(r - x)}{M(x - w)}$

[Casex (ii)]

in the case of Proposition 2. At the same time, by having a slightly higher wholesale price than the competitor, a manufacturer can still get the retailer-loyal and price-sensitive segments with positive probability. Because of this, the manufacturer equilibrium is more complicated than the standard two-sellers model considered in Proposition 2, but it can be shown that the manufacturers, in equilibrium, get an expected payoff equal to $(2l + M)r$. Proposition 4 states the result.

PROPOSITION 4. If the incentives for a retailer to lower the price of the lowest priced brand are greater than the incentives to lower the price of the highest priced brand (i.e., $(M + S)/(l + R) > M/I$), the manufacturers equilibrium is in mixed strategies, but different than the two sellers model. The expected payoff for each manufacturer is equal to $r(2l + M)$.

The most interesting result in this proposition is that the manufacturer equilibrium is now different from the one in the standard two-sellers model. This is because the manufacturer with the lowest wholesale price is not assured anymore of getting the retailer-loyal and price-sensitive segments. Similarly, the manufacturer with the highest wholesale price may get the retailer-loyal and price-sensitive segments with positive probability. The expected payoff for each manufacturer is still equal to $(2l + M)r$ (which was the expected payoff in Proposition 2), but if a manufacturer charges a wholesale price equal to $r$, the other manufacturer charging a lower wholesale price gets the retailer-loyal and price-sensitive segments with probability one.

One example for the price and trade promotions equilibrium strategies for the case of Propositions 3 and 4 (numerical example 2) is presented in Figures 5 through 7. The example has $R = 10, M = 10, I = 20, S = 20, r = 8$.

As in Varian (1980), the results above can be interpreted through time with draws from the mixed strategies being obtained in every period. This is, in
Inward competition of retailers, this results in a substantial change in the equilibrium, where we have an asymmetric equilibrium in the case of retailer forward buying with only one retailer see Lal et al. (1996). The results in this paper suggest that with competing retailers, forward buying may cause less competition at the manufacturer level.

6. Conclusions

In this paper we present a general model of price promotions and trade deals, where there are competing retailers carrying several brands, manufacturers behave optimally, and there is an extensive specification of consumer heterogeneity. Consumer heterogeneity is specified in terms of nine different segments, including those who are completely price sensitive, those who may be loyal to either of the two manufacturer brands or the two competing retailers, and those who are extremely loyal in the sense that they buy only the preferred brand at the preferred retail outlet. We show that modeling retail competition and multibrand (i.e., category) management offers valuable insights.

We find that the promotions across brands are not independent and that the structure of the market is crucial for the characterization of the equilibrium. More specifically, this structure can be expressed in terms of the relative magnitude of the incentives for the retailer to lower the price of the lowest priced brand and to lower the price of the highest priced brand.

Another important possibility to be considered is allowing the retailers and manufacturers to be in an asymmetric position in the market. As in Narasimhan (1988) and given our preliminary analysis, we expect the equilibrium then to have the seller which has a larger number of loyal consumers to price its product (or products) with positive mass at the reservation price. For example, if the retailer-loyal segment is greater for Retailer 1, this retailer will then charge both its products with a positive mass at the reservation
price. Similarly also to Narasimhan (1988), we also expect that when the difference in the number of loyal consumers across sellers goes to zero, the equilibrium converges to the symmetric equilibrium described above.

An important extension that requires study in the spirit of this model is to consider complementary products (consumers buying baskets of products). Another extension that is being carried out is the comparison of the level of price promotions with and without exclusive dealing (see Lal and Villas-Boas 1993).22

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Appendix

In this appendix we prove the lemma.

Proof. Suppose there is an equilibrium where Retailer 1 sets one of its prices (say $P_{A1}$) with positive probability $\alpha$ at a certain level $p$. Then, with probability $\alpha^2$, $P_{A1} = P_{A2} = p$, and both retailers share the $M$ or the $S + M$ consumers (depending on the relation between $w_A$ and $w_B$). Then, each retailer gets $\frac{1}{2}$ of the manufacturer-loyal consumers of manufacturer $A$, or $\frac{1}{2}$ of the manufacturer-loyal consumers of manufacturer $A$ plus $\frac{1}{2}$ of the price-sensitive customers. But then Retailer 2 would be better off by changing its strategy in the following way: the same strategy as the proposed equilibrium but every time that strategy prescribed playing $P_{A2} = p$, Retailer 2 sets $P_{A2} = p - \epsilon$, where $\epsilon$ is a positive real number close to zero. Then, the payoff of retailer 2 is exactly like before (because $\epsilon$ is very close to zero) except that with probability $\alpha^2$ it now gets $M/2$ or $(M + S)/2$ more consumers than before (because, now, at price $p$, all consumers go to retailer 2). Then, in a Nash symmetric equilibrium of the retail market no price is set with positive probability. □

References


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