Optimal Reverse Channel Structure for Consumer Product Returns

by

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Abstract

Consumers often return a product to a retailer because they learn after purchase that the product does not match as well with preferences as had been expected. This is a costly issue for retailers and manufacturers. In fact, it is estimated that the U.S. electronics industry alone spent $13.8 billion dollars in 2007 to restock returned products (Lawton 2008). The bulk of these returns were non-defective items that simply weren’t what the consumer wanted. To eliminate returns and/or to recoup the cost of handling returns, many retailers today are adopting the practice of charging restocking fees to consumers as a penalty for making returns. This paper employs an analytical model of a bilateral monopoly to examine the impact of reverse channel structure on the equilibrium return policy and profit. More specifically, we examine how the return penalty is affected by whether returns are salvaged by the manufacturer or by the retailer. Interestingly, we find that the return penalty may be more severe when returns are salvaged by a channel member who derives greater value from a returned unit. Also, the manufacturer may earn greater profit by accepting returns even if the retailer has a more efficient outlet for salvaging units.

Keywords: Channels of Distribution, Pricing, Reverse Logistics
1. Introduction and Literature Review

This paper models channel management issues of consumer product returns. Product returns occur for a number of reasons. Unsold products are returned at the end of a selling season from retailers to manufacturers due to overstocking of inventory.\(^1\) Consumers return defective units under warranty to resellers for replacement or repair.\(^2\) Some returns might be the result of “opportunistic behavior,” where a consumer buys a product, uses it temporarily, and then returns it for a refund.\(^3\) But even without inventory overstocks, defective products, or opportunistic behavior, products are also returned after purchase because consumers learn that the product does not match as well with preferences as had been expected. The management of this category of product returns is the focus of this research.

In 2007 alone, the United States electronics industry spent $13.8 billion to repackage and re-sell returned products (Lawton 2008). Of those returned products, 95% were non-defective items that weren’t what the consumer was expecting. Returns occur at rates of at least 6% for electronics retailers (Strauss 2007) and as high as 35% for catalog retailers (Rogers and Tibben-Lembke 1998). It is clear that product returns from consumers are costing companies a substantial amount of money. What is not as clear is who should pay for the cost and who should take responsibility for the returned units.

Many retailers have adopted restocking fees by which consumers pay a fee to return non-defective items.\(^4\) For example, Best Buy charges a 15% restocking fee on returns of opened electronics items and 25% for appliances, while many wallpaper retailers charge restocking fees

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\(^1\) For example, see the review article by Cachon (2003).
\(^2\) For example, see Chao, Iravani, Savaskan (2009).
\(^3\) For example, see Chu, Gerstner, and Hess (1998).
\(^4\) In practice, if the product is returned because a wrong order is delivered or the unit is damaged or is defective, consumers are not charged a restocking by the seller. Such returns are not the focus of this research.
as high as 30%. While penalizing returns via a restocking fee may benefit the firm by extracting revenue and reducing return volumes, such a policy may also harm the firm by discouraging consumers from trying the product in the first place. In many instances, retailers explain that they charge return penalties to consumers because they themselves receive only partial refunds from the manufacturer. However, while there are manufacturers that offer full or partial refunds for returned non-defective items, not all returns are sent back to the manufacturer. It is sometimes possible for the retailer to put a returned unit back on the shelf and sell it as new. Other times retailers get a fraction of the new product selling price by selling open box items or by liquidating returned inventory (Strauss 2007). One of the research questions addressed in this paper is how reverse channel structure (that is, who salvages returned units) affects the return policy offered to consumers.

Danze, a manufacturer of kitchen and bath accessories such as faucets and showerheads, represents an interesting case. Danze uses a third-party liquidator to extract value from returned units. The retailer could use the same third-party liquidator, but instead the returned units are shipped from the retailer back to Danze, creating an extra cost. Although salvaging by the retailer would save the shipping cost from the retailer to the manufacturer, Danze takes on the task of salvaging returned units. PetSafe is another manufacturer that uses the third-party liquidator Channel Velocity even though the extra costs associated with getting consumer returns

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6 As of December, 2008, http://www.rehabmart.com/returns.asp tells consumers: “Healthcraft Corporation and others listed below charges RehabMart an exceptional re-stocking fee on all items….This is why we must pass these restocking fees along to our customers.” In May, 2008, technology retailer http://www.enablemart.com/Return-Policy informed consumers “the customer will be held accountable for any product restocking fee that the manufacturer will charge EnableMart.”
7 For example, a representative from Saks Fifth Avenue was quoted as saying, “if merchandise looks like it’s in saleable condition, and has not been worn, we do put it back on the sales floor” (Shame on You 2003).
8 Danze uses Channel Velocity to salvage returned units. From its website, www.channelvelocity.com, this company offers its services to retailers as well.
from the retailer to the manufacturer could be saved by having the retailer use Channel Velocity
directly. Our research helps to explain why manufacturers such as Danze and PetSafe take
returns from the retailer even though it could be more cost-efficient for the retailer to salvage the
returned units. We summarize our research objectives with the following research questions:

1. How is the equilibrium return policy offered to consumers affected by the reverse
channel structure in processing product returns?

2. Why might product returns be processed by the manufacturer even if doing so
decreases the net value of a returned unit?

Our analysis provides an explanation for observed differences in the handling of product
returns and leads to insights about how manufacturers and retailers should set prices in forward
and reverse channels. Previous research has focused on managing overstock returns from the
retailer to the manufacturer that arise because of inaccurate stocking decisions with unknown
demand. Treating price as exogenous, Pasternack (1985) finds that in order to induce the order
quantity of a vertically-integrated channel, the manufacturer should offer partial refunds on all
returns or full refunds on a fraction of returns. Kandel (1996) examines how the manufacturer’s
decision to take back returns depends on its relative advantage in disposing of the unsold
inventory, optimal risk allocation, channel members’ influence over demand, and information
asymmetry. Padmanabhan and Png (1997, 2004) show that a manufacturer’s decision to take
back returns depends on the degree of competition at the retail level, the level of uncertainty
surrounding demand, and the marginal cost of production. Allowing the demand distribution to
be price-dependent, Emmons and Gilbert (1998) demonstrate that the manufacturer can increase
both channel members’ profits by offering a positive refund for overstock returns. Iyer and
Villas-Boas (2003) prove that the manufacturer is less likely to offer a return policy for
overstock returns when the retailer has greater bargaining power. Arya and Mittendorf (2004) find that when the retailer has an information advantage, a manufacturer return policy can be used to elicit a retailer’s private information. He, Marklund, and Vossen (2008) show that the manufacturer can use a return policy to convey information about product demand. Cachon (2003) provides an extensive review of the literature examining inventory decisions and return contracts between the retailer and the manufacturer.

Our research focuses on returns of a different nature: product returns of experience goods (Nelson 1970) from consumers to the retailer that are discovered to be a poor match with consumer preferences. In contrast to returns generated by overstocks, we model a retailer that sets a price and a return penalty (i.e., a restocking fee and/or shipping charges) charged to consumers which affects both sales quantity and the number of units returned. In contrast to an overstock returns setting where retail prices are found to be decreasing with the manufacturer’s refund (e.g. Padmanabhan and Png 1997, Emmons and Gilbert 1998, He et al. 2008), we find the equilibrium retail price in dealing with consumer returns is increasing with the manufacturer’s refund.

Previous research on return policies for consumer returns abstracts away from channel relationships (Davis, Gerstner, and Hagerty 1995; Che 1996; Chu, Gerstner, and Hess 1998; Davis, Hagerty, and Gerstner 1998; Matthews and Persico 2005; Shulman, Coughlan, and Savaskan 2009; Ofek, Katona, and Sarvary 2008; Anderson, Simester, and Hansen 2009). Our research examines settings where the retailer and the manufacturer are independent channel members. As such, we take account of the fact that the manufacturer’s return policy affects the retailer’s pricing strategy, which in turn drives demand as well as the consumer’s product return behavior.
We examine three reverse channel structures. As a benchmark, we consider a vertically-integrated system (VI) in which both the forward (selling) and the reverse (salvaging) channel decisions are made by a single agent, i.e. the manufacturer. We also model a decentralized channel in which there are two possible reverse channel structures. In the first structure, the retailer sells the product as well as processing and salvaging returned units (CR). In the second structure, the manufacturer accepts returns from the retailer and salvages the returned units (CRM). For each of these reverse channel structures, we examine two contracting structures: when the manufacturer is able to charge a fixed fee or quantity discount schedule, and when the manufacturer is limited to constant per-unit wholesale and refund rates. Our model examines the interaction between contract structure and reverse channel structure, as well as how each of these affects the retail price, return penalty, sales quantity and exchanges.

Interestingly, we find that the return penalty may be higher when the retailer salvages the goods than if the manufacturer does so – even if the retailer has greater value for the returned units. Moreover, the manufacturer may earn greater profits from taking back returns itself, even if the retailer has an advantage in salvaging the units. These results are driven by the fact that the retailer’s return policy has an impact on demand as well as on the number of units returned. The retailer has an incentive to use return penalties to reduce costs and increase revenue. Such penalties hurt the manufacturer by reducing quantity demanded without enhancing wholesale revenue per unit. The manufacturer’s ability to align incentives through the joint use of a wholesale price and refund may outweigh any efficiency loss due to handling returned units.

\[9\] Kandel (1996) finds that the manufacturer may accept overstock returns even if the retailer has a salvaging advantage. The driver in that result is that by taking back returns, the manufacturer is able to take all channel profit (leaving the retailer with zero profit), essentially becoming a retailer by selling on consignment (the manufacturer’s refund is greater than or equal to its wholesale price). The result is limited by the assumption that the equilibrium order quantity is the same whether or not the manufacturer accepts returns and the assumption that retail price is an exogenous variable. In our model, the manufacturer’s return policy has an impact on demand and retail price, leaves the retailer with positive profit, and does not serve as a mechanism equivalent to selling directly to consumers.
itself. In fact, when returns pass from the retailer to the manufacturer, the manufacturer is able to induce the retailer to charge the same return penalty to consumers as would be chosen if the manufacturer sold directly to consumers.

The paper is organized as follows. In the following section, we describe the model. Next we present the model analysis and results concerning the combined optimal reverse channel structure, equilibrium pricing policies, and refund policies throughout the channel. We then provide a discussion of the model’s intuition and directions for future research.

2. The Model

2.1. Manufacturer

The manufacturer produces two horizontally differentiated products (denoted by subscript \( j; j=0,1 \)) located at 0 and 1 on a Hotelling unit line. The manufacturer acts as a Stackelberg leader, producing each product at the same per-unit marginal cost \( c \) and charging a per-unit wholesale price for each product \( w_j \) to the retailer. If the manufacturer accepts returns from the retailer arising from consumer returns, the manufacturer chooses a refund \( r_j \) for each product \( j \) to pay to the retailer per unit returned. Each product has the same per-unit salvage value \( s \) for each unit the manufacturer takes back as a return. A positive value of \( s \) reflects the manufacturer’s ability to resell a returned product through secondary channels at a price higher than the reverse logistics cost of getting back the return and remarketing it. A negative value of \( s \) represents a situation where the reverse logistics cost to the manufacturer of remarketing the returned product exceeds any resale value for the good, or the returned product is not resalable and the manufacturer disposes of it at some positive cost. A more responsive and operationally

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10 Modeling two products, rather than a single product, allows for the possibility that consumers exchange an initial purchase for a product more suited to preferences. As shown in the On-Line Supplement, the main results of the paper carry over to a single product setting.
efficient reverse logistics process ensures a higher value for $s$. If product $j$ is exchanged at the retailer level, the replacement good must be produced by the manufacturer at the marginal production cost $c$ and is sold to the retailer at the wholesale price $w_{e,j}$. We assume that $c$ is greater than or equal to $s$.\textsuperscript{11} This assumption prevents the unrealistic situation from occurring in which the manufacturer has an incentive to encourage returns to profitably salvage returned units and make an additional sale on the exchange.

### 2.2. Retailer

For each product $j$, the retailer chooses a retail price, $p_j$, and a return penalty, $f_j$, including any consumer payments for shipping, to charge consumers for a return. The return penalty, $f_j$, represents the total financial loss experienced by the consumer for buying and returning product and thus includes any shipping payments (in purchase or return) that are not reimbursed by the retailer. We examine a scenario in which the retailer may salvage returned units at a net value $s$, (symmetric for each product) and another scenario in which the retailer may send returned units to the manufacturer for a net refund equal to $r_j$. The retailer sends returned units upstream to the manufacturer if and only if $r_j \geq s_r$.

We assume that salvage value is net of all costs associated with getting the product from the consumer to the retailer (and potentially on to the manufacturer), such as shipping paid to third parties, because these costs affect the value of a return. This can be mathematically transformed into a model in which the consumer pays for shipping separately (meaning $f_j$ captures only the literal restocking fee penalty paid to the retailer). Alternatively, we could define $h$ as the exogenous fee paid to third party shippers, $s_r$ as the value to the retailer of having the unit in their possession and $f_r$ as the restocking fee paid by the consumer directly to the

\textsuperscript{11} As shown in Shulman et al. (2009), this assumption implies that the manufacturer can do no better than to treat the returned unit as new and use it to satisfy demand for new units.
retailer. By the definitions in our model, $f_j = f_j + h$ and $s_r = s_r - h$. Consequently, choosing $f_j$ given $s_r$ and $h$, is equivalent to our model specification of choosing $f_j$ given $s_r$.\textsuperscript{12} While this parametric transformation reduces the parameter space and simplifies the exposition of our analysis, it has no impact on our results. Furthermore, this simplification captures the fact that retailers ultimately choose how much it costs a consumer to return a product, and in practice retailers such as www.zappos.com choose to pay for the shipping of returned units.

2.3. Consumers

Each risk-neutral consumer is assumed to keep at most one unit of one of the products.\textsuperscript{13} In owning a product located at $x_j$ on the Hotelling line, consumer $i$ experiences consumption value equal to $u_i - d \cdot |x_j - \theta_i|$, where $d > 0$ measures the disutility per unit deviation from $i$’s ideal taste parameter $\theta_i$, and $u_i$ measures the value consumer $i$ obtains from owning a unit in the product category that perfectly matches his preferences.\textsuperscript{14} Consumers are thus differentiated on two dimensions, with $u_i$ and $\theta_i$ independently distributed. The taste parameter is distributed $\theta_i \sim U[0,1]$, and the parameter $u_i$ takes values between 0 and $\bar{u}$ with an equal density (normalized to 1) at each $u_i$. Modeling a range of $u_i$ values captures the market-expanding effect of relaxing return penalties. In other words, we will show that as the return penalty decreases, the number of consumers who initially purchase the product increases (i.e. the expected utility of buying a product is non-negative for a greater number of consumers).

\textsuperscript{12} Equivalently, the same logic follows if the retailer pays shipping charges $h$ to move the returns to the manufacturer. Defining the net refund $r = r - s - h$ with the net salvage value $s = s - h$ makes choosing the actual refund $r$ given $s$ and $h$ equivalent to our model specification of choosing $r$ given $s$.

\textsuperscript{13} Risk-neutrality by consumers is a common assumption in the literature (e.g. Davis et al. 1998, Matthews and Persico 2005, Shulman et al. 2008). Risk-averse consumers could be modeled in a mean-variance linear utility function with a risk aversion coefficient. This would serve to amplify the existing negative effect of restocking fee on initial sales. While it would affect the specific return policy, it would not qualitatively alter our results. In the interest of parsimony, risk-aversion is excluded from the model.

\textsuperscript{14} Clearly, the consumer may be male or female; for ease of exposition, we characterize the consumer as “he.”
Consumers are equally and completely uninformed about the value of $|x_j - \theta|$ for each product before purchase. However, they share a common and known distribution of product fit before the initial purchase: $|x_j - \theta| \sim U[0, 1]$. Therefore, consumers who decide to make a purchase in the product category will randomly decide between the two products if the prices and return penalties are symmetric. Otherwise, consumers will purchase the product that offers the greatest expected utility. We recognize that consumers, in reality, may have some prior knowledge about which product they will prefer. We make this assumption to simplify an already complex problem in which the important element is that consumers have uncertainty about product fit, which in turn may trigger a return. If, instead, consumers were partially informed about their product fit, there could be fewer returns by infra-marginal consumers, but the price and return penalty would still have an impact on the marginal consumer and thus on the total number of initial purchases and product returns. We also allow for the possibility (occurring with probability $(1-\alpha)$) that a consumer, whose initial beliefs lead him/her to initially buy, discovers after purchase that neither product fits with preferences. In this case, although the a priori expected utility from purchase is positive, the ex post utility to this consumer of owning either good is in fact revealed to be zero. Meanwhile, with probability $\alpha$, the consumer discovers after purchase that his/her utility for the product category is indeed positive, although it is possible that the initially-purchased product is not the best fit to this consumer’s preferences.

Table 1 summarizes our model’s variables and definitions.
Table 1: Parameters and decision variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>Manufacturer’s marginal cost of product</td>
</tr>
<tr>
<td>$s$</td>
<td>Manufacturer’s salvage value of a returned unit (net of costs)</td>
</tr>
<tr>
<td>$s_r$</td>
<td>Retailer’s salvage value of a returned unit (net of costs)</td>
</tr>
<tr>
<td>$d$</td>
<td>Consumer disutility per unit of deviation from match with preferences</td>
</tr>
<tr>
<td>$u_i$</td>
<td>Consumer $i$’s reservation utility for perfect match</td>
</tr>
<tr>
<td>$\bar{u}$</td>
<td>Upper bound on $u_i$ (i.e. the maximum possible reservation utility)</td>
</tr>
<tr>
<td>$x_j$</td>
<td>Location of product $j$</td>
</tr>
<tr>
<td>$\theta_i$</td>
<td>Consumer $i$’s ideal taste parameter</td>
</tr>
<tr>
<td>$(1-\alpha)$</td>
<td>Probability that consumer’s ex-post utility equals zero for each product</td>
</tr>
<tr>
<td>$p_j$</td>
<td>Retail price for product $j$</td>
</tr>
<tr>
<td>$f_j$</td>
<td>Consumers’ return penalty for product $j$ including shipping costs</td>
</tr>
<tr>
<td>$w_j$</td>
<td>Manufacturer’s wholesale price for product $j$</td>
</tr>
<tr>
<td>$r_j$</td>
<td>Retailer’s net refund for product $j$ (the difference between the refund paid by manufacturer and the shipping costs paid by retailer)</td>
</tr>
<tr>
<td>$T$</td>
<td>Fixed fee paid by the retailer in a two-part tariff</td>
</tr>
</tbody>
</table>

After making a purchase at the retail price, the consumer learns how well it fits his/her preferences. Among the fraction $\alpha$ of consumers who have positive product-category utility, a return and exchange is triggered if a consumer realizes that the actual utility of keeping the initially-purchased product is less than can be obtained from returning it and buying an alternative product.\(^{15}\) The fraction $(1-\alpha)$ of consumers who discover after initial purchase that their category consumption utility is zero, meanwhile, will return the initially-purchased product without buying an alternative product.\(^{16}\)

\(^{15}\) In the interest of parsimony, we examine scenarios in which a fraction of the $\alpha$ consumers who do not lose their base valuation for the product offering will choose to purchase the other, better-matching product. We show in the On-Line Supplement that the “return and exchange” behavior is the optimal consumer strategy for a wide array of parameter values (exchanging offers greater utility than returning without subsequent purchase, for those consumers with positive category consumption utility).

\(^{16}\) Davis et al (1998) consider the possibility that consumers may act opportunistically and return a product after extracting value from its use before the return. It is estimated that returns resulting from this “free renting” account for less than 5 percent of all returns. (Middleton 2007). In the interest of parsimony, we abstract away from this possibility.
The sequence of events is depicted in Figure 1 below.

**Figure 1: Sequence of Events and Payoffs**

For a given consumer $i$ who initially purchases product $j$, we can write the probability $\phi_{eq}(f_j, p_j, p_{eq}; d, \alpha)$ that the consumer exchanges this purchase for the product located at $x_{eq}$, the probability $\phi_{ej}(f_j, p_j, p_{ej}; d, \alpha)$ that the consumer keeps this purchase, the probability $\phi_r$ that the consumer returns this purchase without exchange, and the expected utility *ex ante* of making this purchase ($E_j(utility)$), as\(^{17}\):

\[
\phi_{eq}(f_j, p_j, p_{eq}; d, \alpha) = \alpha \left(\frac{1}{2} + \frac{f_j - p_j + p_{eq}}{2d}\right)
\]

\[
\phi_{ej}(f_j, p_j, p_{ej}; d, \alpha) = \alpha \left(\frac{1}{2} - \frac{f_j - p_j + p_{ej}}{2d}\right)
\]

\[
\phi_r = 1 - \alpha
\]

\[
E_j(utility) = \alpha \left(u_i - \frac{p_j + p_{eq} + f_j}{2} - \frac{d}{4} + \frac{(f_j - p_j + p_{eq})^2}{4d}\right) - (1 - \alpha) f_j
\]

\(^{17}\) Proof of this equation is in the On-Line Supplement.
Notice from equation (1) that a higher return penalty increases the likelihood that consumers who make a purchase will keep it, rather than exchanging it. A higher return penalty also decreases the expected utility of initially making a purchase (if the exchange probability is non-negative), because consumers anticipate that this penalty will either induce them to keep a product that doesn’t match as well with preferences or be charged when a return or exchange is made.\(^{18}\) Consumers will purchase the product that offers the greatest utility and will make a purchase initially if and only if their expected utility is greater than or equal to zero, that is, if \(u_i\) is sufficiently high. Given the inherent symmetry of the products’ demand and cost structures, we present only the symmetric model structure and results here \((w_j = w_{x_j} = w, \ r_j = r_{x_j} = r, \ p_j = p_{x_j} = p, \ \text{and} \ f_j = f_{x_j} = f)\).\(^{19}\) Therefore, the initial sales, \(q_b(f, p; d, \bar{u}, \alpha)\), can be written as

\[
q_b(f, p; d, \bar{u}, \alpha) = \bar{u} - p - \frac{d}{4} - \frac{(2 - \alpha)f}{2\alpha} + \frac{f^2}{4d}.
\]  

(2)

Equation 2 captures an important phenomenon related to product return policies. Although a lower return penalty (lower \(f\)) may result in an increase in returns, there is a market expansion effect of the lenient return policy because more people may be willing to initially try the product, knowing they can return it later.\(^{20}\) Therefore, a penalty intended to reduce and recoup costs associated with returns may also reduce revenue by forgoing the market expansion effect of a lenient return policy.

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\(^{18}\) \(\frac{\partial E_j(\text{utility})}{\partial f_j} = \frac{2d - \alpha(f_j - p_j + p_{x_j} + d)}{2d} < 0 \text{ for } d \text{ such that exchanges are non-negative } (d > f_j - p_j + p_{x_j}).\)

\(^{19}\) We show in the On-Line Supplement that allowing the manufacturer to set asymmetric prices does not alter the equilibrium profit, sales quantity, and return quantity from symmetric levels, under either vertical integration or a decentralized channel.

\(^{20}\) \(\frac{\partial q_b(f, p; d, \bar{u}, \alpha)}{\partial f} = -\frac{2d - \alpha(f + d)}{2ad} < 0 \text{ for } d \text{ such that exchanges are non-negative } (\text{which holds for } f < d).\)
3. Model Analysis and Results

As a benchmark to the later analysis of decentralized channel systems, we first model pricing and return policy decisions in a vertically-integrated system that can choose the salvaging technology of the manufacturing level \( (s) \) or the retailing level \( (s_r) \), whichever is greater. Assuming symmetry in prices and return penalties, the vertically-integrated system’s objective function can be written as:

\[
\max_{p,f} \pi_{VI} = q_s(f, p; d, \bar{u}, \alpha)((p-c) + (-p + f + \max\{s, s_r\})(\phi_e(f; d, \alpha) + \phi_r) + (p-c)\phi_r(f; d, \alpha)),
\]

where

\[
q_s(f, p; d, \bar{u}, \alpha) = \bar{u} - p - \frac{d}{4} - \frac{(2-\alpha)f}{2\alpha} + \frac{f^2}{4d} , \quad \phi_e = (1-\alpha) \text{ and } \phi_r(f; d, \alpha) = \alpha(1 - \frac{f}{2d}).
\]

The first expression in the profit function of the vertically-integrated system denotes the number of customers who have high enough expected valuation for the product category to make an initial purchase. This is multiplied by the average margin that a purchase generates for the vertically-integrated system. The margin expression explicitly takes into account the possibility of returns with and without subsequent product purchase (i.e. exchange).

We compare and contrast the optimal channel decisions of the vertically-integrated systems to those in the decentralized systems. We examine the optimal reverse channel structure in a decentralized channel (i.e., whether the manufacturer or the retailer should be responsible for extracting value from returned units) under two scenarios: (i) when the manufacturer charges the retailer a fixed fee and per-unit wholesale price (or equivalently offers a quantity discount schedule) along with a per-unit refund, and (ii) when the manufacturer is limited just to the per-unit wholesale price and refund components in its contract with the retailer. In each structure,

\[\text{21 This is derived from equation (1) by setting } f = f_r = f, \text{ and } p_j = p_{s_j} = p.\]
both the retailer and the manufacturer are assumed to be risk-neutral and can rationally forecast the number of returns. Hence, we assume that the manufacturer and retailer have enough production capacity and inventory, respectively, to supply demand. Therefore, in this case, the manufacturer’s production technology can be either make-to-stock or make-to-order without affecting model results. The vertically-integrated structure and the two decentralized channel structures are depicted in Figure 2.

**Figure 2: Reverse Channel Structures**

3.1. Fixed Fees or Quantity Discount Schedules

First, we examine a two-part tariff wholesale contract, under which the manufacturer charges the retailer a fixed fee \( T \) and a wholesale price \( w \) to acquire units of product to resell to consumers. We allow as well for the manufacturer to offer a refund rate for each unit returned from the retailer to the manufacturer \( r \). Under this contracting structure, the retailer’s objective function can be written as follows:
\[
\max_{p,f} \pi_{\text{ret}} = q_b(f, p; d, \overline{u}, \alpha)((p - w) + (-p + f + \max\{s_r, r\})(\phi_r(f; d, \alpha) + \phi) + (p - w)\phi_r(f; d, \alpha)) - T.
\]

This formulation inherently captures two possible reverse channel structures. When \( r^* \geq s_r \), returns are accepted by the manufacturer; when \( r^* < s_r \), the retailer assumes the product salvaging responsibility. When the manufacturer takes back returns and offers \( r^* \geq s_r \), the manufacturer’s objective function is given by:

\[
\max_{w,T} \pi_{\text{mfgr}}^{\text{CRM}} = T + q_b(f(w, r), p(w, r); d, \overline{u}, \alpha) \cdot ((w - c) + (w - c - (r - s)) \cdot \phi_r(f(w, r); d, \alpha) - \phi_r \cdot (r - s))
\]

s.t. \( \pi_{\text{ret}} \geq 0 \)

If the manufacturer does not accept returns (\( r^* < s_r \)), then the manufacturer’s objective function is given by:

\[
\max_{w,T} \pi_{\text{mfgr}}^{\text{CR}} = T + q_b(f(w), p(w); d, \overline{u}, \alpha) \cdot ((w - c) + (w - c) \cdot \phi_r(f(w); d, \alpha)).
\]

s.t. \( \pi_{\text{ret}} \geq 0 \)

With a fixed fee and wholesale price (but no returns policy from the retailer to the manufacturer, as depicted in the “CR” function above), the manufacturer is able to essentially “sell the firm” to the retailer; when the retailer places higher salvage value on returned units than does the manufacturer (\( s_r > s \)), the pure two-part tariff (without any refund policy offered by the manufacturer to the retailer) achieves the same outcome as with vertical integration. If the manufacturer is also able to offer a refund to the retailer for returned units in addition to using the two-part tariff in wholesale pricing, the outcome of a vertically-integrated system can be achieved for all salvage values (\( s_r \) greater than, equal to, or less than \( s \)). The equilibrium outcome is described in Table 2 and proven in the On-Line Supplement.
Table 2. Equilibrium price, return penalty, quantities and profit in a Vertically-Integrated (VI) System; Also achievable under a \{fixed fee, wholesale price, refund rate\} Wholesale Contract

<table>
<thead>
<tr>
<th>Term</th>
<th>Equilibrium Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retail Price</td>
<td>( p^\text{VI} = \frac{\bar{u}}{2} + \frac{(c-\max{s,s_r})^2}{8d} + \frac{2(c+\max{s,s_r})-d-(1-\alpha)(c-\max{s,s_r})}{2\alpha} )</td>
</tr>
<tr>
<td>Return Penalty</td>
<td>( f^\text{VI} = c - \max{s,s_r} )</td>
</tr>
<tr>
<td>Quantity Sold Initially</td>
<td>( q_b^\text{VI} = \frac{\bar{u}}{2} + \frac{(c-\max{s,s_r})^2}{8d} - \frac{2(3c-\max{s,s_r})+d-(1-\alpha)(c-\max{s,s_r})}{2\alpha} )</td>
</tr>
<tr>
<td>Exchange Probability</td>
<td>( \phi_e^\text{VI} = \alpha \left( \frac{1}{2} - \frac{(c-\max{s,s_r})}{2d} \right) )</td>
</tr>
<tr>
<td>Channel Profit</td>
<td>( \pi^\text{VI} = \alpha(c-\max{s,s_r})^2 - \frac{(4+2\alpha)c+d+2d((2-\alpha)(\alpha^2+2\alpha))}{64\alpha^2} )</td>
</tr>
<tr>
<td>Wholesale Price</td>
<td>( w^\text{VI} = c )</td>
</tr>
<tr>
<td>Refund Rate</td>
<td>( r^\text{VI} = s )</td>
</tr>
</tbody>
</table>

Because our focus is on developing an understanding of which channel member should be responsible for salvaging returned units, we restrict our attention to parameter values such that exchanges are positive: \( d > c - \max\{s,s_r\} \).\(^{22}\) Interestingly, the optimal return penalty in the vertically-integrated system simply passes on all costs associated with the exchange to consumers, i.e. \( f^\text{VI} = (c-\max\{s,s_r\}) \). The optimal reverse channel structure with a manufacturer-to-retailer contract involving a fixed fee, per-unit wholesale price, and per-unit refund rate is described in Proposition 1.

**PROPOSITION 1:** Consider a manufacturer who can charge a fixed fee, a per-unit wholesale price to the retailer, and a per-unit refund rate. If and only if \( s > s_r \), then the manufacturer earns greater profit from the reverse channel structure in which the manufacturer salvages

---

\(^{22}\) From equation (1), exchanges are positive if \( d > f \); and, given the equilibrium values in Table 2, this implies immediately that exchanges are positive if \( d > c - \max\{s,s_r\} \).
returns than the reverse channel structure in which the retailer salvages returns. The equilibrium is described in Table 2.

Proof: see the On-Line Supplement.

Proposition 1 confirms the intuition that when full coordination is achievable, the manufacturer will choose to allocate the responsibility for salvaging returns to the channel member who can extract the greatest salvage value. That is, the manufacturer will accept returns from the retailer if and only if the manufacturer has greater value for the returned units than does the retailer. This contracting form implies that:

• the responsibility for returns would be efficiently allocated, with the manufacturer accepting returns only if it can extract greater value from returned units than can the retailer;

• as such, we would not observe manufacturers selling returned units to salvagers or liquidators, because allocating this activity to the retailer instead would save the cost of shipping returned units from the retailer to the manufacturer;

• the retailer would earn zero profit; and

• the manufacturer’s wholesale price would equal the marginal production cost, and the refund rate offered to the retailer would equal the manufacturer’s salvage value for returned units.

In practice, the manufacturer may not always offer fixed fees or quantity discounts. Therefore, we examine the equilibrium in a decentralized channel when wholesale and refund contracts involve only a per-unit price, but no fixed fee. We identify the optimal reverse channel choice of a manufacturer selling through an independent retailer, and characterize the equilibrium outcome in each reverse channel structure.

23 In personal interviews, store managers from a large department store and a large discount retailer claimed that none of their suppliers offered quantity discounts or charged a fixed fee. These stores carry clothing, food, electronics, and home furnishing items.
3.2. Wholesale Contracts with Per-Unit Wholesale Price and Per-Unit Refund

In a decentralized channel with a wholesale contract involving a per-unit wholesale price charged to the retailer, the manufacturer chooses between two reverse channel structures. In either case, consumers return unwanted products to the retailer; but from that point on, the manufacturer may allocate responsibility for salvaging returned units to the retailer, or choose to salvage the returned units itself, offering the retailer a refund per unit returned upstream. In each case, the manufacturer is the Stackelberg leader choosing a per-unit wholesale price $w$ to charge the retailer and, when accepting returns, a per-unit refund $r$ for each unit returned from the retailer to the manufacturer. When the manufacturer does not accept returns from the retailer, the effective refund rate is $r=0$. Ultimately, the manufacturer chooses the optimal reverse channel structure and sets the wholesale price (and the refund rate to the retailer), taking the outcomes in each reverse channel structure into account.

For each unit initially sold, the retailer earns the unit margin $(p-w)$. Each consumer who buys initially faces a probability $\phi_e(f;d,\alpha)$ that he will exchange the unit and a probability $\phi_r$ that he will return the unit without exchanging it. In expectation, total exchanges therefore equal $\phi_e(f;d,\alpha)q_e(f;p;d,\bar{u},\alpha)$ and total returns (without an exchange) are equal to $\phi_r(f;d,\alpha)q_r(f;p;d,\bar{u},\alpha)$. For each unit returned, the retailer pays out a consumer refund equal to $(p-f)$, and then returns it to the manufacturer for a refund $r$ or salvages the returned unit at a net value $s_r$ (if the manufacturer does not accept returns or if $r<s_r$). If the return is part of an exchange, the retailer also sells the unit for which the initial purchase is exchanged at a unit margin $(p-w)$.

The retailer chooses price and return penalty to maximize its profit. The retailer’s objective function can be written as:
\[
\max_{p,f} \pi_{ret} = q_b(f, p; d, \alpha; \omega, \alpha) \cdot ((p - w) + (-p + f + \max\{s_r, r\}) \cdot (\phi_r(f; d, \alpha) + \phi_r) + (p - w) \cdot \phi_r(f; d, \alpha)),
\]

where \(\phi_r(f; d, \alpha)\), \(\phi_r\), and \(q_b(f, p; d, \alpha; \omega, \alpha)\) are defined in equations (2) and (3).

The retailer will only return units to the retailer if the refund to the retailer, \(r\), is at least as large as the retailer’s own salvage value; hence the term \(\max\{s_r, r\}\) in the retailer’s optimization problem. This implies that the reaction functions defining the retailer’s profit-maximizing price and return penalty are functions of \(w\) and \(r\):

\[
p(w, r) = \frac{\bar{\alpha} + (w - \max\{s_r, r\})^2}{2d} + \frac{2(w + \max\{s_r, r\}) - d}{8} - \frac{(1 - \alpha)(w - \max\{s_r, r\})}{2\alpha}
\]

\(f(w, r) = w - \max\{s_r, r\}\).

The retailer’s best response function illustrates a key difference between consumer returns and overstock returns. Holding wholesale price constant, the retail price is increasing in the refund offered by the manufacturer to the retailer.\(^{24}\) This is in contrast to the finding that retail prices are decreasing in the manufacturer’s refund for overstock returns (e.g. Padmanabhan and Png 1997, Emmons and Gilbert 1998, He et al. 2008). The manufacturer’s refund to the retailer has a different impact on retail price in consumer returns than overstock returns, because of its impact on the return penalty that the retailer charges to consumers. The retailer facing a lower cost of returns (due to a greater refund from the manufacturer for items returned up channel) in turn charges a lower return penalty to consumers. This raises consumers’ expected utility of purchase, and thereby their willingness to pay, allowing the retailer to optimally charge a higher price in the presence of a higher refund.

\(^{24}\) For \(r \geq s_r\), \(\partial p(w, r \mid r > s_r) / \partial r = (2d - \alpha(w - r + d)) / 4ad\). Note that at \(f(w, r) = w - r\), the probability of exchange from equation 1 is equal to \(\phi_r = \alpha(d - (w - r)) / 2d\) which is greater than zero iff \(d > w - r\). Therefore \(\partial p(w, r \mid r > s_r) / \partial r > 0\) for equilibrium values for which the exchange probability is non-negative.
Whether the manufacturer accepts responsibility for returned units or not, the manufacturer earns its margin \((w-c)\) on units sold initially as well on units for which consumers exchange their original purchases. If it is optimal to accept responsibility for returns, the manufacturer also pays out a refund \(r\) and earns the salvage value \(s\) for each exchange. The manufacturer’s objective function is then:

\[
\max_{w,r,I_z} \pi_{mfg} = q_y(f(w,r), p(w,r); d, \bar{u}, \alpha) \cdot ((w-c) + (w-c - I_z \cdot (r-s)) \cdot \phi_y(f(w,r); d, \alpha) - I_z \cdot \phi_r \cdot (r-s))
\]

where \(I_z\) is the manufacturer’s choice of reverse channel structure; \(I_z\) is an indicator variable equal to one if the manufacturer accepts and salvages returned units from the retailer (CRM reverse channel structure) and equal to zero otherwise (CR reverse channel structure). For \(I_z\) equal to zero, the refund rate is effectively equal to zero. The manufacturer’s choice of reverse channel structure is a choice variable rather than a corner solution of the condition \(r<s_r\) because the two optimization problems are not equivalent. The refund rate \(r\) is chosen under the assumption that the returned unit is worth \(s\) and is salvaged by the manufacturer. When the manufacturer chooses to have the retailer salvage units, each returned unit has a value \(s_r\) which is captured by the retailer (rather than the manufacturer) and the wholesale price is set accordingly.

It may appear that returns clearly benefit both the manufacturer and retailer if the manufacturer allocates responsibility for salvaging the units to the retailer, because the retailer gets the salvage value \(s_r\) from a returned unit and the manufacturer gets to make another sale on exchanged units. However, absent a return penalty, returns create a cost for the retailer. The retailer’s profit from selling a good that is kept by a consumer is \(p-w\). The retailer’s profit from selling a good that is exchanged (with a full refund) is \((p-w) - (p-s_r) + (p-w) = p - 2w + s_r\).

Thus, for any \(s_r < w\), the retailer would earn less profit from a unit exchanged than a unit kept by
the consumer, unless this cost is passed to consumers in the form of a return penalty. When consumers are charged a return penalty, the manufacturer’s initial sales go down (because all consumers’ expected utility of making an initial purchase goes down) and thus the manufacturer is not strictly better off if there are more exchanges unless the retailer does not charge a return penalty.

The model is solved recursively; equilibrium results are as presented in Table 3:

Table 3. Equilibrium under a \{Wholesale Price, Refund Rate\} Wholesale Contract

<table>
<thead>
<tr>
<th>Term</th>
<th>Equilibrium Value †</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retail Price</td>
<td>[ \rho^* = \frac{\alpha}{2} \left( w^* - \max{r^<em>, s_r} \right)^2 + \frac{2(w^</em> + \max{r^<em>, s_r}) - d}{8} - \frac{(1 - \alpha)(w^</em> - \max{r^*, s_r})}{2\alpha} ]</td>
</tr>
<tr>
<td>Return Penalty</td>
<td>[ f^* = w^* - \max{r^*, s_r} ]</td>
</tr>
<tr>
<td>Quantity Sold Initially</td>
<td>[ q_b^* = \frac{\alpha}{2} \left( w^* - \max{r^<em>, s_r} \right)^2 + \frac{d^2 + 2d(w^</em> + \max{r^<em>, s_r}) - (w^</em> - \max{r^*, s_r})^2}{8d} ]</td>
</tr>
<tr>
<td>Exchange Probability</td>
<td>[ \phi_e^* = \alpha \left( \frac{1}{2} - \frac{w^* - \max{r^*, s_r}}{2d} \right) ]</td>
</tr>
<tr>
<td>Retailer Profit</td>
<td>[ \pi^<em>_{\text{rel}} = \frac{(d^2\alpha - \alpha(w^</em> - \max{r^<em>, s_r})^2 + 2d((2 + \alpha)w^</em> - (2 - \alpha)\max{r^*, s_r} - 2\alpha\bar{u}))^2}{64\alpha d^2} ]</td>
</tr>
<tr>
<td>CRM Manufacturer Profit ((I_z=1))</td>
<td>[ \pi^*_{\text{mfr}} = \frac{\alpha(c - s)^2 + d(2s(2 - \alpha) + \alpha(4\bar{u} - d) - 2c(2 + \alpha))}{128\alpha d^2} ]</td>
</tr>
<tr>
<td>CRM Wholesale Price ((I_z=1))</td>
<td>[ w^*_{\text{CRM}} = c + \frac{\bar{u}}{2} + \frac{(c - s)^2}{8d} - \frac{2(3c - s) + d}{8} - \frac{(1 - \alpha)(c - s)}{2\alpha} ]</td>
</tr>
<tr>
<td>CRM Refund Rate ((I_z=1))</td>
<td>[ r^*_{\text{CRM}} = s + \frac{\bar{u}}{2} + \frac{(c - s)^2}{8d} - \frac{2(3c - s) + d}{8} - \frac{(1 - \alpha)(c - s)}{2\alpha} ]</td>
</tr>
</tbody>
</table>
While the reverse channel structure is in itself a choice of the manufacturer, we examine how this choice will affect the equilibrium return penalty charged to consumers. The following observation is derived from comparing the equilibrium return penalty, \( f \), when the manufacturer accepts responsibility for returned units to that when the retailer salvages returns.

**OBSERVATION 1:** Consider a situation in which the manufacturer charges the retailer a per-unit wholesale price but not a fixed fee. If \( s > s_r \), then the return penalty, \( f \), charged to consumers is greater when the retailer salvages returned units than when the manufacturer accepts and salvages returned units from the retailer. There exists a critical salvage value for the manufacturer, \( \tilde{s} < s_r \), such that \( \tilde{s} < s < s_r \) also implies the return penalty, \( f \), charged to consumers is greater when the retailer salvages returned units than when the manufacturer accepts and salvages returned units from the retailer. If \( s \leq \tilde{s} \), then the return penalty, \( f \),
charged to consumers is lower when the retailer salvages returned units than when the manufacturer accepts and salvages returned units from the retailer.

Proof: See On-Line Supplement.

Observation 1 shows that it is possible for the equilibrium return penalty to be higher in the reverse channel structure for which the returned units have greater value – specifically, it is possible for the retailer to charge a higher return penalty when salvaging returns than if the manufacturer salvaged returns, even when the retailer’s salvage value exceeds that of the manufacturer. In contrast, previous research (Davis et al. 1995, Matthews and Persico 2005, Shulman et al. 2009) shows the more expected result that sellers with higher salvage values charge lower return penalties. Holding constant the channel member who handles returns, the current model also predicts a negative relationship between salvage value and return penalty. Further, if the manufacturer has a higher salvage value than the retailer ($s > s_r$), this negative relationship between salvage value and return penalty also holds when comparing a channel where the manufacturer handles returns to one where the retailer handles returns. However, the negative relationship between salvage value and return penalty does not always hold when comparing the identity of the channel member handling returns, if $s < s_r$; a retailer whose salvage value for returned units exceeds that of the manufacturer may actually charge a higher return penalty when it is the channel member responsible for returns processing than when the less-efficient manufacturer is responsible for salvaging returned units (i.e., when $\tilde{s} < s < s_r$). To understand the intuition, consider the manufacturer’s refund when salvaging returned units. The manufacturer pays more to the retailer for a return than the product’s salvage value. Therefore, there is a range of potential salvage values for which the refund offered by the manufacturer to the retailer is greater than the retailer’s own salvage value and the retailer responds by offering a
more generous return penalty to consumers. In the following proposition, we describe when the manufacturer would choose to accept and salvage returned units from the retailer and when responsibility for returns will be allocated to the retailer.

PROPOSITION 2: Consider the situation in which the manufacturer charges the retailer a per-unit wholesale price but not a fixed fee. If $s > s_r$, then the manufacturer accepts and salvages returned units from the retailer. There exists a critical salvage value for the manufacturer, $\overline{s} < s_r$, such that $\overline{s} < s < s_r$ also implies the manufacturer will accept and salvage returned units from the retailer. If $s \leq \overline{s}$, then the manufacturer chooses to allocate responsibility for salvaging returned units to the retailer.

Proof: See On-Line Supplement.

As one might expect, Proposition 2 shows that if the manufacturer is more efficient than the retailer in salvaging returned units ($s > s_r$), then the manufacturer would optimally take responsibility for this task.25 Surprisingly, the manufacturer may find it optimal to salvage returned units even if the retailer can extract greater value from returned units ($s < s_r$), as long as the difference in salvage values is not too great. The manufacturer will choose to have the retailer salvage returned units only if the retailer has a sufficiently significant advantage in salvage value over the manufacturer. The driver of this result is demonstrated by the following proposition comparing the equilibrium outcome with a contract involving a per-unit wholesale price (but no fixed fee) to that of a vertically-integrated system.

PROPOSITION 3: Consider a situation in which the manufacturer charges the retailer a per-unit wholesale price but not a fixed fee. If the manufacturer’s salvage value is greater than the

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25 If the manufacturer were able to re-sell a returned product from one region into a different market region as a new product, this could be a reason for the manufacturer’s salvage value, $s$, to be greater than the retailer’s salvage value, $s_r$. We thank the Editor for this insight.
retailer’s salvage value \( s \geq s_r \) the manufacturer can replicate the return penalty charged to consumers in a vertically-integrated channel by accepting product returns from the retailer. The retail price charged to consumers will be distorted upward and manufacturer profit will be distorted downward from a vertically-integrated channel. Otherwise, when \( s < s_r \), the return penalty charged to consumers will be greater in a decentralized channel than in a vertically-integrated system.

Proof: See On-Line Supplement.

Proposition 3 shows that total channel profit with per-unit wholesale pricing to the retailer (but no fixed fee) is less than in a vertically-integrated system achievable with the addition of a fixed fee to the retail contract. While the per-unit wholesale pricing contract results in double marginalization on initial sales \( (p^* > p^{VI}) \), the manufacturer may eliminate double marginalization on returns \( (f^* = f^{VI}) \) by accepting returned units when the manufacturer’s salvage value is the same as in the vertically-integrated channel \( (s > s_r) \). The manufacturer essentially polices itself and the system by choosing \( w \) and \( r \) so as not to distort the retailer’s return penalty choice; specifically, when the manufacturer buys back the returned units, the manufacturer refunds to the retailer the margin earned on new units \( w^* - c \) in addition to its own salvage value for the returned unit \( s \). That is, \( r^* = (w^* - c) + s \). This refund level induces the retailer to set its return penalty so that the return rate is the same as in the vertically-integrated channel. The higher price dampens initial sales \( (q_b^* < q_b^{VI}) \), and therefore fewer total unit returns occur than in the vertically-integrated system \( (\phi_e q_b^* < \phi_e^{VI} q_b^{VI}) \).

Therefore, the apparently counterintuitive result of Proposition 2 – in which the manufacturer may choose to accept returns even if its salvage value is lower than the retailers’ –
stems from the negative externality created by the retailer’s return policy. A restrictive return penalty charged by the retailer provides revenue to the retailer, but adversely affects the total number of units sold. When the retailer salvages returns, the equilibrium values of $p$, $f$, and $w$ are distorted relative to the vertically-integrated channel. The manufacturer can internalize this externality by accepting responsibility for returns through its joint use of wholesale price and a generous refund rate for returned units. Of course, it could still be more efficient to allocate the salvaging task to the retailer, but only if the retailer’s salvage value superiority is great enough. If the retailer’s salvage advantage is not too great, the gain due to the manufacturer’s ability to strategically set the wholesale price and refund can more than compensate for the efficiency loss due to moving the function to the manufacturer. The manufacturer optimally encourages the retailer to salvage returned units only if the retailer has a significant advantage in its ability to extract value from returned units.\footnote{Iyer and Villas-Boas (2003) find that the manufacturer may increase profit by taking back overstock returns because it reduces extreme double marginalization in price. From Propositions 2 and 3, we see that taking back consumer returns may also be profitable for the manufacturer through a related mechanism: the reduced marginalization on product returns (i.e., the reduced return penalty).}

In summary, when fixed fees or quantity discount schedules are not used in the wholesale contract, we have defined conditions under which the manufacturer may better approximate the outcome of a coordinated channel by choosing a reverse channel structure in which it takes over ultimate responsibility for processing consumer returns, rather than leaving this responsibility to the retailer. The reason for this is that the gain in coordination of channel decision-making can outweigh the efficiency loss that can result from the manufacturer’s performance of this important channel function. Our results show that taking back the returns-processing function and combining this channel structure decision with an appropriate channel contracting decision is a partial substitute for using a fixed fee or equivalent quantity discount schedule.
4. Discussion

Product returns represent an enormous cost, greatly affecting manufacturers and retailers alike. Learning to manage product returns successfully can greatly increase a company’s profitability. Our research examines consumer returns of non-defective products that are discovered by consumers not to match with preferences. These returns are different from overstock returns in that the return policy is interrelated with product demand itself. The return penalty charged to consumers has several effects: it recoups costs associated with returns from consumers; it prevents a number of returns from occurring; but it also reduces consumers’ willingness to pay for initial purchases because of uncertainty about whether or not the product matches with preferences. In a channel context, the choice of return penalty adds an additional layer of complexity because it may asymmetrically affect the retailer and the manufacturer. The retail price and return penalty charged to consumers are set to maximize retailer profit, but the return penalty shifts the demand curve as well, thereby affecting the manufacturer’s quantity sold.

One of our key findings is that even if the retailer is more efficient than the manufacturer at salvaging returned units, it is possible for the retailer to charge a higher return penalty when salvaging returned units than if the salvaging were done by the manufacturer. When the retailer salvages returned units without the help of the manufacturer, the retailer misses out on the generous refund from the manufacturer. While a greater salvage value generally implies a more generous return policy, this effect may be reversed by the manufacturer’s incentive to coordinate the pricing in the reverse channel.

Intuition would suggest that the salvaging of returned units should be done as efficiently as possible, but our analysis shows in contrast that the manufacturer may earn greater profit by taking back product returns even if the retailer is more efficient at salvaging the units. The
ensuing loss in efficiency can be more than compensated for by the gain in profitability due to the manufacturer’s internalization of the negative externality that is otherwise created when the retailer handles returns. A retailer that handles returns penalizes consumers for returning products than the manufacturer would; this can reduce the manufacturer’s profit, even if the retailer has the advantage in terms of salvage value.

If the manufacturer is able to “sell the firm” to the retailer through the use of a fixed fee or perfectly developed quantity discount schedule, then the manufacturer will be able to extract all profit from the retailer regardless of reverse channel structure. The subsequently fully-coordinated channel will charge a return penalty to consumers exactly equal to the cost to the channel of a product return. However, it may not always be feasible for the manufacturer to extract all profit through the use of a fixed fee. If fixed fees (or the corresponding quantity discount schedule) cannot be implemented, the manufacturer can use a return policy to approximate the results of a vertically-integrated channel. When offering per-unit wholesale and refund rates, the manufacturer offers a generous refund above its own value for the returned unit in order to induce the retailer to set the optimal return penalty. While the return penalty of the vertically-integrated channel can be replicated when the manufacturer accepts returns, double marginalization in pricing persists, reducing not only total quantity sold, but also total product returns and channel profit relative to a vertically-integrated channel outcome.

The model’s results do not lend themselves to straightforward interpretation of comparative-static effects, and we therefore have refrained from making predictive statements concerning the effects of consumer disutility for a mismatch ($d$), consumer utility from owning a product in the category ($\bar{u}$), marginal production cost ($c$), manufacturer salvage value ($s$), or retailer salvage value ($sr$) on the equilibrium reverse channel structure in our model. However, in
the On-Line Supplement we provide some illustrative examples to show that across a broad spectrum of parameter space, the manufacturer is more likely to optimally handle returns when \( d \) or \( s_r \) are lower, or when \( u \), \( c \), or \( s \) are higher.

Several facets of our model merit further discussion. These findings were developed in a model of heterogeneous consumers (in terms of their reservation utility \( u_i \)) who are completely uninformed \textit{a priori} about their preferences between two products. Alternatively, one could model a situation where some consumers are more informed than others or some have a stronger preference for one of the products than for the other. When such asymmetries are introduced into our modeling framework, one would expect to see fewer returns. Furthermore, had we instead abstracted from reality further by assuming that a single product was offered to consumers who are homogeneous and uninformed before purchase (but vary in their valuation discovered after purchase), then the results would be different: namely, the return penalty would be lower (and the manufacturer would earn the greatest profit) in the channel structure that allocates returns to the channel member with the greatest salvage value.\textsuperscript{27} Our results, on the other hand, are preserved when consumers are heterogeneous in their pre-purchase valuation \( (u_i) \), or when consumers can choose between two products. In either of these cases, the retailer’s choice of return penalty has an asymmetric impact on the retailer and the manufacturer. We have chosen to incorporate both complexities because this not only more accurately depicts reality, but it also generates a more parsimonious model. The added parsimony is due to the fact that the return probability becomes the same for each consumer when consumers choose whether or not to exchange products. In the On-Line Supplement, we show how the qualitative results of Observation 1 and Proposition 2 are preserved in a single-product setting.

\textsuperscript{27} This can easily be shown by extending the model of Matthews and Persico (2005).
In reality, there is even further complexity that suggests avenues for future research in this area. For example, consumers may vary in their return probabilities. This variance can be due to variation in their initial level of information, or due to the possibility that a return will be made without a subsequent purchase. With either of these modifications, however, the same forces are at play as in our model. A return penalty would still reduce returns, pass the costs of returns on to consumers, and have a negative effect on demand. Without a manufacturer’s return policy, the retailer’s incentives to reduce costs associated with returns would still ignore the resulting impact on the manufacturer’s profit from initial sales. It would logically follow that in this more complex setting, the manufacturer would still offer a generous return policy that has the same effect as in Observation 1 and Proposition 2. The additional complexities may lead to distortion of the return penalty in a CRM reverse channel structure relative to the solution in a vertically-integrated system. However, it logically follows that the return penalty in the CRM reverse channel structure would be closer to that in the vertically-integrated channel than is the penalty in the CR reverse channel structure.

The model abstracts away from the possibility that there may be loss of goodwill, future sales, or accessorial sales that a retailer incurs by charging a consumer a return penalty. In a companion model available from the authors, we capture a per-unit-returned loss in retailer profit as a function of the return penalty. Allowing for this possibility changes the specific prices and return penalties chosen in equilibrium. It yields the additional result that the return penalty in both the vertically-integrated structure and the structure where the manufacturer handles returns can actually be higher when retailer’s future or accessorial profit is diminished by the return penalty (if and only if the disutility of mismatch is sufficiently low). However, our qualitative results still hold.
There are also other contracting structures that could be examined. For example, a manufacturer could seek to alter the retailer’s incentive to set a high return penalty by making the per-unit wholesale price an increasing function of the return penalty the retailer charges.\textsuperscript{28} While we could not find instances of the use of such contingent contracts in real-world channels with product returns, this or various other contracting options could be examined in future research.

In sum, the current paper adds to our understanding of optimal product returns management in a marketing channel. The reverse channel structure has a substantive impact on the price and return penalty ultimately charged to consumers. In addition to cost structure considerations, the strategic interaction between the manufacturer and the retailer plays an important role in determining how consumer returns should be managed. Manufacturers cannot ignore the consumer returns facing a retailer that occur through no fault of the manufacturer. Our model highlights the fact that the reverse channel and the forward channel are closely related and both should be carefully considered in determining the firm’s profit-maximizing channel strategy.

\textsuperscript{28} We thank the Area Editor for suggesting this possibility to us.
References


Middleton, D. 2007. Stores Cracking Down on Returns; Target is one of many retailers tightening policies and implementing restocking fees. Florida Times-Union May 19, 2007 pg A-1.


A.1. Proof of Equation 1 (See Section 2)

\[ \phi_o(f_j, p_j, p_{xj}; d, \alpha) = \alpha \left( \frac{1}{2} \frac{f_j - p_j + p_{xj}}{2d} \right) \]

\[ \phi_e(f_j, p_j, p_{xj}; d, \alpha) = \alpha \left( \frac{1}{2} \frac{f_j - p_j + p_{xj}}{2d} \right) \]

\[ \phi_e = 1 - \alpha \]

\[ E_j(utility) = \alpha (u_i - \frac{p_j + p_{xj} + f_A}{2} - \frac{d}{4} + \frac{(f_j - p_j + p_{xj})^2}{4d}) - (1 - \alpha) f_j \]

Proof:

Consider two horizontally differentiated products located at 0 and 1 on a Hotelling unit line. Suppose the consumer located at \( \theta_i \) purchases \( x_j = 0 \). Let the product located at 0 be denoted by \( j = 0 \) and the product located at 1 be denoted by 1. Upon purchasing the product and learning the value of product fit \( |x_j - \theta_i| \), with probability \( \alpha \) the consumer will get utility equal to \( u_i - p_0 - d\theta_i \) from keeping this product and utility equal to \( -f_0 + u_i - p_i - d(1 - \theta_i) \) from exchanging it for \( x_j = 1 \). Comparing these two utilities, consumers who purchase \( x_0 \), will prefer to keep it if \( \theta_i < \frac{1}{2} \frac{f_0 - p_0 + p_1}{2d} \) and will prefer to exchange it for \( x_i = 1 \) if \( \theta_i > \frac{1}{2} \frac{f_0 - p_0 + p_1}{2d} \).

With probability \( (1 - \alpha) \), the consumer will discover the product category offers zero utility and make a return. With probability \( \alpha \), the consumer will have positive utility from owning a product in the category. Because \( \theta_i \sim U[0,1] \), the probability that someone who buys product \( x_0 = 0 \) decides to keep it is equal to \( \alpha \left( \frac{1}{2} \frac{f_0 - p_0 + p_1}{2d} \right) \) and the probability that someone who buys
product \( x_0 = 0 \) decides to exchange it is equal to \( \frac{\alpha(1 - \frac{f_0 - p_0 + p_1}{2d})}{2} \). For a given consumer \( i \) who purchases \( x_0 = 0 \) initially, we may write the probability that the consumer keeps this purchase, \( \phi_{x_0}(f_0, p_0, p_1; d, \alpha) = \alpha(1 + \frac{f_0 - p_0 + p_1}{2d}) \), the probability that the consumer exchanges this purchase for product \( B \phi_{x_0}(f_0, p_0, p_1; d, \alpha) = \alpha(1 - \frac{f_0 - p_0 + p_1}{2d}) \).

We derive the expected utility of purchasing \( x_0 = 0 \) for a consumer with reservation utility \( u_i \). With probability \( \phi_{x_0}(f_0, p_0, p_1; d, \alpha) = \alpha(1 + \frac{f_0 - p_0 + p_1}{2d}) \), the consumer will keep the \( j=0 \) product. The unit will be kept if and only if \( |0 - \theta_i| < \frac{1}{2} + \frac{f_0 - p_0 + p_1}{2d} \). Therefore, the average utility derived from keeping the unit will be equal to \( u_i - p_0 - \frac{d}{2} \left( \frac{1 + f_0 - p_0 + p_1}{2d} \right) \). The unit will be exchanged for product \( j=1 \) if and only if \( \theta_i > \frac{1}{2} + \frac{f_0 - p_0 + p_1}{2d} \), which implies \( |1 - \theta_i| < \frac{1}{2} - \frac{f_0 - p_0 + p_1}{2d} \). Therefore, the average utility derived from exchanging a unit of product \( j=0 \) for a unit of product \( j=1 \) will be \( u_i - p_1 - f_0 - \frac{d}{2} \left( \frac{1 - f_0 - p_0 + p_1}{2d} \right) \). With probability \( (1-\alpha) \), the consumer will realize after purchase that neither product is a match. In this case, the consumer will return the initial purchase at a penalty \( f_0 \) and not make a subsequent purchase.

We may now write the expected utility \textit{ex ante} of purchasing \( x_0 = 0 \) as:

\[
E_{\theta}(utility) = \phi_{x_0}(f_0, p_0, p_1; d)(u_i - p_0 - \frac{d}{2} \left( \frac{1 + f_0 - p_0 + p_1}{2d} \right)) + \phi_{x_0}(f_0, f_1, p_0, p_1; d)(u_i - p_1 - f_0 - \frac{d}{2} \left( \frac{1 - f_0 - p_0 + p_1}{2d} \right)) - (1 - \alpha) f_0
\]

, which simplifies to

\[
E_{\theta}(utility) = \alpha(u_i - p_0 + p_1 + f_0 - \frac{d}{4} + \frac{(f_0 - p_0 + p_1)^2}{4d}) - (1 - \alpha) f_0.
\]
Suppose the consumer located at $\theta_i$ purchases $x_i = 1$. Upon purchasing the product and learning the value of product fit $|x_j - \theta_j|$, with probability $\alpha$ the consumer will get utility equal to $u_i - p_i - d(1 - \theta_i)$ from keeping this product and utility equal to $-f_i + u_i - p_o - d\theta_i$ from exchanging it for $x_0 = 0$. Comparing these two utilities, consumers who purchase $x_i = 1$, will prefer to keep it if $\theta_i > \frac{1}{2} - \frac{f_i - p_i + p_o}{2d}$ and will prefer to exchange it for $x_0 = 0$ if $\theta_i < \frac{1}{2} - \frac{f_i - p_i + p_o}{2d}$. With probability $\alpha$, the consumer will have positive utility from owning a product in the category. Because $\theta_i \sim U[0,1]$, the probability that someone who buys product $x_i = 1$ decides to keep it is equal to $\alpha\left(\frac{1}{2} + \frac{f_i - p_i + p_o}{2d}\right)$ and the probability that someone who buys product $x_i = 1$ decides to exchange it is equal to $\alpha\left(\frac{1}{2} - \frac{f_i - p_i + p_o}{2d}\right)$. For a given consumer $i$ who purchases product $j=1$ initially, we may write the probability that the consumer keeps this purchase as $\phi_{k_1}(f_i, p_o, p_i; d, \alpha) = \alpha\left(\frac{1}{2} + \frac{f_i - p_i + p_o}{2d}\right)$, and the probability that the consumer exchanges this purchase for product 0 as $\phi_{e_1}(f_i, p_o, p_i; d, \alpha) = \alpha\left(\frac{1}{2} - \frac{f_i - p_i + p_o}{2d}\right)$.

We derive the expected utility of making a purchase for a consumer with reservation utility $u_i$. With probability $\phi_{k_1}(f_i, p_o, p_i; d, \alpha) = \alpha\left(\frac{1}{2} + \frac{f_i - p_i + p_o}{2d}\right)$, the consumer will keep the unit $I$. The unit $I$ will be kept if and only if $|1-\theta_i| < \frac{1}{2} + \frac{f_i - p_i + p_o}{2d}$. Therefore, the average utility derived from keeping the unit will be equal to $u_i - p_i - \frac{d}{2} \left(\frac{1}{2} + \frac{f_i - p_i + p_o}{2d}\right)$. The unit will be exchanged for product 0 if and only if $\theta_i < \frac{1}{2} + \frac{f_i - p_i + p_o}{2d}$, which implies
Therefore, the average utility derived from exchanging a unit of product 1 for a unit of product 0 will be 

\[ u_i - p_0 - f_1 - \frac{d}{2} \left( \frac{1}{2} - \frac{f_1 - p_1 + p_0}{2d} \right) \]

With probability \((1-\alpha)\), the consumer will realize after purchase that neither product is a match. In this case, the consumer will return the initial purchase at a penalty \(f_1\) and not make a subsequent purchase.

We may now write the expected utility \(E_1(utility)\) as:

\[
E_1(utility) = 
\phi_i(f_1, p_o, p_i; d, \alpha)(u_i - p_0 - f_1 - \frac{d}{2} \left( \frac{1}{2} - \frac{f_1 - p_1 + p_0}{2d} \right)) + 
\phi_o(f_1, p_o, p_i; d, \alpha)(u_i - p_0 - f_1 - \frac{d}{2} \left( \frac{1}{2} - \frac{f_1 - p_1 + p_0}{2d} \right)) - (1-\alpha)f_1
\]

which simplifies to

\[
E_1(utility) = \alpha(u_i - p_1 + p_o + f_1 - \frac{d}{4} + \frac{(f_1 - p_1 + p_o)^2}{4d}) - (1-\alpha)f_1.
\]

We may therefore generalize that for a given consumer \(i\) who initially purchases product \(j\), the probability that the consumer exchanges this purchase for the product located at \(x_{x_j}\), the probability that the consumer keeps this purchase, the probability that the consumer returns this purchase without exchange, and the expected utility \(E_j(utility)\) can be written as follows:

\[
\phi_{ij}(f_j, p_j, p_{x_j}; d, \alpha) = \alpha \left( \frac{1}{2} + \frac{f_j - p_j + p_{x_j}}{2d} \right)
\]

\[
\phi_{ij}(f_j, p_j, p_{x_j}; d, \alpha) = \alpha \left( \frac{1}{2} - \frac{f_j - p_j + p_{x_j}}{2d} \right)
\]

\[
\phi_i = 1-\alpha
\]

\[
E_j(utility) = \alpha(u_i - \frac{p_j + p_{x_j} + f_j}{2} - \frac{d}{4} + \frac{(f_j - p_j + p_{x_j})^2}{4d}) - (1-\alpha)f_j
\]

\(Q.E.D.\)
A.2. Proof of Equation 2 (See Section 2)

Initial sales are

\[ q_b(f, p; d, \bar{u}, \alpha) = \bar{u} - p - \frac{d}{4} - \frac{(2 - \alpha)f}{2\alpha} + \frac{f^2}{4d} \]

Consumers are differentiated in their value of \( u_i \) which takes values between 0 and \( \bar{u} \) with an equal density (normalized to 1) at each \( u_i \) and is independent of \( \theta_i \). If prices and return penalties are symmetric for each product, then the expected utility of making an initial purchase from equation 1 simplifies to

\[ E(\text{utility}) = \alpha(u_i - p - \frac{f}{2} - \frac{d}{4} + \frac{f^2}{4d}) - (1 - \alpha)f. \]

A consumer with \( u_i \) will make a purchase initially if expected utility is greater than the outside option, where the outside option is normalized to zero. That is, all consumers with

\[ u_i \in [p + \frac{f}{2} + \frac{d}{4} + \frac{f^2}{4d} + \frac{(1 - \alpha)f}{\alpha} \bar{u}] \]

will buy initially. Therefore, the initial sales will be.

\[ q_b(f, p; d, \bar{u}, \alpha) = \bar{u} - p - \frac{d}{4} - \frac{(2 - \alpha)f}{2\alpha} + \frac{f^2}{4d}. \]

Q.E.D.

A.3. Condition for all consumers with positive utility (a proportion \( \alpha \) of the population) to prefer to keep or exchange rather than return and make no other purchase

In this Appendix, for given values of retailer decisions (\( p \) and \( f \)), we identify when all consumers who have positive utility from owning one of the two products (the proportion \( \alpha \) of the population) and make a purchase initially will prefer to keep or exchange it, rather than return the good and leave the market. More specifically, we find that this returns and exchange behavior by the \( \alpha \) consumers is feasible if and only if

\[ \frac{4df - \alpha(d^2 + f^2)}{2d\alpha} \geq 0. \]

We first show that this returns and exchange behavior is feasible for a consumer located at \( u_i \). if
2(u_i - p) + f - d \geq 0$, and then show that this implies the condition $\frac{4df - \alpha(d^2 + f^2)}{2d\alpha} \geq 0$. The derivations in this Appendix are later used to determine parameter values for which our assumption that all units sold are kept or exchanged holds even if consumers were given the option to return without subsequent purchase.

Consider consumers who can engage in one of three possible consumption behaviors. First, the consumer could **buy a product and keep it** (i.e., not return it). Such a consumer located at $\theta_i$, who purchases and keeps a product priced at $p$ and located at $x_j$, receives a utility of $u_i - p - d \mid x_j - \theta_i \mid$.

A second possibility is that the consumer purchases a product, then **returns and exchanges it** for a more preferred product after initial purchase. This consumer, located at $\theta_i$, who purchases a product priced at $p$ and located at $x_j$, receives total utility of

$$u_i - p - f - \text{Min} \{d \mid x_j - \theta_i \mid\}.$$ 

Finally, a consumer may purchase a product, then **return it and not exchange it** (i.e., **leave the market**). The consumer located at $\theta_i$ who purchases a product priced at $p$ and located at $x_j$ receives total utility of $-f$.

In what follows, we model the returns behavior of a consumer who initially bought the product located at $x_j = 0$. Note that since a consumer is equally likely to buy either product initially when prices and restocking fees are constant across products, the derivation below does not depend on consideration of the product located at $x_j = 0$.

Let $\theta_i$ denote the consumer between 0 and 1 who is indifferent between keeping the product located at $x_j = 0$ after purchase and exchanging it for the product located at $x_j = 1$. Buying
and keeping the product located at \( x_j = 0 \) yields a utility equal to \( u_i - p - d \mid \theta_i - 0 \mid \). Returning the product and buying the product located at \( x_j = 1 \) yields utility equal to \( u_i - p - f - d \mid 1 - \theta_i \mid \).

Therefore, one can easily show that the marginal consumer is located at \( \theta_i = \frac{f + d}{2d} \) such that

\[
u_i - p - d \theta_i = u_i - p - f - d (1 - \theta_i) .
\]

All consumers with \( \theta \in [0, \theta_i] \) who initially buy the product located at \( x_j = 0 \) earn greater utility from keeping it than from exchanging it.

Let \( \theta_2 \) denote the consumer between 0 and 1 who is indifferent between keeping the product located at \( x_j = 0 \) and returning it to opt out of the market (i.e., not replacing the product located at \( x_j = 0 \)). For this consumer, the utility of buying and keeping the product equals the utility of buying and returning the product located at \( x_j = 0 \) for a refund, \( u_i - d \mid \theta_2 - 0 \mid - p = -f \).

Therefore, \( \theta_2 = \frac{u_i - p + f}{d} \). All consumers with \( \theta \in [0, \theta_2] \) have greater value from keeping the product than from returning it.

Let \( \theta_3 \) denote the consumer between 0 and 1 who has returned the product located at \( x_j = 0 \) and is indifferent between purchasing the product located at \( x_j = 1 \) and opting out of the market.

For this consumer, \( u_i - p - f - d \mid 1 - \theta_3 \mid = -f \). Therefore, \( \theta_3 = \frac{d - (u_i - p)}{d} \). All consumers with \( \theta \in [\theta_3, 1] \) who initially purchase the product located at \( x_j = 0 \) enjoy greater utility from exchanging the product located at \( x_j = 0 \) for the product located at \( x_j = 1 \) than from opting out of the market.

By comparing the values of \( \theta_1, \theta_2, \) and \( \theta_3 \), we can order them \( \theta_2 \geq \theta_1 \geq \theta_3 \) if \( 2(u_i - p) + f - d \geq 0 \). Otherwise, the inequalities are reversed and \( \theta_2 < \theta_1 < \theta_3 \). The definitions of \( \theta_1, \theta_2, \) and \( \theta_3 \) imply several choice rules. For consumers with:
• $1 > \theta > \theta_1$, returning the initial purchase and buying the other product (RX) $\succ$ keeping purchase (K),
• $1 > \theta > \theta_2$, returning the initial purchase and owning nothing (R) $\succ$ keeping purchase (K),
• $1 > \theta > \theta_3$, returning the initial purchase and buying the other product (RX) $\succ$ returning and owning nothing (R).

When $2(u_i - p) + f - d < 0$ (which implies $\theta_2 < \theta_1 < \theta_3$), we have:

• for $0 < \theta < \theta_2$, K $\succ$ R $\succ$ RX;
• for $\theta_2 < \theta < \theta_1$, R $\succ$ K $\succ$ RX;
• for $\theta_1 < \theta < \theta_3$, R $\succ$ RX $\succ$ K;
• for $1 > \theta > \theta_3$, RX $\succ$ R $\succ$ K.

Thus, if $2(u_i - p) + f - d < 0$, keeping (K) is the dominant choice for $0 < \theta < \theta_2$; returning and owning nothing (R) is the dominant choice for $\theta_2 < \theta < \theta_3$; and returning the initial purchase to buy the other product (RX) is the dominant choice for $\theta > \theta_3$. Therefore, if $2(u_i - p) + f - d < 0$, there are consumers whose $\theta$ value lies in $(\theta_2, \theta_3)$ who will return the initially purchased product for a refund and opt out of the market.

When $2(u_i - p) + f - d \geq 0$ (which implies $\theta_2 > \theta_1 > \theta_3$), we have:

• for $0 < \theta < \theta_3$, K $\succ$ R $\succ$ RX;
• for $\theta_3 < \theta < \theta_1$, K $\succ$ RX $\succ$ R;
• for $\theta_1 < \theta < \theta_2$, RX $\succ$ K $\succ$ R;
• for $1 > \theta > \theta_3$, RX $\succ$ R $\succ$ K.
Therefore, if $2(u_i - p) + f - d \geq 0$ keeping (K) is the dominant choice for $0 < \theta < \theta_i$, and returning the initial purchase to buy the other product (RX) is the dominant choice for $\theta_i < \theta < 1$. There does not exist a group of consumers who would prefer to leave the market without owning a product rather than keeping or exchanging the initial purchase.

Because the condition $2(u_i - p) + f - d \geq 0$ is less strict for higher $u_i$, we may say that if it holds for the lowest $u_i$ such that the consumer is indifferent between buying and not buying initially, then it will hold for all higher $u_i$ values. From A.2., this marginal consumer is located at $u_i = p - \frac{f}{2} + \frac{d}{4} + \frac{f^2}{4d} + \frac{(1-\alpha)}{\alpha}f^{-}$. Thus if $\frac{4df^{-} - \alpha(d^2 + f^2)}{2d\alpha} \geq 0$, all consumers will keep or exchange even with the available option to return a product for a refund with no exchange.

**A.4. Proof of Entries in Table 2 (See Section 3)**

We first solve for the equilibrium decisions of a vertically integrated system with net salvage value $\max\{s, s_r\}$. We will then show that the same outcome can be achieved in a decentralized channel in which the manufacturer uses a fixed fee, wholesale price equal to marginal cost and a refund rate equal to the net salvage value to coordinate the channel.

The vertically integrated system’s objective function can be written as

$$\max_{p,f} \pi = q_b(f, p; d, \bar{u}, \alpha)((p-c) + (-p + f + \max\{s, s_r\})(\phi_f(f; d, \alpha) + \phi_r) + (p-c)\phi_r(f; d, \alpha))$$

where,

$$q_b(f, p; d, \bar{u}, \alpha) = \bar{u} - p - \frac{d}{4} - \frac{(2-\alpha)f}{2\alpha} + \frac{f^2}{4d}, \quad \phi_f = (1-\alpha) \text{ and } \phi_r(f; d, \alpha) = \alpha\left(\frac{1}{2} - \frac{f}{2d}\right).$$

The first order conditions,
\[
\frac{\partial \pi}{\partial p} = \alpha (\bar{u} - (p - c) + \frac{(d - f)(c - \max\{s, s_r\} - f)}{2d}) - (1 - \alpha)(2f + \max\{s, s_r\} - c) \\
+ \frac{\alpha}{4} (-d + \frac{f^2}{d} - 2(f + 2p)) = 0
\]

\[
\frac{\partial \pi}{\partial f} = (\alpha(d^2 - f^2) + 2d (f - 2\alpha + 4\alpha (p - \bar{u}))(\alpha (2f - c + \max\{s, s_r\} - (2 - \alpha)d) \\
+ (2d - \alpha d - \alpha f)(2d - \alpha d - \alpha f)(f + \max\{s, s_r\} - c) - 2\alpha c + 2\alpha dp) = 0'
\]

have three solutions:

I) \( p = \frac{\bar{u}}{2} + \frac{(c - \max\{s, s_r\})^2}{8d} + \frac{2(c + \max\{s, s_r\}) - d - (1 - \alpha)(c - \max\{s, s_r\})}{8} \]

\[ f = c - \max\{s, s_r\} \]

Defining \( X_{VI} \equiv \alpha^2 ((c - \max\{s, s_r\})^2 - 2d(c + \max\{s, s_r\} - 2\bar{u}) - d^2) - 4\alpha (c - \max\{s, s_r\}) \), we have

II)

\[
p = 2\bar{u} + \frac{(c - \max\{s, s_r\})^2 - d(d + 2\max\{s, s_r\})}{2d} - \frac{\sqrt{X_{VI}}}{\alpha^2} + \frac{4d (\max\{s, s_r\} - c) + (c + d - \max\{s, s_r\})\sqrt{X_{VI}}}{2d\alpha}
\]

\[ f = c - \max\{s, s_r\} - \sqrt{X_{VI}} \]

III)

\[
p = 2\bar{u} + \frac{(c - \max\{s, s_r\})^2 - d(d + 2\max\{s, s_r\})}{2d} + \frac{\sqrt{X_{VI}}}{\alpha^2} + \frac{4d (\max\{s, s_r\} - c) - (c + d - \max\{s, s_r\})\sqrt{X_{VI}}}{2d\alpha}
\]

\[ f = c - \max\{s, s_r\} + \sqrt{X_{VI}} \]

The initial sales quantity \( q_b \) when evaluated at solutions I, II and III is

\[
q_b = \frac{\bar{u}}{2} + \frac{(c - \max\{s, s_r\})^2}{8d} - \frac{2(3c - \max\{s, s_r\}) + d - (1 - \alpha)(c - \max\{s, s_r\})}{2\alpha}, \quad q_b = 0, \quad \text{and} \quad q_b = 0
\]
respectively. We therefore rule out potential solutions II and III. We verify below that the second order conditions are satisfied.

\[
\frac{\partial^2 \pi}{\partial \alpha \partial p} = -2\alpha < 0.
\]

\[
\frac{\partial^2 \pi}{\partial \alpha \partial f} = \frac{\alpha \tilde{u}}{d} - \frac{3\alpha^2 f (2f - c + \max\{s, s_r\}) + d\alpha (6\alpha p + 3f + \max\{s, s_r\} - c) - 8\alpha f - 3\alpha \max\{s, s_r\} - d^2 (\alpha^2 + 8(1 - \alpha))}{4\alpha d^2}.
\]

We show that \(\frac{\partial^2 \pi}{\partial \alpha \partial f} < 0\) at solution I. Evaluated at solution I and the minimum value for \(\tilde{u}\) such that quantity is non-negative, the expression \(\frac{\partial^2 \pi}{\partial \alpha \partial f} \bigg|_{\pi = \pi_0, (\pi) = 0} = -\frac{(\alpha(c - \max\{s, s_r\}) - d(2 - \alpha))^2}{2\alpha d^2} < 0\).

Evaluated at solution I, the second derivative with respect to \(f\) is decreasing in \(\tilde{u}\):

\[
\frac{\partial(\frac{\partial^2 \pi}{\partial \alpha \partial f})}{\partial \tilde{u}} = -\frac{\alpha}{4d} < 0.
\]

Therefore, \(\frac{\partial^2 \pi}{\partial \alpha \partial f} < 0\) at solution I. The determinant of the hessian at solution I is

\[
|H| = \frac{\alpha}{d} \left(\frac{\tilde{u}}{2} + \frac{(c - \max\{s, s_r\})^2}{8d} - \frac{2(3c - \max\{s, s_r\}) + d}{8} - \frac{(1 - \alpha)(c - \max\{s, s_r\})}{2\alpha}\right) > 0
\]

if and only if \(\tilde{u}\) is such that

\[
q_b = \frac{\tilde{u}}{2} + \frac{(c - \max\{s, s_r\})^2}{8d} - \frac{2(3c - \max\{s, s_r\}) + d}{8} - \frac{(1 - \alpha)(c - \max\{s, s_r\})}{2\alpha} > 0.
\]

Because

\[
\lim_{f \to +\infty} \pi = -\infty, \quad \lim_{p \to +\infty} \pi = -\infty,
\]

the only local maximum of profit (solution I) is also a global maximum. Therefore,

\[
p_{\text{VI}} = \frac{\tilde{u}}{2} + \frac{(c - \max\{s, s_r\})^2}{8d} + \frac{2(c + \max\{s, s_r\} - d)}{8} - \frac{(1 - \alpha)(c - \max\{s, s_r\})}{2\alpha}
\]
\[ f(w,r) = c - \max\{s, s_r\}. \] Substituting this solution into the demand/return equations and the profit expression yields

\[ q_{br}^{r} = \frac{\bar{u}}{2} + \frac{(c - \max\{s, s_r\})^2 - 2(3c - \max\{s, s_r\}) + d}{8} - \frac{(1 - \alpha)(c - \max\{s, s_r\})}{2\alpha}, \]

\[ f_{er}^{r} = \frac{1}{2} - \frac{(c - \max\{s, s_r\})}{2d} \]

and

\[ \pi_{wr}^{r} = \frac{(\alpha(c - \max\{s, s_r\})^2 - (4 + 2\alpha)cd + 2d((2 - \alpha)\max\{s, s_r\} + 2\alpha\bar{u}) - \alpha d^2)^2}{64d^2}. \]

We now show that this outcome can be achieved in a decentralized channel in which the manufacturer charges a fixed fee, wholesale price equal to marginal cost, and a refund rate equal to the net salvage value.

A retailer’s objective function can be written as

\[ \max_{p,f} \pi_{ret} = q_b(f, p; d, \bar{u}, \alpha)((p - w) + (-p + f + \max\{s_r, r\})(\phi_{r}(f; d, \alpha) + \phi_{r}(\alpha)) + (p - w)\phi_{r}(f; d, \alpha)) - T \]

where \( T \) is the fixed fee. The first order conditions are

\[ \frac{\partial \pi_{ret}}{\partial p} = \alpha(\bar{u} - (p - w) + (d - f)(w - \max\{s_r, r\} - f)) - (1 - \alpha)(2f + \max\{s_r, r\} - w) \]

\[ + \frac{\alpha}{4}(-d + \frac{f^2}{d} - 2(f + 2p)) = 0 \]

\[ \frac{\partial \pi_{ret}}{\partial f} = \frac{(\alpha d^2 - f^2) + 2d(f - 2\alpha + 4\alpha(p - \bar{u})))(\alpha(2f - w + \max\{s_r, r\}) - (2 - \alpha)d)}{4\alpha d^2} \]

\[ + \frac{(2d - \alpha d - \alpha f)((2d - \alpha d - \alpha f)(f + \max\{s_r, r\} - w) - 2daw + 2adp)}{4\alpha d^2} = 0 \]

There are three possible solutions that satisfy the first order conditions.

I) \( p(w, r) = \frac{\bar{u}}{2} + \frac{(w - \max\{s_r, r\})^2}{8d} + \frac{2(w + \max\{s_r, r\}) - d}{8} - \frac{(1 - \alpha)(w - \max\{s_r, r\})}{2\alpha} \)

\[ f(w, r) = w - \max\{s_r, r\} \]

Defining \( X \equiv \alpha^2((w - \max\{s_r, r\})^2 - 2d(w + \max\{s_r, r\} - 2\bar{u}) - d^2) - 4d\alpha(w - \max\{s_r, r\}) \), we have
II)

\[ p(w, r) = 2\bar{u} + \left( w - \max\{s_r, r\} \right)^2 - d(d + 2\max\{s_r, r\}) \frac{\sqrt{X}}{2d} + 4d\left(\max\{s_r, r\} - w\right) + \left( w + d - \max\{s_r, r\} \right) \frac{\sqrt{X}}{2d\alpha} \]

\[ f = w - r - \sqrt{X} \]

III)

\[ p(w, r) = 2\bar{u} + \left( w - \max\{s_r, r\} \right)^2 - d(d + 2\max\{s_r, r\}) \frac{\sqrt{X}}{2d} + 4d\left(\max\{s_r, r\} - w\right) - \left( w + d - \max\{s_r, r\} \right) \frac{\sqrt{X}}{2d\alpha} \]

\[ f(w, r) = w - \max\{s_r, r\} + \sqrt{X} \]

The initial sales quantity \( q_b \) when evaluated at solutions I, II and III is

\[ q_b = \frac{\bar{u}}{2} - \frac{w - \max\{s_r, r\} - d^2 + 2d\left( w + \max\{s_r, r\} \right) - \left( w - \max\{s_r, r\} \right)^2}{8d}, \quad q_b = 0, \quad \text{and} \quad q_b = 0 \]

respectively. We therefore rule out potential solutions II and III. Therefore, the retailer’s reaction functions are

\[ p(w, r) = \bar{u} + \left( w - \max\{s_r, r\} \right)^2 + \frac{d(2\max\{s_r, r\}) - d}{8d} - \frac{(1 - \alpha)(w - \max\{s_r, r\})}{2\alpha} \]

\[ f(w, r) = w - \max\{s_r, r\} \]

We now examine two cases separately (when \( r > s_r \) and when \( r < s_r \)) and identify which will hold in equilibrium.

Assuming the manufacturer accepts returns and offers \( r > s_r \)

The manufacturer’s objective is

\[ \max_{w, r, T} \pi_{\text{mfr}} = T + q_b(f(w, r), p(w, r); d, \bar{u})(w - c) + (w - c - (r - s))\phi_c(f(w, r); d) - \phi_r(r - s) \]

s.t. \( \pi_{\text{ret}} \geq 0 \)
Using a Lagrangian multiplier $\lambda$ on the constraint, we have the following Kuhn-Tucker conditions:

\[
\frac{\partial \pi_{nfr}}{\partial w} = \frac{(2d + \alpha(d+r-w))(2cd + \alpha(c+d+r-w) + (r-w-d)(r-s-w)) - 2d(w+s-r))}{8\alpha d^2} + \frac{(\alpha + 2r-s-2w) + d(2+\alpha))(\alpha(w-r)^2 - d^2\alpha - 2d(w(2+\alpha) - r(2-\alpha) - 2\alpha \overline{\alpha}))}{16\alpha d^2} + \lambda \frac{(2d + \alpha(r+d-w))(\alpha(d^2 - (w-r)^2 + 2d(w+r) - 2\overline{\alpha}) + 4d(w-r))}{16\alpha d^2} = 0
\]

\[
\frac{\partial \pi_{nfr}}{\partial r} = \frac{(-2d + \alpha(d-r-w))(2cd + \alpha(c+d-r-w) + (r-w-d)(r-s-w)) - 2d(w+s-r))}{8\alpha d^2} + \frac{(d(2-\alpha) + \alpha(c + 2r-s-2w))(\alpha(w-r)^2 - d^2\alpha - 2d(w(2+\alpha) - r(2-\alpha) - 2\alpha \overline{\alpha}))}{16\alpha d^2} + \frac{\lambda (\alpha(d+w-r) - 2d)(\alpha(d^2 - (w-r)^2 + 2d(w+r) - 2\overline{\alpha}) + 4d(w-r))}{16\alpha d^2}
\]

\[
\frac{\partial \pi_{nfr}}{\partial T} = 1 - \lambda
\]

\[
\lambda \pi_{nfr} = 0, \lambda > 0
\]

There are three potential solutions that satisfies these conditions.

1. \(w=c, \quad r=s, \quad \lambda = 1, \quad T = \frac{(\alpha(c-s)^2 - d^2\alpha - 2d(c(2+\alpha) - s(2-\alpha) - 2\alpha \overline{\alpha}))(\alpha(c-s) - d(2+\alpha))^2}{64d^2}\)

Defining \(Y_M = (\alpha(c-s)^2 - d^2\alpha - 2d(c(2+\alpha) - s(2-\alpha) - 2\alpha \overline{\alpha}))\):

2. \(T = 0, \lambda = 1, \quad r = s - \frac{Y_M}{2d\alpha} + \frac{\sqrt{-\alpha^3 Y_M(\alpha(c-s) - d(2+\alpha))^2}}{2d\alpha^3}\),

\[
w = c - \frac{Y_M}{2d} - \frac{(d(2-\alpha) - \alpha(c-s))\sqrt{-\alpha^3 Y_M(\alpha(c-s) - d(2+\alpha))^2}}{2d\alpha^3(\alpha(c-s) - d(2+\alpha))}
\]

3. \(T = 0, \lambda = 1, \quad r = s - \frac{Y_M}{2d\alpha} - \frac{\sqrt{-\alpha^3 Y_M(\alpha(c-s) - d(2+\alpha))^2}}{2d\alpha^3}\),

\[
w = c - \frac{Y_M}{2d} + \frac{(d(2-\alpha) - \alpha(c-s))\sqrt{-\alpha^3 Y_M(\alpha(c-s) - d(2+\alpha))^2}}{2d\alpha^3(\alpha(c-s) - d(2+\alpha))}\)
The retailer’s reaction to possible solutions 2 and 3 leads to $q_b = 0$ and zero profit for both the retailer and the manufacturer. Therefore, the first possibility is the only solution for which profit and $q_b$ are potentially positive. As such, assuming the manufacturer accepts returns and offers $r > s_r$ (which holds true in equilibrium for $s > s_r$), the equilibrium is described by

$$p^{\text{ret}} = \frac{\bar{u}}{2} + \frac{(c-s)^2}{8d} + \frac{2(c+s) - d}{8} - \frac{(1-\alpha)(c-s)}{2\alpha}$$

$$f^{\text{ret}} = c - s,$$

$$q_b^{\text{ret}} = \frac{\bar{u}}{2} + \frac{(c-s)^2}{8d} - \frac{2(3c-s) + d}{8} - \frac{(1-\alpha)(c-s)}{2\alpha},$$

$$\phi_e^{\text{ret}} = \alpha \left( \frac{1}{2} - \frac{(c-s)}{2d} \right)$$

and $\pi^{\text{ret}}_{mgf} = \frac{(\alpha (c-s)^2 - (4 + 2\alpha)cd + 2d((2-\alpha)s + 2\alpha \bar{u}) - \alpha d^2)^2}{64d^2}$ which is equivalent to a vertically integrated system when $s > s_r$.

If the manufacturer does not accept returns ($r < s_r$):

The manufacturer’s objective is

$$\max_{w,T} T + q_b (f(w,r), p(w,r); d, \bar{u})((w-c) + (w-c)\phi_e (f(w,r); d))$$

s.t. $\pi_{ret} \geq 0$

Defining $Y_R \equiv \alpha((w - s_r)^2 - 2d(w + s_r - 2\bar{u}) - d^2) - 4d(w - s_r)$, the Kuhn-Tucker conditions (with a Lagrangian multiplier $\lambda$) are

$$\frac{\partial \pi_{mgf}}{\partial w} = \frac{1}{16\alpha d^2} (2(c-w)(\alpha(s_r - w) + d(2+\alpha))^2 + (\alpha(c+s_r - 2w) + d(2+\alpha))Y_R)$$

$$+ \lambda \left( (2d + \alpha(s_r + d - w))(\alpha(d^2 - (w - s_r)^2) + 2d(w + s_r) - 2\bar{u}) + 4d(w - s_r) \right) = 0$$

$$\frac{\partial \pi_{mgf}}{\partial T} = 1 - \lambda = 0, \quad \lambda \pi_{mgf} \geq 0, \quad \lambda > 0.$$
There are three possible solutions:

A. \( w = c, \lambda = 1, \quad T = \frac{(\alpha(c - s_r) - (4 + 2\alpha)c+d + 2d((2 - \alpha)s_r + 2\alpha s - \alpha d^2)^2)}{64d^2} \)

\[ w = \frac{3\alpha s_r + 3d\alpha(2 + \alpha) - \sqrt{6d\alpha^2(2\alpha^2(s_r - \overline{u}) + d(2 + 2\alpha + \alpha^2))}}{3\alpha^2}, \lambda = 1 \]

B. \[ T = \frac{(2\alpha^2(s_r - \overline{u}) + d(2 + 2\alpha + \alpha^2))^2}{36\alpha^2} \]

\[ w = \frac{3\alpha s_r + 3d\alpha(2 + \alpha) + \sqrt{6d\alpha^2(2\alpha^2(s_r - \overline{u}) + d(2 + 2\alpha + \alpha^2))}}{3\alpha^2}, \lambda = 1 \]

C. \[ T = \frac{(2\alpha^2(s_r - \overline{u}) + d(2 + 2\alpha + \alpha^2))^2}{36\alpha^2} \]

For possibilities B and C, the quantity sold simplifies to \( \frac{2\alpha^2(s_r - \overline{u}) + d(2 + 2\alpha + \alpha^2)}{6\alpha^2} \) which is positive if and only if \( 2\alpha^2(s_r - \overline{u}) + d(2 + 2\alpha + \alpha^2) < 0 \). However, if \( 2\alpha^2(s_r - \overline{u}) + d(2 + 2\alpha + \alpha^2) < 0 \), then the wholesale price in possible solutions B and C will be imaginary. Thus we rule out these possibilities because either quantities will be non-positive or wholesale price will be imaginary. The quantity for solution A simplifies to

\[ q_b^{vt} = \frac{\overline{u}}{2} + \frac{(c - s_r)^2}{8d} - \frac{2(3c - s_r) + d}{8} - \frac{(1 - \alpha)(c - s_r)}{2\alpha} \] which is positive for sufficiently high \( \overline{u} \).

The resulting equilibrium is:

\[ p^{vt} = \frac{\overline{u}}{2} + \frac{(c - s_r)^2}{8d} + \frac{2(c + s_r) - d}{8} - \frac{(1 - \alpha)(c - s_r)}{2\alpha} \]

\[ f^{vt} = c - s_r, \]

\[ q_b^{vt} = \frac{\overline{u}}{2} + \frac{(c - s_r)^2}{8d} - \frac{2(3c - s_r) + d}{8} - \frac{(1 - \alpha)(c - s_r)}{2\alpha}, \]

\[ \phi_e^{vt} = \alpha\left(1 - \frac{(c - s_r)}{2d}\right) \]
We compare manufacturer profit when the manufacturer salvages returned units to when the retailer salvages returned units. At \( s = s_r \), the two profits are equal. Claim: The manufacturer’s profit when the manufacturer salvages returns is increasing in \( s \) for all parameters such that exchanges and quantity are non-negative. Proof of Claim:

$$\frac{\partial \pi_{\text{mfr}}^{\text{opt}}}{\partial s} = \frac{d(2 - \alpha) - \alpha(c - s)}{2d} \left( \frac{\bar{u}}{2} + \frac{(c - s)^2}{8d} - \frac{2(3c - s) + d}{8} - \frac{(1 - \alpha)(c - s)}{2\alpha} \right).$$

Exchanges are non-negative if and only if \( d > c - s \) which implies \( \frac{d(2 - \alpha) - \alpha(c - s)}{2d} > 0 \). Quantity is non-negative if and only if 

$$\frac{\bar{u}}{2} + \frac{(c - s)^2}{8d} - \frac{2(3c - s) + d}{8} - \frac{(1 - \alpha)(c - s)}{2\alpha} > 0.$$ 

Therefore, the manufacturer’s profit when the manufacturer salvage returns is increasing in \( s \) for all parameters such that exchanges and quantity are non-negative.

Since the manufacturer’s profit when the retailer salvages returns is invariant with respect to \( s \), the manufacturer’s profit when the manufacturer salvages returns is increasing with respect to \( s \) and the two profits are equal when \( s = s_r \), it can be concluded that the manufacturer earns greater profit from taking back returns if and only if \( s > s_r \).

Thus, when the manufacturer may offer a fixed fee in addition to a wholesale price and refund rate, the equilibrium can be described as:

<table>
<thead>
<tr>
<th>Retail Price</th>
<th>( p^{\text{opt}} = \frac{\bar{u}}{2} + \frac{(c - \max{s, s_r})^2}{8d} + \frac{2(c + \max{s, s_r}) - d - (1 - \alpha)(c - \max{s, s_r})}{8} - \frac{2\alpha}{2\alpha} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return Penalty</td>
<td>( f^{\text{opt}} = c - \max{s, s_r} )</td>
</tr>
<tr>
<td>Quantity Sold Initially</td>
<td>( q^{\text{opt}} = \frac{\bar{u}}{2} + \frac{(c - \max{s, s_r})^2}{8d} - \frac{2(3c - \max{s, s_r}) + d - (1 - \alpha)(c - \max{s, s_r})}{8} - \frac{2\alpha}{2\alpha} )</td>
</tr>
</tbody>
</table>
Exchange Probability

\[ \phi^v_i = \frac{1}{2} \left(1 - \frac{c - \max\{s, s_i\}}{2d}\right) \]

Channel Profit

\[ \pi^v_{vir} = \left(\alpha(c - \max\{s, s_i\}) - (4 + 2\alpha)cd + 2d((2 - \alpha)\max\{s, s_i\} + 2\alpha\bar{u}) - \alpha d\right)^i \]

\[ \frac{64d^i}{64d^i} \]

Wholesale Price

\[ W^v = c \]

Refund Rate

\[ r^v = s \]

This is equivalent to a vertically integrated system in which the greatest net salvage value for returned units \( \max\{s, s_i\} \) is attainable.

\[ Q.E.D. \]

A.5. Proof that Asymmetric Prices and Restocking Fees Yield Equivalent Profit to Symmetric Equilibrium of Vertically-Integrated System.

In this Appendix, we show that allowing for asymmetric equilibria in a vertically-integrated channel system will produce the same profit, total number of returns, total number of exchanges, and total number of initial sales as the symmetric equilibrium.

Note from equation (1) in the paper’s text that the difference between expected utility from buying product \( j=0 \) and the expected utility from buying product \( j=1 \) is invariant with respect to \( u_i \). Therefore, if product 0 is preferred \( \text{ex ante} \) over product 1 by one consumer, it is preferred for all consumers (and vice versa). Also, notice that the expected utility from buying product 0 (and equivalently product 1) is increasing in \( u_i \). Therefore, if a consumer located at \( u_i = \hat{u} \) has non-negative expected utility from purchasing product 0 (and equivalently product 1), then all consumers with \( u_i > \hat{u} \) will have positive expected utility from purchasing the product.

Suppose that there is a \( \hat{u} \) for which prices and restocking fees are chosen such that all consumers with \( u_i < \hat{u} \) do not buy either product initially, and all consumers with \( u_i \in [\hat{u}, \bar{u}] \) do initially buy one of the two products. We will prove that all resulting asymmetric equilibria will...
produce the same profit, total number of returns, total number of exchanges, and total number of initial sales as the symmetric equilibrium. This implies that for any given population size that is optimal to serve (\(\bar{u} - \hat{u}\) where \(\hat{u}\) can be any value between 0 and \(\bar{u}\)), wholesale prices, refund rates, and retail prices and restocking fees will produce the same outcome as the symmetric equilibrium. Therefore, making asymmetric choices will not improve profit nor change initial sales or total returns. Therefore, the symmetric equilibrium established in Appendix A.4. which also identifies the optimal population to serve initially, will also produce the same profit, returns, exchanges, and initial sales as when allowing for asymmetric choices.

The form of the profit expression will depend on whether the expected utility of buying product 0 is equal or unequal to that of buying product 1. Three possibilities exist: (1)

\[ E_{j=0}(utility \mid u_i = \hat{u}) = E_{j=1}(utility \mid u_i = \hat{u}) \geq 0 \]; (2) \( E_{j=0}(utility \mid u_i = \hat{u}) > E_{j=1}(utility \mid u_i = \hat{u}) \geq 0 \); or (3) \( E_{j=0}(utility \mid u_i = \hat{u}) \geq 0 \) and \( E_{j=1}(utility \mid u_i = \hat{u}) < 0 \). We next explore each of these possibilities.

**Possibility 1: Product 0 offers expected utility equal to the expected utility offered by product 1.**

If Product 0 offers the same expected utility as product 1, then consumers are indifferent between buying 0 and buying 1. Therefore consumers for whom \(u_i \in [\hat{u}, \bar{u}]\) will randomly choose between initial purchase of product 0 or 1, resulting in half the market (of total size \(\bar{u} - \hat{u}\)) buying product 0 and the other half buying product 1. Assuming that \(s > s_n\), the profit can be written as the sum of profits from initially-bought products that are kept (subscript \(k\)), those that are exchanged (subscript \(e\)), and those that are returned without exchange (subscript \(r\)), summed across the two product offerings (see equation (1) in the paper for definitions of the \(\phi\) expressions):

\[
\pi = \frac{\bar{u} - \hat{u}}{2} \left( (p_0 - c)\phi_{k0} + (p_1 - c + f_0 - c + s)\phi_{k1} + (f_0 - c + s)\phi_{r0} \right) + (p_1 - c)\phi_{k1} + (p_0 - c + f_1 - c + s)\phi_{k1} + (f_1 - c + s)\phi_{r1}
\]
The seller’s objective is to maximize profit subject to the constraint that consumers with 

\( u_i = \hat{u} \) have non-negative expected utility from making an initial purchase. We also have the constraint that expected utility from product 0 is equal to that of product 1. The constrained optimization problem can be described with Lagrangian multipliers \( \lambda_1 \) and \( \lambda_2 \):

\[
\max_{p_0, p_1, \hat{u}, \lambda_1, \lambda_2} \quad L = \pi + \lambda_1 (E_{j=0}(utility \mid u_i = \hat{u}) - E_{j=1}(utility \mid u_i = \hat{u})) + \lambda_2 E_{j=0}(utility \mid u_i = \hat{u})
\]

\[
\text{s.t.} \quad E_{j=0}(utility \mid u_i = \hat{u}) - E_{j=1}(utility \mid u_i = \hat{u}) = 0
\]

\[
\lambda_2 E_{j=0}(utility \mid u_i = \hat{u}) = 0
\]

\[
\lambda_1 \geq 0, \quad \lambda_2 \geq 0
\]

The Kuhn-Tucker conditions are

\[
\frac{dL}{dp_0} = (\bar{u} - \hat{u}) \frac{\alpha(d + f_0 - f_1 - 2(p_0 - p_1) + \lambda_1 (f_1 + f_0) - \lambda_2 (d + f_0 + p_0 - p_1))}{2d} = 0,
\]

\[
\frac{dL}{dp_1} = (\bar{u} - \hat{u}) \frac{\alpha(d + f_1 - f_0 - 2(p_1 - p_0) - \lambda_1 (f_1 + f_0) - \lambda_2 (d - f_0 + p_0 - p_1))}{2d} = 0,
\]

\[
\frac{dL}{df_0} = (\bar{u} - \hat{u}) \frac{\alpha(c - s + 2(p_0 - f_0 - p_1)(1 + \lambda_1 - \lambda_2)) + (2 - \alpha)(1 + 2\lambda_1 - 2\lambda_2)}{4d} = 0,
\]

\[
\frac{dL}{df_1} = (\bar{u} - \hat{u}) \frac{(2 - \alpha)(1 - 2\lambda_1)}{4d} + \alpha(c - s + 2(p_1 - f_1 - p_0)(1 - \lambda_1)) = 0
\]

\[
\frac{\partial L}{\partial \lambda_1} = 2 \frac{(2 - \alpha)(f_1 - f_0)}{4d} + \alpha(f_1 + f_0)(f_1 - f_0 - 2(p_0 - p_1)) = 0
\]

\[
\lambda_2 (\alpha(\hat{u} - p_0 + p_1) + \frac{d}{4} + \frac{(f_0 - p_0 + p_1)^2}{4d}) - (1 - \alpha)f_0 = 0
\]

\[
\lambda_1 \geq 0, \quad \lambda_2 \geq 0
\]

The unique solution to the Kuhn-Tucker conditions is:

\[
p_0 = p_1 = \hat{u} + \frac{(c - s)^2}{4d} - \frac{(c - s)(2 - \alpha)}{2\alpha} - \frac{d}{4}
\]

\[
f_0 = f_1 = c - s
\]

\[
\lambda_1 = (\bar{u} - \hat{u}) / 2, \quad \lambda_2 = (\bar{u} - \hat{u})
\]

The equilibrium results in the following outcome.

Total quantity sold initially: \((\bar{u} - \hat{u})\)
Total quantity returned (without subsequent exchange): \( \frac{(\bar{u} - \hat{u})}{2} \phi_{i,0} + \frac{(\bar{u} - \hat{u})}{2} \phi_{i,1} = (1 - \alpha)(\bar{u} - \hat{u}) \)

Total quantity of exchanges:

\[ \frac{(\bar{u} - \hat{u})}{2} \phi_{e,0} + \frac{(\bar{u} - \hat{u})}{2} \phi_{e,1} = \alpha(\bar{u} - \hat{u})(d - c + s) + \frac{\alpha(\bar{u} - \hat{u})(d - c + s)}{4d} = \alpha(\bar{u} - \hat{u})(\frac{1}{2} - \frac{c - s}{2d}) \]

Manufacturer profit: \( \alpha(\bar{u} - \hat{u})(\bar{u} - c + \frac{(c - s)^2}{4d} - \frac{(c - s)(2 - \alpha)}{2\alpha} - \frac{d}{4}) \).

Possibility 2: \( E_{j=0}(utility \mid u_i = \hat{u}) > E_{j=1}(utility \mid u_i = \hat{u}) \geq 0 \).

If Product 0 offers greater expected utility than product 1, all consumers who buy initially (consumers with \( u_i \in [\hat{u}, \bar{u}] \)) will initially buy product 0. In this case, the seller’s profit can be written as

\( \pi = (\bar{u} - \hat{u})(p_0 - c)\phi_{i,0} + (p_1 - c + f_0 - c + s)\phi_{i,0} + (f_0 - c + s)\phi_{i,0} \).

The seller’s objective is to maximize profit subject to the constraint that consumers with \( u_i = \hat{u} \) have \( E_{j=0}(utility \mid u_i = \hat{u}) > E_{j=1}(utility \mid u_i = \hat{u}) \geq 0 \). The constrained optimization problem can be described with Lagrangian multipliers \( \lambda_1 \) and \( \lambda_2 \):

\[
\begin{align*}
\max_{p_0, p_1, \lambda_1, \lambda_2} & \quad L = \pi + \lambda_1 (E_{j=0}(utility \mid u_i = \hat{u}) - E_{j=1}(utility \mid u_i = \hat{u})) + \lambda_2 E_{j=1}(utility \mid u_i = \hat{u}) \\
\text{s.t.} & \quad \lambda_1 (E_{j=0}(utility \mid u_i = \hat{u}) - E_{j=1}(utility \mid u_i = \hat{u})) = 0 \\
& \quad \lambda_2 E_{j=1}(utility \mid u_i = \hat{u}) = 0 \\
& \quad \lambda_1 = 0, \quad \lambda_2 \geq 0
\end{align*}
\]

The Kuhn-Tucker conditions are
There are no solutions that satisfy $\lambda_1=0$ and all of the first-order conditions jointly; therefore, the firm will never choose to set prices and restocking fees so that $E_{j=0}(utility \mid u_j = \hat{u}) > E_{j=1}(utility \mid u_i = \hat{u}) \geq 0$. In other words, the firm will not choose to sell only product 0 and to give the marginal consumer strictly positive utility. We therefore rule out Possibility 2.

Possibility 3: $E_{j=0}(utility \mid u_j = \hat{u}) \geq 0$ and $E_{j=1}(utility \mid u_i = \hat{u}) < 0$.

We now turn our attention to potential equilibria in which the seller sets prices and restocking fees such that $E_{j=0}(utility \mid u_j = \hat{u}) \geq 0$ and $E_{j=1}(utility \mid u_i = \hat{u}) < 0$. The objective function becomes:

$$\max_{p_0, p_1, f_0, f_1, \lambda_1, \lambda_2} \pi + \lambda_2 E_{j=0}(utility \mid u_i = \hat{u})$$

subject to:

$$\lambda_2 (E_{j=0}(utility \mid u_j = \hat{u})) = 0$$

$$\lambda_2 \geq 0$$

$$E_{j=1}(utility \mid u_i = \hat{u}) < 0$$

The Kuhn-Tucker conditions are:
\[
\frac{\partial L}{\partial p_0} = (\bar{u} - \hat{u}) \frac{\alpha(d + 2(f_0 - p_0 + p_1)) + s - c - \lambda_2(d + f_0 - p_0 + p_1))}{2d} = 0
\]
\[
\frac{\partial L}{\partial p_1} = (\bar{u} - \hat{u}) \frac{\alpha(d - 2(f_0 - p_0 + p_1)) - s + c - \lambda_2(d - f_0 + p_0 - p_1))}{2d} = 0
\]
\[
\frac{\partial L}{\partial f_0} = (\bar{u} - \hat{u})(2 - \alpha)(1 - \lambda_2) + \alpha(c - s - (2 - \lambda_2)(p_1 - p_0 + f_0)) = 0
\]
\[
\lambda_2 (\alpha(\hat{u} - p_0 + p_0 - f_0) - \frac{d}{4} + \frac{(f_0 - p_0 + p_1)^2}{4d}) - (1 - \alpha)f_0 = 0
\]
\[
\lambda_2 \geq 0
\]

The Kuhn-Tucker conditions are satisfied at any price and restocking fee combination such that
\[
p_0 = \hat{u} + \frac{(c - s)^2}{4d} - \frac{\alpha(c - s)}{\alpha} - \frac{d}{4}, \quad p_1 = \hat{u} + \frac{(c - s)^2}{2} - \frac{d}{4} \quad \text{and}
\]
\[
e_{j=1}(utility | u_i = \hat{u}) = \frac{(2 - \alpha)(f_0 - f_1)}{2} + \frac{\alpha(f_1 + f_0)(f_1 + f_0 - 2(c - s))}{4d} < 0 \quad \text{(which implies } \lambda_2 = \bar{u} - \hat{u}).
\]

The equilibrium outcome in terms of profit, total returns, total sales, and total exchanges will be the same as in the symmetric solution where \(E_{j=0}(utility | u_i = \hat{u}) = E_{j=1}(utility | u_i = \hat{u}).\)

Specifically,

Total quantity sold initially: \((\bar{u} - \hat{u})\)

Total quantity returned (without subsequent exchange): \((\bar{u} - \hat{u})\phi_{\phi_0} + 0 \cdot \phi_{\phi_1} = (1 - \alpha)(\bar{u} - \hat{u})\)

Total quantity of exchanges: \((\bar{u} - \hat{u})\phi_{\phi_0} + 0 \cdot \phi_{\phi_1} = \alpha(\bar{u} - \hat{u})(\frac{1}{2} - \frac{c - s}{2d})\)

Manufacturer profit: \(\alpha(\bar{u} - \hat{u})(\hat{u} - c + \frac{(c - s)^2}{4d} - \frac{(c - s)(2 - \alpha)}{2\alpha} - \frac{d}{4}).\)

Moreover, if a consumer initially purchases product 0, the out-of-pocket marginal expense to return it and buy product 1 is also the same as in the symmetric case, namely:
\[
p_1 - (p_0 - f_0) = (c - s).
\]
Summarizing the insights from analyzing Possibilities 1, 2, and 3, we have: (a) Possibility 2 cannot exist; (b) Possibility 1 produces symmetric solution values; and (c) Possibility 3 produces the same profit, total returns, total sales, and total exchanges as in the symmetric solution. Thus, allowing asymmetry in the vertically-integrated channel structure results in the same outcomes as assuming symmetry.

In particular, given that the vertically-integrated channel structure can choose the most efficient salvage technology, we can replace $s$ with $\max\{s, s_r\}$ in the above expressions for the asymmetric vertically-integrated case. From Appendix A.4., we know that $\hat{q}_b = \bar{u} - \hat{u}$ implies at the optimum,

$$\hat{u} = \frac{\bar{u}}{2} - \frac{(c - \max\{s, s_r\})^2}{8d} + \frac{2(3c - \max\{s, s_r\}) + d}{8} + \frac{(1 - \alpha)(c - \max\{s, s_r\})}{2\alpha}.$$

Substituting this expression into the above solution values in Possibilities 1 and 3 shows that a symmetric solution and an asymmetric solution both yield the same values for total quantity sold, total quantity returned, total exchanges, and total channel profit, as follows:

Total quantity sold:

$$q_b^{VI} = \frac{\bar{u}}{2} + \frac{(c - \max\{s, s_r\})^2}{8d} - \frac{2(3c - \max\{s, s_r\}) + d}{8} - \frac{(1 - \alpha)(c - \max\{s, s_r\})}{2\alpha}.$$

Total quantity returned: 

$$(1 - \alpha)q_b^{VI}.$$

Total exchanges: 

$$\alpha q_b^{VI} \left(\frac{1 - c - s}{2d}\right).$$

Channel profit: 

$$\pi_{\text{ofr}}^{VI} = \frac{(\alpha(c - \max\{s, s_r\})^2 - (4 + 2\alpha)c + 2d((2 - \alpha)\max\{s, s_r\} + 2\alpha\bar{u}) - \alpha d)^2}{64d^2}.$$

Q.E.D.
A.6. Proof of Proposition 1

PROPOSITION 1: Consider a manufacturer who can charge a fixed fee, a per-unit wholesale price to the retailer, and a per-unit refund rate. If and only if \( s > s_r \), then the manufacturer earns greater profit from the reverse channel structure in which the manufacturer salvages returns than the reverse channel structure in which the retailer salvages returns. The equilibrium is described in Table 2.

From Appendix A.4., the manufacturer’s profit from accepting and salvaging returns is greater than the manufacturer’s profit from the retailer salvaging returns if and only if \( s > s_r \).

Q.E.D.

A.7. Parameter values for which all consumers who buy initially prefer to keep or exchange, rather than return without subsequent purchase in the VI system

From A.3., we know that all consumers who initially buy a product will prefer to keep or exchange it rather than return it without subsequent purchase if \( df - \frac{d^2}{2f} > 0 \). Substituting the equilibrium value of \( f \) from Table 2, this implies that \( 2(2 + 4\alpha)(c-s) \alpha \alpha + \alpha \alpha = \alpha \alpha \) is sufficient to ensure that consumers who buy initially will prefer to keep or exchange their initial purchase even if they have the option to return without subsequent purchase.

A.8. Proof of Retailer’s Reaction Functions (See Equation 3)

Given a wholesale price and a refund, the retailer has the following objective function:

\[
\max_{q_b(f, p; d, \bar{u})} ((p-w) + (-p + f + \max\{s, r\})(\phi_r(f; d) + \phi_i) + (p-w)\phi_r(f; d)) \to \max
\]

The first order conditions are
\begin{align*}
\frac{\partial \pi_{ret}}{\partial p} &= \alpha(\bar{u} - (p-w) + \frac{(d-f)(w-\max\{s_r, r\} - f)}{2d} - (1-\alpha)(2f + \max\{s_r, r\} - w) \\
&\quad + \frac{\alpha}{4}(-d\frac{f^2}{d} - 2(f + 2p)) = 0
\end{align*}
\begin{align*}
\frac{\partial \pi_{ret}}{\partial f} &= \frac{\alpha(d^2 - f^2) + 2d(f(2-\alpha) + 4\alpha(p-\bar{u}))(\alpha(2f - w + \max\{s_r, r\}) - (2-\alpha)d)}{8\alpha d^2} \\
&\quad + \frac{(2d - \alpha d - \alpha f)((2d - \alpha d - \alpha f)(f + \max\{s_r, r\} - w) - 2d\alpha + 2\alpha dp)}{4\alpha d^2} = 0
\end{align*}

There are three possible solutions that satisfy the first order conditions.

I) \( p(w, r) = \frac{\bar{u}}{2} + \frac{(w-\max\{s_r, r\})^2}{8d} + \frac{2(w + \max\{s_r, r\}) - d}{8} - \frac{1-\alpha)(w-\max\{s_r, r\})}{2\alpha} \)
\[
f(w, r) = w - \max\{s_r, r\}
\]

Defining \( X \equiv \alpha^2 ((w-\max\{s_r, r\})^2 - 2d(w + \max\{s_r, r\} - 2\bar{u}) - d^2) - 4d\alpha(w - \max\{s_r, r\}) \), we have

II)
\begin{align*}
p(w, r) &= 2\bar{u} + \frac{(w-\max\{s_r, r\})^2}{2d} - d(2 + \max\{s_r, r\}) - \frac{\sqrt{X}}{\alpha^2} + \frac{4d\max\{s_r, r\} - w + (w-d + \max\{s_r, r\})\sqrt{X}}{2d\alpha}
\end{align*}
\[
f = w - r - \sqrt{X}
\]

III)
\begin{align*}
p(w, r) &= 2\bar{u} + \frac{(w-\max\{s_r, r\})^2}{2d} - d(2 + \max\{s_r, r\}) + \frac{\sqrt{X}}{\alpha^2} + \frac{4d\max\{s_r, r\} - w - (w-d + \max\{s_r, r\})\sqrt{X}}{2d\alpha}
\end{align*}
\[
f(w, r) = w - \max\{s_r, r\} + \sqrt{X}.
\]

The initial sales quantity \( q_b \) when evaluated at solutions I, II and III is
\[
q_b = \frac{\bar{u}}{2} - \frac{w - \max\{s_r, r\}}{2\alpha} - \frac{d^2 + 2d(w + \max\{s_r, r\}) - (w - \max\{s_r, r\})^2}{8d}, \quad q_b = 0, \text{ and } q_b = 0
\]
respectively. We therefore rule out potential solutions II and III. Therefore, the retailer’s reaction functions are

\[
p(w, r) = \frac{\bar{w}}{2} + \frac{(w - \max\{s_r, r\})^2}{8d} + \frac{2(w + \max\{s_r, r\}) - d}{8} - \frac{(1 - \alpha)(w - \max\{r, s_r\})}{2\alpha}
\]

\[
f(w, r) = w - \max\{s_r, r\}
\]

Q.E.D.

**A.9. Proof of Entries in Table 3 (See Section 3)**

The manufacturer’s objective function takes into account the retailer’s reaction (from Appendix A.8.) and can be described as

\[
\max_{w, r, z} q_w(f(w, r), p(w, r); d, \bar{u})(w - c) + (w - c - I_z(r - s))\phi_z(f(w, r); d) - I_z\phi_z(r - s)
\]

where \( I_z \) is an indicator variable equal to one if the manufacturer accepts and salvages returned units from the retailer (CRM reverse channel structure) and equal to zero otherwise (CR reverse channel structure). We solve the model recursively, first identifying the wholesale price and refund for each reverse channel structure. We will then identify when each reverse channel structure will give the manufacturer greater profit. The model cannot simply be solved by finding the optimal \( w \) and \( r \) when the manufacturer accepts returns and identifying parameters for which \( r^* < s_r \).

Such a solution method operates under the assumption that the value of the returned unit is \( s \) and the refund rate \( r \) factors into the retailer’s reaction function. However, when return responsibility is allocated to the retailer, the returned units are valued at \( s_r \) (though by the retailer) and the wholesale price is the only element in retailer’s decision. The wholesale price in each reverse channel structure will be set differently because of where units are salvaged, at what value they are salvaged, and the retailer’s decisions based on these facts. Thus, we solve for the equilibrium wholesale price and refund rate in each reverse channel structure separately.
First, we examine when the retailer salvages returned units.

Defining \(Y_r = \alpha((w-s_r)^2 - 2d(w+s_r - 2u) - d^2) - 4d(w-s_r)\), the first order condition is

\[
\frac{\partial \pi_{mfr}}{\partial w} = \frac{1}{16\alpha d^2}(2(c-w)(\alpha(s_r-w) + d(2+\alpha))^2 + (\alpha(c+s_r-2w) + d(2+\alpha))Y_r) = 0
\]

Defining \(A = 44d^2 + 4\alpha d(-3c + 11d + 3s_r) + \alpha^2(3(c-s_r)^2 + 19d^2 + 38ds_r - 6c - 32u)\) and

\(B = -9\alpha^3(-2d + a(c - d - s_r))(-12d^2 - 4ad(c + 3d - s_r) + a^2((c-s)^2 - 7d(d + 2s_r) - 2cd + 16u))\),

the first order condition is satisfied at:

I) \(w = \frac{1}{4}(c + 3(s_r + d + \frac{2d}{\alpha})) - \frac{3^{7/3}}{12} \left( \frac{A}{12(B + \sqrt{B^2 - 3\alpha^6 A^3})^{1/3}} + \frac{(B + \sqrt{B^2 - 3\alpha^6 A^3})^{1/3}}{3^{1/3}\alpha^2} \right)\)

II) \(w = \frac{1}{4}(c + 3(s_r + d + \frac{2d}{\alpha})) + \frac{3^{7/3}(1 + i\sqrt{3})A}{24(B + \sqrt{B^2 - 3\alpha^6 A^3})^{1/3}} + \frac{3^{7/3}(1 - i\sqrt{3})(B + \sqrt{B^2 - 3\alpha^6 A^3})^{1/3}}{24\alpha^2}\)

III) \(w = \frac{1}{4}(c + 3(s_r + d + \frac{2d}{\alpha})) + \frac{3^{7/3}(1 - i\sqrt{3})A}{24(B + \sqrt{B^2 - 3\alpha^6 A^3})^{1/3}} + \frac{3^{7/3}(1 + i\sqrt{3})(B + \sqrt{B^2 - 3\alpha^6 A^3})^{1/3}}{24\alpha^2}\)

We restrict our attention to real solutions and therefore we may discard solutions II and III.

Therefore, the manufacturer’s wholesale price when returns responsibility is allocated to the retailer \((r^* = 0)\) will be:

\[
w^* = \frac{1}{4}(c + 3(s_r + d + \frac{2d}{\alpha})) - \frac{3^{7/3}}{12} \left( \frac{A}{12(B + \sqrt{B^2 - 3\alpha^6 A^3})^{1/3}} + \frac{(B + \sqrt{B^2 - 3\alpha^6 A^3})^{1/3}}{3^{1/3}\alpha^2} \right).
\]

Secondly we solve for when the manufacturer accepts returns

If the manufacturer chooses to take back returns \((I_z = 1)\), the manufacturer’s objective function is given by

\[
\max_{w,r} q_b(f(w,r), p(w,r); d, \bar{u})(w-c) + (w-c-r+s))\phi_e(f(w,r); d) \quad \text{where the retailer’s reaction}
\]

functions are defined in Appendix A.6. The first order conditions for the manufacturer are
There are three possible solutions that satisfy the first order conditions for the manufacturer.

**A)**

\[
\begin{align*}
  w^* &= c + \frac{\bar{u}}{2} + \frac{(c-s)^2}{8d} - \frac{2(3c-s) + d}{8} - \frac{(1-\alpha)(c-s)}{2\alpha}, \\
  r^* &= s + \frac{\bar{u}}{2} + \frac{(c-s)^2}{8d} - \frac{2(3c-s) + d}{8} - \frac{(1-\alpha)(c-s)}{2\alpha}
\end{align*}
\]

Defining \( Y_M = (\alpha(c-s)^2 - d^2\alpha - 2d(c+\alpha) - s(2-\alpha) - 2\alpha \bar{p}) \)

**B)**

\[
\begin{align*}
  r &= s - \frac{Y_M}{2d\alpha} + \frac{\sqrt{-\alpha^3 Y_M(\alpha(c-s) - d(2+\alpha))^2}}{2d\alpha^3}, \\
  w &= c - \frac{Y_M}{2d} - \frac{(d(2-\alpha) - \alpha(c-s))\sqrt{-\alpha^3 Y_M(\alpha(c-s) - d(2+\alpha))^2}}{2d\alpha^3(\alpha(c-s) - d(2+\alpha))}
\end{align*}
\]

**C)**

\[
\begin{align*}
  r &= s - \frac{Y_M}{2d\alpha} - \frac{\sqrt{-\alpha^3 Y_M(\alpha(c-s) - d(2+\alpha))^2}}{2d\alpha^3}, \\
  w &= c - \frac{Y_M}{2d} + \frac{(d(2-\alpha) - \alpha(c-s))\sqrt{-\alpha^3 Y_M(\alpha(c-s) - d(2+\alpha))^2}}{2d\alpha^3(\alpha(c-s) - d(2+\alpha))}
\end{align*}
\]

The retailer’s reaction to possible solutions B and C leads to \( q_b = 0 \) and zero profit for both the retailer and the manufacturer. Therefore, A is the only solution for which profit and \( q_b \) are potentially positive. Defining \( q_b^{\text{CRM}} \) as the equilibrium quantity sold initially when the wholesale and refund rate are given by solution A with the manufacturer accepting returns:
We verify that the solution to the first order conditions also satisfies the second order conditions for any parameters such that \( q_{CRM}^b > 0 \).

Evaluated at the \( w, r \) of solution A, 

\[
\frac{\partial^2 \pi_{mgfr}}{\partial w \partial u} = -\frac{1}{2d} \left( \frac{\alpha u}{4} + \frac{2d(1+\alpha)}{\alpha} - \frac{9\alpha(c-s)^2}{4} - \frac{9(c-s)}{16d} - \frac{\alpha(18c-7(d+2s))}{16} \right),
\]

which is less than zero for all \( u \), such that 

\[
\bar{u} + \frac{(c-s)^2}{4} - \frac{c-s}{4\alpha} - \frac{2(c+s)+d}{16} > 0 \quad \text{(i.e., } q_{CRM}^b > 0 \text{ holds when)}
\]

\[
\bar{u} > \frac{c-s}{\alpha} + \frac{2(c+s)+d}{4} - \frac{(c-s)^2}{4d},
\]

by the fact that

\[
\frac{c-s}{\alpha} + \frac{2(c+s)+d}{4} - \frac{(c-s)^2}{4d} > \frac{9(c-s)}{\alpha} + \frac{\alpha(18c-7(d+2s))}{4\alpha} - \frac{8d(1+\alpha)}{\alpha^2} - \frac{9(c-s)^2}{4d}.
\]

The latter inequality holds because

\[
\frac{c-s}{\alpha} + \frac{2(c+s)+d}{4} - \frac{(c-s)^2}{4d} > \frac{9(c-s)}{\alpha} + \frac{\alpha(18c-7(d+2s))}{4\alpha} - \frac{8d(1+\alpha)}{\alpha^2} - \frac{9(c-s)^2}{4d}.
\]

\[
\frac{2(d(2+\alpha)-(c-s))^2}{d\alpha^2} > 0.
\]

\[
\frac{\partial^2 \pi_{mgfr}}{\partial r \partial u} = -\frac{1}{2d} \left( \frac{\alpha u}{4} + \frac{2d(1-\alpha)}{\alpha} - \frac{9\alpha(c-s)^2}{4} - \frac{9(c-s)}{16d} - \frac{\alpha(14c-18s+7d)}{16} \right),
\]

which is less than zero for all \( u > \frac{9(c-s)}{\alpha} - \frac{8d(1-\alpha)}{\alpha^2} - \frac{9\alpha(c-s)^2}{4\alpha d} - \frac{(14c-18s+7d)}{4} \). This is true for all \( u \) such that

\[
q_{CRM}^b = \frac{\bar{u} + (c-s)^2}{4} - \frac{c-s}{16d} - \frac{2(c+s)+d}{4\alpha} > 0 \quad \text{(i.e., } q_{CRM}^b > 0 \text{ holds when)}
\]

\[
\bar{u} > \frac{c-s}{\alpha} + \frac{2(c+s)+d}{4} - \frac{(c-s)^2}{4d},
\]

by the fact that
\[
\frac{c-s}{\alpha} + \frac{2(c+s)+d}{4} - \frac{(c-s)^2}{4d} > \frac{9(c-s)}{\alpha} - \frac{8d(1-\alpha)}{\alpha^2} - \frac{9\alpha(c-s)^2}{4\alpha d} - \frac{(14c-18s+7d)}{4}.
\]

The latter inequality holds because

\[
\frac{c-s}{\alpha} + \frac{2(c+s)+d}{4} - \frac{(c-s)^2}{4d} - \frac{9(c-s)}{\alpha} + \frac{8d(1-\alpha)}{\alpha^2} + \frac{9\alpha(c-s)^2}{4\alpha d} + \frac{(14c-18s+7d)}{4}
\]

is equal to

\[
2(d(2-\alpha) - \alpha(c-s))^2 > 0.
\]

The determinant of the Hessian

\[
|H_{\text{mfr}}| = \frac{\alpha^2 (\frac{\bar{u}}{4} + \frac{(c-s)^2}{16d} - \frac{c-s}{4\alpha} - \frac{2(c+s)+d}{16}) > 0 \text{ if and only if } \bar{u} \text{ is such that}
\]

\[
q_{\text{CRM}} = \frac{\bar{u}}{4} + \frac{(c-s)^2}{16d} - \frac{c-s}{4\alpha} - \frac{2(c+s)+d}{16} > 0.
\]

Because \(\lim_{w \to +\infty} \pi_{\text{mfr}} = -\infty\) and \(\lim_{r \to +\infty} \pi_{\text{mfr}} = -\infty\), the only local maximum of manufacturer’s profit (solution A) is also a global maximum.

In conclusion

Making simplifications using the retailer’s reaction functions, we may characterize the equilibrium as in Table 3 from the paper:

Table 3. Equilibrium under a \{Wholesale Price, Refund Rate\} Wholesale Contract

<table>
<thead>
<tr>
<th>Term</th>
<th>Equilibrium Value †</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retail Price</td>
<td>(p^* = \frac{\bar{u}}{2} \frac{(w^* - \max{r^<em>, s_1})^2}{8d} + \frac{2(w^</em> + \max{r^<em>, s_1}) - d}{8} - \frac{(1-\alpha)(w^</em> - \max{r^*, s_1})}{2\alpha})</td>
</tr>
<tr>
<td>Return Penalty</td>
<td>(f^* = w^* - \max{r^*, s_1})</td>
</tr>
<tr>
<td>Quantity Sold Initially</td>
<td>(q_b^* = \frac{\bar{u}}{2} \frac{w^* - \max{r^<em>, s_1}}{2\alpha} - \frac{d^2 + 2d(w^</em> + \max{r^<em>, s_1}) - (w^</em> - \max{r^*, s_1})^2}{8d})</td>
</tr>
<tr>
<td>Exchange Probability</td>
<td>(\phi_e^* = \alpha(\frac{1}{2} - \frac{(w^* - \max{r^*, s_1})}{2d}))</td>
</tr>
<tr>
<td>Retailer Profit</td>
<td>(\pi_{\text{ret}}^* = \frac{(d^2\alpha - \alpha(w^* - \max{r^<em>, s_1})^2 + 2d((2+\alpha)w^</em> - (2-\alpha)\max{r^*, s_1} - 2\alpha\bar{u}))^2}{64\alpha d^2})</td>
</tr>
</tbody>
</table>
\[
\text{CRM Manufacturer Profit (} I_z=1 \text{)}
\]
\[
\rho_{\text{CRM}}^{\text{mfgr}} = \frac{(\alpha (c - s)^2 + d (2s(2 - \alpha) + \alpha (4\bar{\mu} - d) - 2c(2 + \alpha)))^2}{128\alpha d^2}
\]

\[
\text{CRM Wholesale Price (} I_z=1 \text{)}
\]
\[
w^*_{\text{CRM}} = c + \frac{\bar{\mu} + (c - s)^2}{2d} - \frac{2(3c - s) + d}{8} - \frac{(1 - \alpha)(c - s)}{2\alpha}
\]

\[
\text{CRM Refund Rate (} I_z=1 \text{)}
\]
\[
r^*_{\text{CRM}} = s + \frac{\bar{\mu} + (c - s)^2}{2d} - \frac{2(3c - s) + d}{8} - \frac{(1 - \alpha)(c - s)}{2\alpha}
\]

\[
\text{CR Manufacturer Profit (} I_z=0 \text{)}
\]
\[
\rho_{\text{CR}}^{\text{mfgr}} = \frac{(w^* - c)(\alpha(s_r - w^*) + d(2 + \alpha))(\alpha(w^* - \max\{r^*,s_r\})^2 - \alpha d^2)}{16\alpha d^2}
\]
\[+ \frac{(w^* - c)(\alpha(s_r - w^*) + (2 + \alpha)d)((2 - \alpha)s_r + 2\alpha\bar{\mu} - (2 + \alpha)w^*)}{8\alpha d}
\]

\[
\text{CR Wholesale Price (} I_z=0 \text{)}
\]
\[
w^*_{\text{CR}} = \frac{1}{4} \left( c + 3(s_r + d + \frac{2d}{\alpha}) \right) - \frac{3^{2/3}}{12} \left( \frac{A}{12(B + \sqrt{B^2 - 3\alpha^2 A^3})^{1/3}} + \frac{B + \sqrt{B^2 - 3\alpha^2 A^3}}{3^{1/3} \alpha^2} \right)
\]

\[
\text{CR Refund Rate (} I_z=0 \text{)}
\]
\[r^*_{\text{CR}} = 0
\]

† In the equations for retail price, return penalty, quantity sold initially, exchange probability, and retailer profit, \(w^* = w^*_{\text{CRM}}\) and \(r^* = r^*_{\text{CRM}}\) when the channel structure is CRM; \(w^* = w^*_{\text{CR}}\) and \(r^* = r^*_{\text{CR}}\) when the channel structure is CR. In the equation for CR manufacturer profit above, \(w^* = w^*_{\text{CR}}\) and \(r^* = r^*_{\text{CR}}\).

†† \(A = 44d^2 + 4\alpha d(-3c + 11d + 3s_r) + \alpha^2(3(c - s_r)^2 + 19d^2 + 38ds_r - 6cd - 32\bar{\mu})\) and \(B = -9\alpha^2(-2d + a(c - d - s_r))(-12d^2 - 4\alpha d(c + 3d - s_r) + a^2((c - s_r)^2 - 7d(d + 2s_r) - 2cd + 16\bar{\mu}))\).

Q.E.D.

A.10. Proof that Asymmetric Prices and Restocking Fees Yield Equivalent Profit to Symmetric Equilibrium in a Decentralized Channel.

Suppose that there is a \(\hat{u}\) such that prices and restocking fees are chosen such that all consumers with \(u_i < \hat{u}\) do not buy either product initially and all consumers with \(u_i \in [\hat{u}, \bar{\mu}]\) buy one of the two products. The form of the profit expression will depend on whether the expected utility of buying product 0 is equal or unequal to that of buying product 1. Three possibilities
exist: (1) $E_{j=0}(utility \mid u_i = \hat{u}) = E_{j=1}(utility \mid u_i = \hat{u}) \geq 0$; (2) $E_{j=0}(utility \mid u_i = \hat{u}) > E_{j=1}(utility \mid u_i = \hat{u}) \geq 0$; or (3) $E_{j=0}(utility \mid u_i = \hat{u}) \geq 0$ and $E_{j=1}(utility \mid u_i = \hat{u}) < 0$. We explore each of these possibilities solving recursively.

**Possibility 1:** Product 0 offers expected utility equal to the expected utility offered by product 1.

If Product 0 offers the same expected utility as product 1, then consumers are indifferent between buying 0 and buying 1. Therefore consumers for whom $u_i \in [\hat{u}, \bar{u}]$ will randomly choose between products resulting in half the market (of total size $\bar{u} - \hat{u}$) buying product 0 and the other half buying product 1. We solve the model recursively, starting with the retailer’s problem. The retailer’s profit can be written as the sum of profits from initially-bought products that are kept (subscript $k$), those that are exchanged (subscript $e$), and those that are returned without exchange (subscript $r$), summed across the two product offerings (see equation (1) in the paper for definitions of the $\phi$ expressions):

$$\pi_{ret} = \frac{(\bar{u} - \hat{u})}{2} ((p_0 - w_0) \phi_0 + (p_1 - w_1 + f_0 - w_0 + \max\{r_0, s_r\}) \phi_0 + (f_0 - w_0 + \max\{r_0, s_r\}) \phi_0 + (p_1 - w_1) \phi_0 + (p_0 - w_0 + f_1 - w_1 + \max\{r_1, s_r\}) \phi_1 + (f_1 - w_1 + \max\{r_1, s_r\}) \phi_1)$$

The retailer’s objective is to maximize profit subject to the constraint that consumers with $u_i = \hat{u}$ have non-negative expected utility from making an initial purchase. We also have the constraint that expected utility from product 0 is equal to that of product 1. The constrained optimization problem can be described with Lagrangian multipliers $\lambda_1$ and $\lambda_2$:

$$\max_{p_0, p_1, \ldots, f_0, h_0, \lambda_0} L_{ret} = \pi_{ret} + \lambda_1 (E_{j=0}(utility \mid u_i = \hat{u}) - E_{j=1}(utility \mid u_i = \hat{u})) + \lambda_2 E_{j=0}(utility \mid u_i = \hat{u})$$

s.t. $E_{j=0}(utility \mid u_i = \hat{u}) - E_{j=1}(utility \mid u_i = \hat{u}) = 0$

$\lambda_2 E_{j=0}(utility \mid u_i = \hat{u}) = 0$

$\lambda_1 \geq 0, \lambda_2 \geq 0$

The Kuhn-Tucker conditions are
The unique solution to these conditions is

\[ p_0 = \tilde{u} - \frac{d}{4} - \frac{w_i - \max\{r_0, s_r\} + (1 - \alpha)(w_0 - \max\{r_1, s_r\}) + (1 - \alpha)(w_0 - \max\{r_1, s_r\})^2 + (\max\{r_0, s_r\} - w_i)^2}{2\alpha} \]

\[ p_1 = \tilde{u} - \frac{d}{4} - \frac{w_0 - \max\{r_0, s_r\} + (1 - \alpha)(w_1 - \max\{r_1, s_r\}) + (w_0 - \max\{r_1, s_r\})^2 + (1 - \alpha)(\max\{r_0, s_r\} - w_i)^2}{2\alpha} \]

\[ f_0 = \frac{(\max\{r_0, s_r\} + \max\{r_1, s_r\} - w_0 - w_i)(\alpha(\max\{r_0, s_r\} - \max\{r_1, s_r\} + w_0 - w_i) - 2d(2 - \alpha))}{4d(2 - \alpha)} \]

\[ f_1 = \frac{(w_0 + w_i - \max\{r_0, s_r\} - \max\{r_1, s_r\})(2d(2 - \alpha) + \alpha(\max\{r_0, s_r\} - \max\{r_1, s_r\} + w_0 - w_i))}{4d(2 - \alpha)} \]

We first examine when the manufacturer accepts returns (CRM reverse channel structure).

The manufacturer takes the retailer’s reaction into account in maximizing its profit

\[ \pi_{s vr}^{CRM} = \frac{(\tilde{u} - \tilde{u})(w_0 - c)\phi_{\tilde{u}0} + (w_i - c + w_0 - c - r_0 + s)\phi_{\tilde{u}0} + (w_0 - c - r_0 + s)\phi_{\tilde{u}0} + (w_i - c + w_0 - c - r_i + s)\phi_{\tilde{u}1} + (w_0 - c - r_i + s)\phi_{\tilde{u}1})}{2} \]

where the expressions \( \phi_{\tilde{u}0}, \phi_{\tilde{u}1}, \phi_{\tilde{u}0}, \phi_{\tilde{u}1}, \phi_{\tilde{u}0}, \phi_{\tilde{u}1} \) from equation (1) in the text are evaluated at the optimal prices and restocking fees as functions of wholesale prices and refunds as described.
above. The manufacturer has the constraint that the contract offered to the retailer must provide non-negative profit. Thus, the manufacturer’s constrained optimization problem is

\[
\max_{w_0, w_1, r_0, r_1} L^{\text{mfg}} = \pi_{\text{mfg}}^{\text{CRM}} + \lambda_3 \pi_{\text{ret}} \\
\text{s.t. } \lambda_3 \pi_{\text{ret}} = 0, \lambda_3 \geq 0
\]

The Kuhn-Tucker conditions are

\[
\frac{\partial L^{\text{mfg}}}{\partial w_0} = \frac{(\bar{u} - \hat{u})}{2} (1 - \lambda_3 + \frac{\alpha((c - s - 2w_0 + 2r_i + d) + \lambda_3(w_0 - r_i - d))}{2d})
\]

\[
\frac{\partial L^{\text{mfg}}}{\partial w_i} = \frac{(\bar{u} - \hat{u})}{2} (1 - \lambda_3 + \frac{\alpha((c - s - 2w_i + 2r_0 + d) + \lambda_3(w_i - r_0 - d))}{2d})
\]

\[
\frac{\partial L^{\text{mfg}}}{\partial r_0} = \frac{(\bar{u} - \hat{u})}{2} ((1 - \alpha)(\lambda_3 - 1) - \frac{\alpha((c - s - 2w_i + 2r_0 + d) + \lambda_3(w_i - r_0 - d))}{2d})
\]

\[
\frac{\partial L^{\text{mfg}}}{\partial r_0} = \frac{(\bar{u} - \hat{u})}{2} ((1 - \alpha)(\lambda_3 - 1) - \frac{\alpha((c - s - 2w_0 + 2r_i + d) + \lambda_3(w_0 - r_i - d))}{2d})
\]

\[
\lambda_3 \pi_{\text{ret}} = 0, \lambda_3 \geq 0
\]

These conditions are satisfied at

\[
r_i = w_0 - c + s
\]

\[
w_i = 2\hat{u} - w_0 + \frac{(c - s)^2}{2d} - \frac{(2 - \alpha)(c - s)}{2\alpha}
\]

\[
r_0 = 2\hat{u} - w_0 + \frac{(c - s)^2}{2d} - \frac{2(c - s)}{2\alpha}
\]

\[
\lambda_3 = 1
\]

Plugging these wholesale prices and refund rates into the retailer’s reaction function, the resulting prices and restocking fees will be

\[
p_0 = p_i = \hat{u} + \frac{(c - s)^2}{4d} - \frac{(c - s)(2 - \alpha)}{2\alpha} - \frac{d}{4}
\]

\[
f_0 = f_i = c - s
\]

This equilibrium results in the following outcome.
Total quantity sold initially: \((\bar{u} - \hat{u})\)

Total quantity returned (without subsequent exchange): \(\frac{(\bar{u} - \hat{u})\phi_0}{2} + \frac{(\bar{u} - \hat{u})\phi_1}{2} = (1 - \alpha)(\bar{u} - \hat{u})\)

Total quantity of exchanges:
\[
\frac{(\bar{u} - \hat{u})}{2}\phi_0 + \frac{(\bar{u} - \hat{u})}{2}\phi_1 = \frac{\alpha(\bar{u} - \hat{u})(d - c + s)}{4d} + \frac{\alpha(\bar{u} - \hat{u})(d - c + s)}{4d} = \frac{\alpha(\bar{u} - \hat{u})(d - c + s)}{2d}
\]

Manufacturer profit:
\[
\alpha(\bar{u} - \hat{u})(\bar{u} - c + \frac{(c - s)^2}{4d} - \frac{(c - s)(2 - \alpha)}{2\alpha} - \frac{d}{4})
\]

We now examine when the manufacturer does not accept returns (CR reverse channel structure).

The manufacturer earns profit equal to
\[
\pi_{\text{CR}} = \frac{(\bar{u} - \hat{u})}{2}((w_0 - c)\phi_0 + (w_1 - c + w_0 - c)\phi_0 + (w_0 - c)\phi_0 + (w_1 - c)\phi_1) + (w_1 - c)\phi_1 + (w_0 - c + w_1 - c)\phi_1 + (w_1 - c)\phi_1)
\]

where the expressions \(\phi_0, \phi_1, \phi_0, \phi_1, \phi_0, \phi_1\) from equation (1) in the text are evaluated at the optimal prices and restocking fees as functions of wholesale prices and refunds as described above. The manufacturer has the constraint that the contract offered to the retailer must provide non-negative profit. Thus, the manufacturer’s constrained optimization problem is
\[
\max_{w_0, w_1} L_{\text{CR}} = \pi_{\text{CR}} + \lambda_3\pi_{\text{ret}}
\]
\[
\text{s.t. } \lambda_3 \pi_{\text{ret}} = 0, \lambda_3 \geq 0
\]

The Kuhn-Tucker conditions are
\[
\frac{\partial L_{\text{CR}}}{\partial w_0} = \frac{(\bar{u} - \hat{u})}{2}(1 - \lambda_3 + \alpha((c - 2w_0 + 2s_1 + d) + \lambda_3(w_0 - s_1 - d)))
\]
\[
\frac{\partial L_{\text{CR}}}{\partial w_1} = \frac{(\bar{u} - \hat{u})}{2}(1 - \lambda_3 + \alpha((c - 2w_1 + s_1 + d) + \lambda_3(w_1 - s_1 - d)))
\]
\[
\lambda_3 \pi_{\text{ret}} = 0, \lambda_3 \geq 0
\]

The three potential solutions are each symmetric:
\[ w_0 = w_i = \frac{c + d + s_r}{2}, \quad \lambda_3 = 0 \]

\[ w_0 = w_i = \frac{(2d + \alpha(-c + d + s_r))(2d + \alpha(d + s_r)) \pm \sqrt{2d(2d + \alpha(-c + d + s_r))^2(d(2 + 2\alpha + \alpha^2) - 2\alpha^2(\hat{u} - s_r))}}{\alpha(2d + \alpha(-c + d + s_r))}, \]

\[ \lambda_3 = \frac{4d^2(2 + 2\alpha + \alpha^2) - 8d\alpha^2(\hat{u} - s_r) \pm \sqrt{2d(2d + \alpha(-c + d + s_r))^2(d(2 + 2\alpha + \alpha^2) - 2\alpha^2(\hat{u} - s_r))}}{2d(2 + 2\alpha + \alpha^2) - 2\alpha^2(\hat{u} - s_r)}. \]

**Possibility 2:** \( E_{j=0}(utility | u_i = \hat{u}) > E_{j=1}(utility | u_i = \hat{u}) \geq 0. \)

If Product 0 offers greater expected utility than product 1, all consumers who buy initially (consumers with \( u_i \in [\hat{u}, \bar{u}] \)) will initially buy product 0. In which case, the seller’s profit can be written as

\[ \pi_{ret} = (\pi - \hat{u})(p_0 - w_0)\phi_{k_0} + (p_1 - w_i + f_1 - w_0 + \max \{r_0, s_r\})\phi_{k_0} + (f_0 - w_0 + \max \{r_0, s_r\})\phi_{f_0}. \]

The retailer’s objective is to maximize profit subject to the constraints that consumers with \( u_i = \hat{u} \) have \( E_{j=0}(utility | u_i = \hat{u}) > E_{j=1}(utility | u_i = \hat{u}) \geq 0. \) The constrained optimization problem can be described with Lagrangian multipliers \( \lambda_1 \) and \( \lambda_2 \).

\[
\max_{p_0, p_1, \lambda_1, \lambda_2, \lambda_3} L_{ret}^{\pi} = \pi_{ret} + \lambda_1(E_{j=0}(utility | u_i = \hat{u}) - E_{j=1}(utility | u_i = \hat{u}))) + \lambda_2E_{j=1}(utility | u_i = \hat{u})
\]

s.t. \( \lambda_1(E_{j=0}(utility | u_i = \hat{u}) - E_{j=1}(utility | u_i = \hat{u}))) = 0 \)

\( \lambda_2E_{j=1}(utility | u_i = \hat{u}) = 0 \)

\( \lambda_1 = 0, \quad \lambda_2 \geq 0 \)

The Kuhn-Tucker conditions are
There are no solutions that satisfy $\lambda_1=0$ and all of the first-order conditions jointly; therefore, the retailer will never choose to set prices and restocking fees so that $E_{j=0}(utility \mid u_i = \hat{u}) > E_{j=1}(utility \mid u_i = \hat{u}) \geq 0$. In other words, the retailer will not choose to sell only product 0 and to give the marginal consumer strictly positive utility. We therefore rule out Possibility 2.

**Possibility 3:** $E_{j=0}(utility \mid u_i = \hat{u}) \geq 0$ and $E_{j=1}(utility \mid u_i = \hat{u}) < 0$.

The objective function with the Lagrangian multiplier on the expected utility from buying product 0 initially becomes

$$
\max_{p_0, p_1, \lambda_0, \lambda_1, \lambda_2} L^{ext} = \pi_{ret} + \lambda_2 E_{j=0}(utility \mid u_i = \hat{u}) \\
s.t. \quad \lambda_2 (E_{j=0}(utility \mid u_i = \hat{u}) = 0 \\
\lambda_2 \geq 0, \quad E_{j=1}(utility \mid u_i = \hat{u}) < 0
$$

The Kuhn-Tucker conditions are
The Kuhn-Tucker conditions are satisfied (and profit is the same) at any price and restocking fee combination such that

\[ p_0 = \hat{u} + \frac{(w_1 - \max \{r_0, s_r\})^2}{4d} - \frac{(w_1 - \max \{r_0, s_r\})}{2} - \frac{f_0(1 - \alpha)}{\alpha} - \frac{d}{4}, \]

\[ p_1 = \hat{u} + \frac{(w_1 - \max \{r_0, s_r\})^2}{4d} + \frac{(w_1 - \max \{r_0, s_r\})}{2} - \frac{d}{\alpha} - \frac{4d}{4} \]

and

\[ E_{u_i}(utility | u_i = \hat{u}) = \frac{(2 - \alpha)(f_0 - f_1) + \alpha(f_1 + f_0)(f_1 + f_0 - 2(w_1 - \max \{r_0, s_r\}))}{2} \times \frac{d}{4d} < 0 \] (with \( \lambda_2 = \tilde{u} - \hat{u} \)).

We first examine when the manufacturer accepts returns (CRM reverse channel structure).

The manufacturer takes the retailer’s reaction into account in maximizing its profit

\[ \pi_{\text{prof}}^{\text{CRM}} = (\tilde{u} - \hat{u})(w_0 - c)\phi_{\hat{u}0} + (w_1 - c + w_0 - r_0 - c + s)\phi_{\hat{u}0} + (w_0 - c - r_0 + s)\phi_{\hat{u}0} \]

where the expressions \( \phi_{\hat{u}0}, \phi_{\hat{u}0}, \phi_{r0} \) from equation (1) in the text are evaluated at the optimal prices and restocking fees as functions of wholesale prices and refunds as described above. The manufacturer has the constraint that the contract offered to the retailer must provide non-negative profit. Thus, the manufacturer’s constrained optimization problem is

\[ \max_{w_0, w_1, r_0, r_i} L^{\text{prof}} = \pi_{\text{prof}}^{\text{CRM}} + \lambda_3 \pi_{\text{ret}}^{\text{ret}} \]

subject to \( \lambda_3 \pi_{\text{ret}}^{\text{ret}} = 0, \lambda_3 \geq 0 \)

The Kuhn-Tucker Conditions are
\[
\frac{\partial L^{mfg}}{\partial w_0} = (\bar{u} - \hat{u})(1 - \lambda_1) = 0
\]
\[
\frac{\partial L^{mfg}}{\partial w_1} = \alpha(\bar{u} - \hat{u})(c + d + 2r_0 - s - 2w_1 - \lambda_1(d + r_0 - w_2)) = 0
\]
\[
\frac{\partial L^{mfg}}{\partial r_0} = \frac{\bar{u} - \hat{u}((2 - \lambda_1)(w_1 - r_0) - c + s) - d(2 - \alpha)(1 - \lambda_3)}{2d} = 0
\]

These conditions are satisfied at
\[
w_0 = r_0 + \alpha(\hat{u} + \frac{(c-s)^2}{4d} - \frac{c-s}{2} - \frac{d}{4} - r_0)
\]
\[
w_1 = r_0 + c - s, \quad \lambda_1 = 1.
\]

Plugging these wholesale prices and refund rates into the retailer’s reaction function, the resulting prices and restocking fees will be any price and restocking fee combination such that
\[
p_0 = \hat{u} + \frac{(c-s)^2}{4d} - \frac{(c-s)}{2} - \frac{d}{4} - \frac{f_0}{\alpha} - \frac{d}{4}
\]
\[
p_1 = \hat{u} + \frac{(c-s)^2}{4d} + \frac{(c-s)}{2} - \frac{f_0}{\alpha} - \frac{d}{4}
\]
\[
E_{u_i}(utility|u_i = \hat{u}) = \frac{(2 - \alpha)(f_0 - f_1)}{2} + \frac{\alpha(f_1 + f_0)(f_1 + f_0 - 2(c-s))}{4d} < 0.
\]

The equilibrium results in the following outcome.

Total quantity sold initially: \((\bar{u} - \hat{u})\)

Total quantity returned (without subsequent exchange): \((\bar{u} - \hat{u})\phi_{r_0} + 0\phi_{r_1} = (1 - \alpha)(\bar{u} - \hat{u})\)

Total quantity of exchanges: \((\bar{u} - \hat{u})\phi_{e_0} + 0\phi_{e_1} = \frac{\alpha(\bar{u} - \hat{u})(d - c + s)}{2d}\)

Manufacturer profit: \(\alpha(\bar{u} - \hat{u})(\hat{u} - c + \frac{(c-s)^2}{4d} - \frac{(c-s)(2-\alpha)}{2\alpha} - \frac{d}{4})\)

Moreover, if a consumer initially purchases product 0, the out-of-pocket marginal expense to return it and buy product 1 is also the same as in the symmetric case, namely:
\[
p_1 - (p_0 - f_0) = (c - s).
\]
We now examine when the manufacturer does not accept returns (CR reverse channel structure).

The manufacturer takes the retailer’s reaction into account in maximizing its profit

\[
\pi_{\text{mfr}}^{\text{CR}} = (\bar{u} - \hat{u})((w_0 - c)\phi_{\text{c}0} + (w_1 - c + w_0 - c)\phi_{\text{c}0} + (w_0 - c)\phi_{\text{r}0})
\]

where the expressions \( \phi_{\text{c}0}, \phi_{\text{c}0}, \text{a}\) and \( \phi_{\text{r}0} \) from equation (1) in the text are evaluated at the optimal prices and restocking fees as functions of wholesale prices and refunds as described above. The manufacturer has the constraint that the contract offered to the retailer must provide non-negative profit. Thus, the manufacturer’s constrained optimization problem is

\[
\max_{w_0, w_1} L_{\text{mfr}}^{\text{CR}} = \pi_{\text{mfr}}^{\text{CR}} + \lambda_3 \pi_{\text{ret}}
\]

s.t. \( \lambda_3 \pi_{\text{ret}} = 0, \lambda_3 \geq 0 \)

The Kuhn-Tucker Conditions are

\[
\frac{\partial L_{\text{mfr}}^{\text{CR}}}{\partial w_0} = (\bar{u} - \hat{u})(1 - \lambda_3) = 0
\]

\[
\frac{\partial L_{\text{mfr}}^{\text{CR}}}{\partial w_1} = \alpha(\bar{u} - \hat{u})(c + d + s_r - 2w_1 - \lambda_3(d + s_r - w_0)) + \frac{2d}{\lambda_3} = 0
\]

\( \lambda_3 \pi_{\text{ret}} = 0, \lambda_3 \geq 0 \)

The solution to these conditions is

\[
w_0 = \frac{2d(\alpha(2\hat{u} - c) + s_r(2 - \alpha)) - \alpha(d^2 - (c - s_r)^2)}{4d}
\]

\[
w_1 = c
\]

\[
\lambda_3 = \bar{u} - \hat{u}
\]

The above wholesale contract would be the manufacturer’s optimal contract if the retailer were in fact to react by setting prices and restocking fees according to Possibility 3 (i.e., such that \( E_{j|a}(utility \mid u_j = \hat{u}) \geq 0, \) and \( E_{j|a}(utility \mid u_j = \hat{u}) < 0 \)). However, the retailer gets to choose its reaction to the wholesale contract. Plugging this wholesale contract into the retailer’s possible reaction from Possibility 3 (\( E_{j|a}(utility \mid u_j = \hat{u}) \geq 0, \) and \( E_{j|a}(utility \mid u_j = \hat{u}) < 0 \)) and into the
retailer’s reaction from Possibility 1 \( (E_{j=0}(utility \mid u_i = \hat{u}) = E_{j=1}(utility \mid u_i = \hat{u}) \) ), the retailer in fact earns greater profit from reacting with the best response function from Possibility 1 (in which \( E_{j=0}(utility \mid u_i = \hat{u}) = E_{j=1}(utility \mid u_i = \hat{u}) \) ) for any value of \( \hat{u} \) that yields non-negative manufacturer profit in Possibility 3 \( (\hat{u} \geq \frac{2cd(2+\alpha) + \alpha d^2 - \alpha(c-s_c)^2 - 2ds_c(2-\alpha)}{4d\alpha}) \). Thus, the retailer’s actual reaction to the manufacturer contract from Possibility 3 (asymmetric choices set assuming \( E_{j=0}(utility \mid u_i = \hat{u}) \geq 0, \) and \( E_{j=1}(utility \mid u_i = \hat{u}) < 0 \) ) will be to offer prices and restocking fees according to Possibility 1 (expected utility from each product is equivalent). The manufacturer would recognize the retailer’s actual reaction. As shown above, when the retailer responds as in Possibility 1, the manufacturer optimally offers symmetric wholesale prices. Therefore, in the CR reverse channel structure, there will only be a symmetric equilibrium.

**Conclusion**

Summarizing the insights from analyzing Possibilities 1, 2, and 3 for the CRM reverse channel structure, we have: (a) Possibility 2 cannot exist; (b) Possibility 1 produces symmetric solution values; and (c) Possibility 3 produces the same profit, total returns, total sales, and total exchanges as in the symmetric solution. Thus, allowing asymmetry in a decentralized channel in which the manufacturer offers a per-unit wholesale price and refund rate results in the same outcomes as assuming symmetry.

When prices and restocking fees are chosen such that all consumers with \( u_i < \hat{u} \) do not buy either product initially and all consumers with \( u_i \in [\hat{u}, \bar{u}] \) buy one of the two products, asymmetric prices, restocking fees, wholesale prices, or refund rates will not affect manufacturer profit, total quantity of exchanges, or the total quantity sold. Therefore, for any given \( \hat{u} \), asymmetric wholesale prices do not improve profit or change the number of product returns. To
increase the number of consumers who buy initially, the prices and restocking fees may be lowered, but there is no additional benefit on sales or profit to charge asymmetric prices (retail or wholesale). Thus we have shown that the asymmetric wholesale prices (and consequently asymmetric retail prices) will not increase profit or change the number of exchanges for any number of initial sales that the manufacturer would like to induce. Q.E.D.

A.11. Proof of Observation 1

OBSERVATION 1: Consider a situation in which the manufacturer charges the retailer a per-unit wholesale price but not a fixed fee. If \( s > s_r \), then the return penalty, \( f \), charged to consumers is greater when the retailer salvages returned units than when the manufacturer accepts and salvages returned units from the retailer. There exists a critical salvage value for the manufacturer, \( \tilde{s} < s_r \), such that \( \tilde{s} < s < s_r \) also implies the return penalty, \( f \), charged to consumers is greater when the retailer salvages returned units than when the manufacturer accepts and salvages returned units from the retailer. If \( s \leq \tilde{s} \), then the return penalty, \( f \), charged to consumers is lower when the retailer salvages returned units than when the manufacturer accepts and salvages returned units from the retailer.

Proof:

We first prove that for \( s = s_r \), the return penalty, \( f \), charged to consumers is greater when the retailer salvages returned units than when the manufacturer accepts and salvages returned units from the retailer. From Table 3, we know that \( f^* = w^* - \max\{s_r, r\} \). When the manufacturer accepts returns responsibility, this simplifies to \( f^* = c - s \). When the manufacturer does not accept returns responsibility, then \( f^* = w^* - s_r \) and it must be that \( w^* > c \) for the
manufacturer to earn positive profit. Thus, when $s = s_r$, the return penalty is greater when the retailer is responsible for salvaging returned units than when the manufacturer accepts this responsibility.

The return penalty charged to consumers when the manufacturer salvages returns is decreasing in $s$. The return penalty charged to consumers when the retailer salvages returns is invariant with respect to $s$. We define $f^{CRM}(s)$ as the equilibrium return penalty charged to consumers when the manufacturer accepts returns responsibility (as a function of $s$) and $f^{CR}(s)$ as the equilibrium return penalty charged to consumers when the retailer salvages returns (as a function of $s$). Because $f^{CRM}(s|_{s=s_r}) < f^{CR}(s|_{s=s_r})$, $\frac{\partial f^{CRM}(s)}{\partial s} < 0$ and $\frac{\partial f^{CR}(s)}{\partial s} = 0$, there exists an $\tilde{s} < s_r$ such that the functions $f^{CRM}(s) = f^{CR}(s)$ at $s = \tilde{s}$. If $s > \tilde{s}$ then $f^{CRM}(s) < f^{CR}(s)$ and if $s \leq \tilde{s}$, then $f^{CRM}(s) \geq f^{CR}(s)$.

Q.E.D.

A.12. Proof of Proposition 2

PROPOSITION 2: Consider the situation in which the manufacturer charges the retailer a per-unit wholesale price but not a fixed fee. If $s > s_r$, then the manufacturer accepts and salvages returned units from the retailer. There exists a critical salvage value for the manufacturer, $\bar{s} < s_r$, such that $\bar{s} < s < s_r$ also implies the manufacturer will accept and salvage returned units from the retailer. If $s \leq \bar{s}$, then the manufacturer chooses to allocate responsibility for salvaging returned units to the retailer.

We first show that the manufacturer earns greater profit from accepting and salvaging returned units from the retailer when $s = s_r$. From Tables 3, we know that
\[ \pi_{mfg}^{CR} = \frac{(w^*-c)(s_r-w^*) + (2+\alpha)\alpha d((w^*-s_r)^2 + 2d((2-\alpha)s_r + 2\alpha \bar{u} - (2+\alpha)w^*) - \alpha d^2)}{16\alpha d^2} \]

where \( w^* \) is defined in Table 3 and
\[ \pi_{mfg}^{CRM} = \frac{(\alpha(c-s)^2 + d(2s(2-\alpha) + \alpha(4\bar{u} - d) - 2c(2+\alpha)))^2}{128d^2} \]. We will show that at \( s = s_r, \pi_{mfg}^{CR} - \pi_{mfg}^{CRM} < 0 \) for all \( w > c \).

We will do this in two parts.

Part 1: We will show that for \( d > \frac{\alpha(w^*-s_r)}{2+\alpha} \), any value of \( \bar{u} \) will make \( \pi_{mfg}^{CR} - \pi_{mfg}^{CRM} < 0 \).

Part 2: We will show that for \( d \leq \frac{\alpha(w^*-s_r)}{2+\alpha} \), the value of \( \pi_{mfg}^{CR} - \pi_{mfg}^{CRM} < 0 \) for \( \bar{u} \) greater than the minimum \( \bar{u} \) for which quantity is positive in the CR model.

We begin with part 1. \( d > \frac{\alpha(w^*-s_r)}{2+\alpha} \). The second derivative of \( (\pi_{mfg}^{CR} - \pi_{mfg}^{CRM}) \) with respect to \( \bar{u} \) is equal to \(-\frac{\alpha}{4}\). Therefore, the expression is concave in \( \bar{u} \) and has a unique maximum.

Evaluated at the expression maximizing value of \( \bar{u} \), the maximum value that \( (\pi_{mfg}^{CR} - \pi_{mfg}^{CRM}) \) can take is \( \frac{(w^*-c)^2(\alpha(w^*-s_r) - d(2+\alpha))}{16d^2} \). This maximum value is negative by fact that \( d > \frac{\alpha(w^*-s_r)}{2+\alpha} \). Since the expression \( (\pi_{mfg}^{CR} - \pi_{mfg}^{CRM}) \) is negative at its maximum value over \( \bar{u} \), it is negative for all values of \( \bar{u} \).

We now show part 2. \( d \leq \frac{\alpha(w^*-s_r)}{2+\alpha} \). From Table 3, the quantity of consumers who buy initially (heretofore defined as \( q_{b}^{CR} \)) is equal to \( q_{b}^{CR} = \frac{\bar{u} - w^*-s_r}{2\alpha} - \frac{d^2 + 2d(w^*+s_r)-(w^*-s_r)^2}{8d} \). This is
positive if and only if \( \bar{u} > \bar{u}^{CR} = \frac{w^* - s_r}{\alpha} + \frac{d^2 + 2d(w^* + s_r) - (w^* - s_r)^2}{4d} \), where we define \( \bar{u}^{CR} \) as the value of \( \bar{u} \) such that \( q_b^{CR} = 0 \).

We can show that \( \bar{u} > \bar{u}^R \) implies that \( \pi_{\text{mfg}}^{CR} - \pi_{\text{CRM}}^{CR} < 0 \) for \( d \leq \frac{\alpha(w^* - s_r)}{2 + \alpha} \). At \( \bar{u} = \bar{u}^{CR} \), the expression \( \pi_{\text{mfg}}^{CR} - \pi_{\text{CRM}}^{CR} = \frac{(w^*-c)(\alpha(c+2s_r-3w^*)+2d(2+\alpha))}{128\alpha d^2} \), which is negative. The expression \( \frac{\partial (\pi_{\text{mfg}}^{CR} - \pi_{\text{CRM}}^{CR})}{\partial \bar{u}} \) evaluated at \( \bar{u} = \bar{u}^{CR} \) is equal to

\[
\frac{\partial (\pi_{\text{mfg}}^{CR} - \pi_{\text{CRM}}^{CR})}{\partial \bar{u}} = \frac{(w^*-c)(\alpha(c+2s_r-3w^*)+2d(2+\alpha))}{16d}.
\]

This derivative is negative for all \( d < \frac{\alpha(2(w^* - s_r) + w^* - c)}{2(2+\alpha)} \).

Because \( \frac{\alpha(2(w^* - s_r) + w^* - c)}{2(2+\alpha)} = \frac{\alpha(w^* - s_r)}{2 + \alpha} + \frac{\alpha(w^* - c)}{2(2 + \alpha)} > \frac{\alpha(w^* - s_r)}{2 + \alpha} \), the derivative is also negative for all \( d < \frac{\alpha(w^* - s_r)}{2 + \alpha} \). By fact that \( \frac{\partial^2 (\pi_{\text{mfg}}^{CR} - \pi_{\text{CRM}}^{CR})}{\partial \bar{u} \partial \bar{u}} = -\frac{1}{4} \), the derivative

\[
\frac{\partial (\pi_{\text{mfg}}^{CR} - \pi_{\text{CRM}}^{CR})}{\partial \bar{u}} < 0 \text{ for all } \bar{u} > \bar{u}^{CR}.
\]

Because \( \pi_{\text{mfg}}^{CR} - \pi_{\text{CRM}}^{CR} < 0 \) at \( \bar{u} = \bar{u}^{CR} \) and \( \frac{\partial (\pi_{\text{mfg}}^{CR} - \pi_{\text{CRM}}^{CR})}{\partial \bar{u}} < 0 \) for all \( \bar{u} > \bar{u}^{CR} \), the expression \( \pi_{\text{mfg}}^{CR} - \pi_{\text{CRM}}^{CR} < 0 \) for all \( \bar{u} > \bar{u}^{CR} \).

Therefore, \( \pi_{\text{CRM}}^{CR} > \pi_{\text{mfg}}^{CR} \) at \( s = s_r \).
We have thus far proven that if \( s = s_r \), then \( \pi_{\text{mfg}}^{\text{CR}} > \pi_{\text{mfg}}^{\text{CR}} \). We now show that \( \frac{\partial \pi_{\text{mfg}}^{\text{CR}}}{\partial s} > 0 \) and \( \frac{\partial \pi_{\text{mfg}}^{\text{CR}}}{\partial s} = 0 \).

\[
\frac{\partial \pi_{\text{mfg}}^{\text{CRM}}}{\partial s} = \frac{d(2-\alpha) - \alpha(c-s)(\overline{u} + \frac{(c-s)^2}{4} - \frac{c-s}{4\alpha} - \frac{2(c+s)+d}{16})}{2d} > 0 \quad \text{for all } \overline{u} \text{ such that}
\]

\[
q_b^M = \frac{\overline{u}}{4} + \frac{(c-s)^2}{16} - \frac{c-s}{4\alpha} - \frac{2(c+s)+d}{16} > 0 \text{ and all } d \text{ such that the equilibrium exchange rate when the manufacturer accepts returns (heretofore defined as } \phi_c^{\text{CRM}} \text{) } \phi_c^{\text{CRM}} = \alpha(\frac{1}{2} - (\frac{c-s}{2d}) > 0. \]

Therefore, when \( \pi_{\text{mfg}}^{\text{CRM}} \geq 0 \) and \( \pi_{\text{mfg}}^{\text{CR}} \geq 0 \), there exists an \( \overline{s} < s_r \) such that the functions \( \pi_{\text{mfg}}^{\text{CRM}}(s) = \pi_{\text{mfg}}^{\text{CR}}(s) \) at \( s = \overline{s} \). If \( \overline{s} < s \) then \( \pi_{\text{mfg}}^{\text{CRM}} > \pi_{\text{mfg}}^{\text{CR}} \) and \( s \leq \overline{s} \), then \( \pi_{\text{mfg}}^{\text{CRM}} \leq \pi_{\text{mfg}}^{\text{CR}} \).

Q.E.D.

A.13. Proof of Proposition 3

PROPOSITION 3: Consider a situation in which the manufacturer charges the retailer a per-
unit wholesale price but not a fixed fee. If the manufacturer’s salvage value is greater than the
retailer’s salvage value \( (s \geq s_r) \) the manufacturer can replicate the return penalty charged to
consumers in a vertically-integrated channel by accepting product returns from the retailer. The
retail price charged to consumers will be distorted upward and manufacturer profit will be
distorted downward from a vertically-integrated channel. Otherwise, when \( s < s_r \), the return
penalty charged to consumers will be greater in a decentralized channel than in a vertically
integrated system.

Proof:
In the vertically integrated system, the return penalty charged to consumers will be equal to

\[ f^{VI} = c - \max\{s, s_r\} \]

and the retail price will be

\[ p^{VI} = \frac{\bar{u}}{2} \left( \frac{c - \max\{s, s_r\}}{8d} \right)^2 + \frac{2(c + \max\{s, s_r\}) - d}{8} - \frac{(1 - \alpha)(c - \max\{s, s_r\})}{2\alpha} . \]

When the manufacturer accepts returns (CRM reverse channel structure), then the price and consumer return penalty from Table 3 simplify to

\[ f^{CRM} = c - s \]

and

\[ p^{CRM} = \frac{3}{2} \left( \frac{\bar{u}}{4} + \frac{(c - s)^2}{16d} - \frac{c - s + d}{4\alpha} \right) + \frac{2(c - s) - d}{8} - \frac{(c - s) + \bar{c}}{6} . \]

If \( s > s_r \):

In this case \( \max\{s, s_r\} = s \). Clearly, the return penalty when the manufacturer accepts returns is the equivalent to the fully-coordinated solution. The price difference is equal to

\[ p^{CRM} - p^{VI} = \frac{\bar{u}}{4} + \frac{(c - s)^2}{16d} - \frac{c - s + d}{4\alpha} + \frac{2(c - s) - d}{16} \]

which is greater than zero for all parameters such that initial quantity sold

\[ q_b^{CRM} = \frac{\bar{u}}{4} + \frac{(c - s)^2}{16d} - \frac{c - s + d}{4\alpha} + \frac{2(c - s) + d}{16} \]

is non-negative.

If \( s < s_r \):

Consider \( \bar{s} < s < s_r \). From Proposition 2, in this region the manufacturer will accept returns from the retailer. However, \( f^{VI} = c - s_r \) which is less than \( f^{CRM} = c - s \) by fact that \( s < s_r \).

If \( \bar{s} < s < s_r \), then the manufacturer does not accept returns. In which case,

\[ f^{CR} = w^{*,CR} - s_r \] which is greater than \( f^{VI} = c - s_r \) for any \( w^* > c \), a condition which is necessary for the manufacturer to earn non-negative profit when not accepting returns (when in the CR reverse channel structure). \( Q.E.D. \)
A.14. Examples of When Manufacturer Should Accept Returns

In this section we examine how the model’s parameters affect whether or not the manufacturer optimally takes product returns from the retailer. While the analytical solutions for the manufacturer’s profit in the CRM channel structure as well as the CR channel structure are presented above, the complexity of these analytical solutions makes examining the comparative statics infeasible without the use of numerical analysis. Figures 14.3 and 14.4 illustrate parametric situations when a manufacturer would earn greater profit from salvaging returned units itself than from allocating this responsibility to the retailer. We have chosen two graphs that represent small numbers and larger numbers respectively. In each graph, we see for a broad range of examples the impact of $d$ and $s_r$ on the optimal reverse channel structure. In the graph below, we also see the impact of marginal cost on the optimal reverse channel structure.

**Figure 14.3** How The Optimal Reverse Channel Structure Varies with $c$, $d$, and $s_r$

Parameters: $a=1$, $s=1.5$, $\bar{u}=15$, Black: $c=1.6$, Red: $c=2.2$, Cyan $c=2.8$, Solid Line: $s=15$, Dashed Line: $s=19$(Horizontal Axis=$d$, Vertical Axis=$s_r$)

Shaded regions denote parameter space for which $\pi_{\text{CR}}^{\text{mfgr}} > \pi_{\text{CRM}}^{\text{mfgr}}$. Outlined regions represent the feasible parameter space. Parameter values outside the outlined regions violate rational conditions on $f_{\text{CR}}$ and $W_{\text{CR}}$. 
In the following graph, we are able to see the impact of $\overline{u}$ and $s$ on the optimal reverse channel structure over a large set of parameters.

**Figure 14.4 How The Optimal Reverse Channel Structure Changes with $\overline{u}$, $s$, $d$, and $s_r$**

(a=1, c=20, Green: $\overline{u} = 100$, Blue: $\overline{u} = 85$, Purple $\overline{u} = 70$, Solid Line: $s=15$, Dashed Line: $s=19$)

(Horizontal Axis=$d$, Vertical Axis=$s_r$)

Shaded regions denote parameter space for which $\Pi^{CR}_{mfgr} > \Pi^{CRM}_{mfgr}$. Outlined regions represent the feasible parameter space. Parameter values outside the outlined regions violate rational conditions on $f^{CR}$ and $w^{CR}$.

Consistent with our analytical results above, these examples show that the manufacturer prefers to salvage the returned units except when the retailer has a significant salvage advantage (high $s_r$) and consumers place a higher value on getting the right product (high $d$). A higher value of $s_r$ makes each returned unit more valuable to the retailer and a higher value of $d$ creates a greater number of returns (making the retailer’s salvage advantage more valuable). In Figure 14.3, we see the effect of the marginal cost of production, $c$, while Figure 14.4 shows the effects of the manufacturer’s salvage value for returned units, $s$, and the highest value a consumer may obtain from owning a unit in the product category, $\overline{u}$. 
The minimum values of $d$ and $sr$, necessary for the manufacturer to prefer that the retailer salvages returns are increasing as $c$ increases (as shown in Figure 14.3). While an increase in $c$ decreases manufacturer profits whether or not the manufacturer accepts returns, the magnitude of the effect is greater when the retailer salvages returns. This comes as a result of the distortion in $f^{CR}$. In either channel structure, the manufacturer increases wholesale price with an increase in the marginal cost of production. However, in the CRM channel structure, the manufacturer also raises the refund for returned units, which dampens the effect on the restocking fee and the resulting equilibrium quantities. So although $\frac{\partial W_{CRM}}{\partial c} > 0$, this leads to $|\frac{\partial q_{CRM}}{\partial c}| < |\frac{\partial q_{CR}}{\partial c}|$, which has a stronger net effect on manufacturer profits in the CR structure than in the CRM structure. Thus an increase in $c$ implies that higher values of $sr$ and $d$ are necessary in order for
\[
\pi_{mfg}^{CR} > \pi_{mfg}^{CRM}.
\]

The minimum values of $d$ and $sr$ necessary for the manufacturer to prefer that the retailer salvages returns are also increasing as $s$ or $\bar{u}$ increases. An increase in $s$ has a positive effect on manufacturer profit in the CRM structure and no effect in the CR structure. An increase in $\bar{u}$ has a positive effect on manufacturer profit regardless of who salvages returned units. However, it has a greater impact on manufacturer profit in a CRM structure than a CR structure, again because of the distortion in restocking fee that occurs in the CR structure. Although equilibrium sales quantity increases with $\bar{u}$ at a faster rate in the CR structure than in the CRM structure, the reverse is true for the equilibrium wholesale price. Overall, manufacturer profit increases with $\bar{u}$ at a faster rate in the CRM structure than the CR structure. Thus, for higher $\bar{u}$, the retailer’s
salvage advantage (and/or the consumer disutility of mismatch) must be greater for the manufacturer to earn greater profit when the retailer salvages returned units.\(^1\)

A.1.5. Single-Product Model

In this Appendix, we generate the analogues to Observation 1 and Proposition 2 in a single-product model.

For a product located at \(x=0\), a consumer with reservation utility \(u_i\) chooses to keep his initial purchase if \(u_i - p - d\theta_i > f\), that is, if \(\theta_i < (u_i - p + f)/d\).\(^2\) Ex ante, consumer \(i\)'s expected utility of making an initial purchase is then equal to

\[
E(u_i) = \left(\frac{u_i - p + f}{d}\right) (u_i - p - \frac{d}{2} (u_i - p + f)) + (1 - \frac{u_i - p + f}{d}) (-f) = \frac{(f - p + u_i)^2 - 2df}{2d}.
\]

Consumers for whom \(E(u_i) \geq 0\) (that is, for whom \(u_i \in [p - f + \sqrt{2df}, \bar{u}]\)) buy initially. Thus, the quantity sold is \(\bar{u} - p + f - \sqrt{2df}\). For each consumer, the probability of a return is equal to \((1-(u_i-p+f)/d)\). Integrating over all \(u_i\) such that an initial purchase is made gives the total expected number of returns as:

\[
\int_{p-f+\sqrt{2df}}^\pi (1 - \frac{u_i - p + f}{d}) du_i = \bar{u} + 2f - \sqrt{2df} - p - \frac{(\bar{u} - p + f)^2}{2d}.
\]

There are several points to note about the demand function and the return function. The quantity sold is decreasing in price. The quantity sold is decreasing in the restocking fee, due to the fact that the derivative with respect to \(f\) is \(1 - \frac{\sqrt{d}}{\sqrt{2f}}\), which is negative if and only if the

---

\(^1\) It should be noted that larger \(\bar{u}\) or \(s\) expands the feasible parameter space and larger \(c\) shrinks the feasible set. A larger \(\bar{u}\) means an expanded market and thus a greater set of \(\{s, r, d\}\) that satisfy the constraints of the model. For larger \(c\) or smaller \(s\), there is greater incentive to shut down returns entirely (a scenario which could not address the issue of optimal reverse channel structure).

\(^2\) If the product is located at \(x_j=1/2\), the probability that it is kept doubles. In such a case, a consumer with reservation utility \(u_i\) would keep the purchase if \(u_i - p - d|1/2 - \theta| > f\), leading to an expected utility equal to \((f - p + u_i)^2 - df/d\). The subsequent analysis for the product located \(x_j=0\) follows for \(x_j=1/2\) as well but with a different scale.
consumer located at \( u_i = p - f + \sqrt{2df} \) has a positive probability of making a return. The expected total number of returns is decreasing in price (the derivative of expected total returns with respect to \( p \) is \(-\frac{1}{d} \left( 1 - \frac{\bar{u} - p + f}{d} \right)\), which is negative if and only if the probability that the consumer with the highest \( u_i = \bar{u} \) has a positive probability of making a return). The expected total number of returns is decreasing in the restocking fee (its derivative with respect to \( f \) is \((2 - \frac{\bar{u} - p + f}{d} - \frac{\sqrt{d}}{\sqrt{2f}})\)). For sales to be positive it must be that \( \bar{u} > \sqrt{2df} - f + p \). Evaluated at this minimum bound on \( \bar{u} \), the derivative of expected total number of returns with respect to the restocking fee is equal to \( \frac{1}{(\sqrt{f} (\sqrt{2d} - \sqrt{f}) + p)} (2 - \frac{(d + 2f)}{\sqrt{2df}}) < 0 \) for all values of \( f \) such that the consumer located at \( u_i = p - f + \sqrt{2df} \) has a positive probability of making a return (by fact that \( 2 - \frac{(d + 2f)}{\sqrt{2df}} \) is equal to zero at \( f = d/2 \) and increasing in \( f \) through this point). Thus, the derivative of the total number of returns with respect to \( f \) is \((2 - \frac{\bar{u} - p + f}{d} - \frac{\sqrt{d}}{\sqrt{2f}})\), which is negative at the minimum bound \( \bar{u} = \sqrt{2df} - f + p \) and is decreasing in \( \bar{u} \). Therefore this derivative is negative for all \( \bar{u} \) such that sales are positive.

Letting \( Q_{sales} \) represent the total initial sales and \( Q_{returns} \) represent the total number of returns (rather than returns probability), it can then be noted that:
On-Line Supplement for Optimal Reverse Channel Structure for Consumer Product Returns

Page 54 of 58

\[ \frac{\partial Q_{sales}}{\partial p} < 0, \quad \frac{\partial Q_{sales}}{\partial f} < 0, \quad \frac{\partial Q_{returns}}{\partial p} < 0, \quad \frac{\partial Q_{returns}}{\partial f} < 0, \text{ and} \]

\[ \frac{\partial Q_{sales}}{\partial p} \cdot \frac{\partial Q_{returns}}{\partial f} - \frac{\partial Q_{sales}}{\partial f} \cdot \frac{\partial Q_{returns}}{\partial p} = \frac{1}{u} \left( \frac{(u - p + f) - \sqrt{2df}}{\sqrt{2df}} \right) > 0 \text{ for all } u \text{ such that sales are positive } u > \sqrt{2df} - f + p. \]

To obtain interpretable results, we use general linear demand and return functions that preserve the directional effects of \( p \) and \( f \). We examine a single product setting with demand and return functions of the form:

\[ Q_{sales} = \alpha - \beta p - \delta f \]

\[ Q_{returns} = z - y \beta - v f \]

Where \( Q_{sales} \) represents the total initial sales and \( Q_{returns} \) represents the total number of returns (rather than returns probability).

We assume \( (v \beta - y \delta) > 0 \), to mirror the result above that

\[ \frac{\partial Q_{sales}}{\partial p} \cdot \frac{\partial Q_{returns}}{\partial f} - \frac{\partial Q_{sales}}{\partial f} \cdot \frac{\partial Q_{returns}}{\partial p} > 0. \]

A15.1. CR Reverse Channel Structure

The retailer chooses price and restocking fee to maximize

\[ \text{profit}_{retCR} = (p - w)Q_{sales} + (f - p + s_r)Q_{returns}. \]

Taking first order conditions, the retailer maximizes profit with the reaction functions:

\[ p(w) = \frac{s_r v(y + y - \delta) + (y + \delta)(z + w \delta) + v(z - 2 \alpha - w(2 \beta + \delta))}{v^2 + 2v(y - 2 \beta - \delta) + (y + \delta)^2} \]

\[ f(w) = \frac{y z + y \alpha + w y \beta - 2 z \beta + v(z - \alpha - w \beta) + 2 wy \delta - z \delta + \alpha \delta - w f \delta - s_r (v(y - 2 \beta) + y(y + \delta))}{v^2 + 2v(y - 2 \beta - \delta) + (y + \delta)^2}. \]

The second order conditions are satisfied if
The manufacturer chooses wholesale price to maximize
\[
profit_{manCR} = (w-c)Q_{sales}(p(w),f(w)).
\]
Since it is a single-product setting, returns do not generate an additional sale for the manufacturer.

Taking first order conditions, the manufacturer maximizes profit at
\[
w_{CR}^* = \frac{1}{4(\beta+\delta)(v\beta-y\delta)} \left( v(2\alpha\beta+2c\beta^2+\gamma(-2\alpha+s_r(\beta-\delta))+\alpha\delta+\beta\delta(2c+s_r)+z(\beta+\delta))-\right.
\]
\[
(v^2(\alpha-\beta s_r)+z\delta(\beta+\delta)+s^2(\alpha+\beta s_r)+y(-z(\beta+\delta)+\delta(\alpha+\delta s_r+2c(\beta+\delta)))) \right).\]

The second order condition for the above wholesale price to be a profit maximum is
\[
\frac{\partial^2 profit_{manCR}}{\partial^2 w} = \frac{4(\beta+\delta)(v\beta-y\delta)}{v^2+2v(y-2\beta-\delta)+(y+\delta)^2} < 0 \text{ by assumption that } (v\beta-y\delta) > 0 \text{ and } -(v^2+2v(y-2\beta-\delta)+(y+\delta)^2) > 0.
\]

The manufacturer’s equilibrium profit in the CR reverse channel structure is given by
\[
profit_{manCR} = -\frac{1}{8(\beta+\delta)(v\beta-y\delta)(v^2+2v(y-2\beta-\delta)+(y+\delta)^2)} \left( v(2\alpha\beta-2c\beta^2+\gamma(-2\alpha+s_r(\beta-\delta))+\alpha\delta+\beta\delta(-2c+s_r)+z(\beta+\delta))-\right.
\]
\[
(v^2(\alpha-\beta s_r)+z\delta(\beta+\delta)+s^2(\alpha+\beta s_r)+y(-z(\beta+\delta)+\delta(\alpha+\delta s_r-2c(\beta+\delta)))) \right)^2.
\]

A15.2. CRM Reverse Channel Structure

The retailer chooses price and restocking fee to maximize
\[
profit_{retCRM} = (p-w)Q_{sales}+(f-p+r)Q_{returns}.
\]
Taking first order conditions, the retailer maximizes profit with the reaction functions:
\[
p(w, r) = \frac{rv(v + y - \delta) + (y + \delta)(z + w\delta) + v(z - 2\alpha - w(2\beta + \delta))}{v^2 + 2v(y - 2\beta - \delta) + (y + \delta)^2}
\]

\[
f(w, r) = \frac{yz + y\alpha + wy\beta - 2z\beta + v(z - \alpha - w\beta) + 2wy\delta - z\delta + \alpha\delta - w\beta\delta - r(v(y - 2\beta) + y(y + \delta))}{v^2 + 2v(y - 2\beta - \delta) + (y + \delta)^2}
\]

The second order conditions are satisfied for the same parameter values as the CR model above.

The manufacturer chooses wholesale price and refund to maximize

\[
profit_{manCRM} = (w-c)Q_{sales}(p(w, r), f(w, r)) + (s-r)Q_{returns}(p(w, r), f(w, r))
\]

Taking first order conditions, the manufacturer’s profit is maximized at

\[
w_{CRM}^* = \frac{v\alpha + cv\beta - cy\delta - z\delta}{2(v\beta - y\delta)}
\]

\[
r_{CRM}^* = \frac{v\alpha + y\alpha + sv\beta - z\beta - sy\delta - z\delta}{2(v\beta - y\delta)}
\]

The equilibrium manufacturer profit is

\[
profit_{manCRM} = \frac{1}{2(v^2 + 2v(y - 2\beta - \delta) + (y + \delta)^2)} \left( \delta[(sv - z)(sv + z) + (-sv + z)\alpha] - \beta(sv + z)^2 + sz\delta^2 - c^2(\beta + \delta)(v\beta - y\delta) + \alpha((v + y)(sv + z) - v\alpha) + c(v^2(s\beta - \alpha) - z\delta(\beta + \delta) - y^2(\alpha + s\delta) + v(-2y\alpha + \beta(sv + z + 2\alpha + s\delta) + \delta(z + \alpha - sy)) + y(z\beta - \delta(\alpha + s\delta - z))) \right).
\]

The second order conditions are satisfied if

\[
\frac{\partial^2 profit_{manCRM}}{\partial^2 w} = \frac{4(\beta + \delta)(v\beta - y\delta)}{v^2 + 2v(y - 2\beta - \delta) + (y + \delta)^2} < 0
\]

\[
\frac{\partial^2 profit_{manCRM}}{\partial^2 r} = \frac{4v(v\beta - y\delta)}{v^2 + 2v(y - 2\beta - \delta) + (y + \delta)^2} < 0
\]

and the determinant of the hessian \[-\frac{4(v\beta - y\delta)^2}{v^2 + 2v(y - 2\beta - \delta) + (y + \delta)^2} > 0\] by assumption that

\[-(v^2 + 2v(y - 2\beta - \delta) + (y + \delta)^2) > 0.\]
A.15.3. Comparing CRM to CR (assuming \(-v^2 + 2v(y - 2\beta - \delta) + (y + \delta)^2 > 0, (v\beta - y\delta) > 0\), and \(y < \beta\))

**PROPOSITION 2:** Consider the situation in which the manufacturer charges the retailer a per-unit wholesale price but not a fixed fee. If \(s > s_r\), then the manufacturer accepts and salvages returned units from the retailer. There exists a critical salvage value for the manufacturer, \(s < s_r\), such that \(s < s < s_r\) also implies the manufacturer will accept and salvage returned units from the retailer. If \(s \leq s\), then the manufacturer chooses to allocate responsibility for salvaging returned units to the retailer.

Proof: For \(s = s_r\),

\[
\text{profitmanCRM} - \text{profitmanCR} = \frac{(v(\alpha - s\beta) - z(\beta + \delta) + y(\alpha + s\delta))^2}{8(\beta + \delta)(v\beta - y\delta)}.
\]

This is greater than zero if and only if \((v\beta - y\delta) > 0\), which we have assumed previously to be true.

\[
\frac{\partial \pi_{mfgr}^{CRM}}{\partial s} = \frac{v(\alpha + \beta(c - 2s)) - \delta(cy - z)(y + \delta) + v(-2z\beta - z\delta - a\delta + c\beta\delta + y(\alpha + c\beta - c\delta + 2s\delta))}{2(v^2 + 2v(y - 2\beta - \delta) + (y + \delta)^2)},
\]

(which equals the equilibrium value of \(Q_{returns} \geq 0\)) and \(\frac{\partial \pi_{mfgr}^{CR}}{\partial s} = 0\). Therefore, when

\[
\pi_{mfgr}^{CRM} \geq 0 \text{ and } \pi_{mfgr}^{CR} \geq 0,
\]

there exists an \(s < s_r\) such that the functions \(\pi_{mfgr}^{CRM}(s) = \pi_{mfgr}^{CR}(s)\) at \(s = s\). If \(s < s\) then \(\pi_{mfgr}^{CRM} > \pi_{mfgr}^{CR}\) and \(s \leq s\), then \(\pi_{mfgr}^{CRM} \leq \pi_{mfgr}^{CR}\). Q.E.D.

Observation 1 can be proven with the additional assumption on the demand and return equations \(v(2\beta - y) - y(y + \delta) > 0\).

**OBSERVATION 1:** Consider a situation in which the manufacturer charges the retailer a per-unit wholesale price but not a fixed fee. If \(s > s_r\), then the return penalty, \(f\), charged to consumers is greater when the retailer salvages returned units than when the manufacturer accepts and salvages returned units from the retailer. There exists a critical salvage value for
the manufacturer, \( \tilde{s} < s_r \), such that \( \tilde{s} < s < s_r \) also implies the return penalty, \( f \), charged to consumers is greater when the retailer salvages returned units than when the manufacturer accepts and salvages returned units from the retailer. If \( s \leq \tilde{s} \), then the return penalty, \( f \), charged to consumers is lower when the retailer salvages returned units than when the manufacturer accepts and salvages returned units from the retailer.

Proof:

If \( s = s_r \), then

\[
\frac{f^{CRM} - f^{CR}}{4(\beta + \delta)(\nu \beta - y \delta)} = \frac{\beta (v(-\alpha + s \beta) + z(\beta + \delta) - y(\alpha + s \delta))}{4(\beta + \delta)(\nu \beta - y \delta)}.
\]

If \( v(-\alpha + s \beta) + z(\beta + \delta) - y(\alpha + s \delta) < 0 \), this difference is negative, implying \( f^{CRM} < f^{CR} \). The derivative

\[
\frac{\partial f^{CRM}}{\partial s} = \frac{\nu(2\beta - y) - y(y + \delta)}{2(v^2 + 2v(y - 2\beta - \delta) + (y + \delta)^2)} < 0
\]

(by the fact that the S.O.C. implies \( v^2 + 2v(y - 2\beta - \delta) + (y + \delta)^2 < 0 \) and under assumption \( v(2\beta - y) - y(y + \delta) > 0 \)). The derivative of the restocking fee in the CR structure with respect to the manufacturer’s salvage value is \( \frac{\partial f^{CR}}{\partial s} = 0 \). Therefore, there exists an \( \tilde{s} < s_r \) such that the functions \( f^{CRM}(s) = f^{CR}(s) \) at \( s = \tilde{s} \). If \( s > \tilde{s} \) then \( f^{CRM} < f^{CR} \) and if \( s \leq \tilde{s} \), then \( f^{CRM} \geq f^{CR} \).

\[Q.E.D.\]