Expertise in Online Markets

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Abstract

The phenomenon of sniping - submitting bids in the last minute - in online auctions has highlighted the existence of knowledgeable buyers or experts in such markets. In this paper, we consider online markets with auctions with hard or soft close and posted prices, and examine the effect of the presence of experts on other buyers, the platform and the sellers. We model buyer expertise as the ability to accurately predict the quality, or condition, of an item. In auctions with a hard close, while sniping emerges as an equilibrium strategy for experts, we show that non-experts bid more aggressively as the proportion of experts increases. As a consequence, we obtain the surprising implication that the auction platform may obtain a higher revenue by enforcing a hard close rather than a soft close in online auctions, thus providing a simple explanation for the prevalence of this format. In online markets where both auctions and posted prices are available, we show that the presence of experts allows the sellers of high quality items to signal their quality by choosing to sell via auctions.

1 Introduction

The advent of online auctions such as those in eBay led to the first massive scale deployment of simple second-price auction mechanisms for consumer products. Even though eBay started as a platform for consumer-to-consumer auctions for selling items off one’s garage, it is now a large selling platform enabling over $200 billion commerce volume and reaching over 200 million users annually. The addition of posted-price sales has fueled this growth by allowing it to serve as a competitor to other online retail sites. The growth of this new segment of online markets that combine auctions with posted prices raises important new questions

1http://venturebeat.com/2013/10/16/ebay-earnings-sales-up-21-revenue-up-14-and-double-digit-paypal-user-growth/
about the optimal strategies for the buyers and sellers to follow in the market as well as for
the platform in its design choices.

The eBay auction format enforces a hard close or ending time at which the item is sold
to the highest (winning) bid. In the hours leading to the closing time, the auction is open
and simulates the open outcry English auction in the online setting. If all bidders had
only private values, traditional auction theory dictates that the dominant strategy for every
bidder is to bid up to her true value. To enable this, eBay also offers a proxy bidding tool
which allows a bidder to specify her maximum value to the tool, and the tool automatically
bids the minimum bid increment above the current highest bid (as long as it is below the
declared value). Thus, it was somewhat of a paradox when a majority of eBay auctions
exhibited sniping – the phenomenon when a bidder submits his only bid at the last few
seconds before the auction close.

While several explanations for this behavior have been advanced, one of the most intuitive
and accepted ones is that of experienced bidders (Wilcox, 2000) or dealers/experts (Roth
and Ockenfels, 2002). For example, Roth and Ockenfels (2002) argue, and provide empirical
evidence, that existence of sniping in online markets is partly due to buyers’ heterogeneity
in their experience with online markets and their expertise in the product category: “...there may be bidders who are dealers/experts and who are better able to identify high-value antiques. These well-informed bidders ... may wish to bid late because other bidders will recognize that their bid is a signal that the object is unusually valuable.”

In this line of reasoning, the item auctioned off is assumed to have a common value
which these experts have a better knowledge of; submitting a sniping bid is a way for
experts to withhold this information to reap the advantage of this information asymmetry
in the resulting price. While several papers have subsequently built upon and refined this
explanation for sniping (Bajari and Hortacsu 2003, Rasmusen 2006, Hossain and Morgan
2006, Ockenfels and Roth 2006, Hossain 2008, and Ely and Hossain 2009), all of them
have examined the phenomenon only from bidders’ perspective. More broadly, to best of
our knowledge, no other paper has studied the strategic impact of buyers’ heterogeneity in expertise (that causes the sniping behavior) on the platform and sellers’ strategies in online markets. In this paper, we examine the effect of the existence of experts buyers on all of the stakeholders in online markets: the expert and non-expert buyers, the sellers and the platform. We discuss the following research questions:

1. How do expert buyers benefit from their expertise in online markets?

2. How do non-expert buyers adjust their strategies to compete with experts?

3. How does the presence of experts affect the platform revenue?

4. How does the presence of experts affect the sellers’ strategies in online markets?

Our Contributions

First, in line with the previous posed explanations in the literature on existence of sniping, we re-create the result that sniping arises as a natural equilibrium strategy for the experts to hide their information about the item until the last minute. While previous papers provided examples of models where sniping emerges under certain conditions, we are able to completely characterize all the equilibria in our model, thus delineating the cases when it is prevalent. By comparing with an auction with soft close (that extends the clock with every submitted bid) we note that expert buyers are able to procure better prices in the auction with hard close by sniping.

Second and more interestingly, we show that the presence of experts encourages the non-experts to bid more aggressively. In particular, we show that because of the sniping strategy of the expert buyers in hard close auctions, non-expert buyers have to bid more than their expected value; otherwise they only win items of low quality value against the expert buyers. Quantifying this, we show in Proposition 1 that the higher the proportion of experts among the bidders, the more aggressively the non-experts bid above their expected value for the item.
Next, we consider the impact of the presence of experts on the platform’s strategies. In particular, should the platform maintain the hard close format for the auction that allows the experts to snipe rather than switch to the soft close format? Also, if the platform knows the quality value of the item and can credibly reveal it to the buyers, should it commit to sharing this information with them? We find interesting answers to these questions. Regarding the first question, at the outset, it appears that the hard close format may hurt platform revenue since without the sniping behavior of experts, non-expert buyers could respond to bids of experts, and the item would sell at a higher price. Since the platform’s fee is usually a fixed fraction of the selling price, the platform would then have an incentive to favor the soft close format. Contrary to this expectation, we show that the aggressive bidding behavior of the non-experts that we describe above implies that the platform’s overall revenue increases in the hard close format for a wide range of parameter values (Proposition 2). This is a very interesting and novel explanation as to why online auction companies such as eBay retain the hard close auction format that allows sniping from a revenue perspective.

This result has another important and interesting implication regarding the second question: the platform can benefit from committing to withholding the quality information (Corollary 1). This is in contrast to the celebrated linkage principle (Milgrom and Weber, 1982), and is driven by buyers’ heterogeneity in their level of expertise. Proposition 1 can also be interpreted as a reverse winner’s curse. In auctions with common values, bidders bid lower than their valuation to avoid winner’s curse. However, this result shows that when bidders are heterogeneous in their level of information, non-informed bidders bid more than their valuation to make up for their lack of information.

Finally, we consider the impact of the presence of expert buyers on the sellers’ strategies. In particular, we investigate the choice of selling mechanisms between the auction and a

\(^2\)In fact, some auction platforms such as the now defunct Amazon Auctions and Trademe, removed sniping by implementing a soft close that automatically extended the auction time whenever a bid is submitted.

\(^3\)EZsniper.com provides an extensive list of auction sites with a hard close.

\(^4\)The linkage principle argues that the auction house always benefits from committing to reveal all available information.
posted price sale when they are both available (as is common in most online auction-houses). In the presence of expert buyers, under certain conditions, we show that by selling in an auction, a seller can credibly signal \(^5\) the quality value of his item (Proposition \(^3\)). By selling in an auction, the seller shows that he can rely on the market (specifically on the expert buyers) to decide the value of the item. This is a risk that a seller with a low quality value item cannot take. Furthermore, this signaling is only possible if there are enough experts, who know the value of the item, in the market. Otherwise, the seller of a high quality-value item will not be able to separate himself from the seller of a low-value item. In other words, the existence of experts in the market allows the sellers of high quality products to separate themselves by selling in auctions. This finding is in line with auction houses’ claim that selling in auction increases buyers’ confidence. For example, Fraise Auction\(^6\) argues that one of the benefits of selling in auction is that “The competitive bidding format creates confidence among the buyers when they see other people willing to pay a similar amount for the property.” To best of our knowledge, this result is a new explanation for popularity of auctions in certain product categories.

Taken together, we initiate the first comprehensive study of the effect of the presence of expert buyers in online markets featuring auctions with a hard close and posted prices, and establish the following results.

1. Expert buyers benefit by sniping in hard close auctions by obtaining lower prices in comparison with the soft close format.

2. Non-expert buyers must adjust their strategies, and, under certain conditions, have to bid more than their expected value in hard close auctions in equilibrium.

3. As a consequence, the platform revenue is higher in the hard close auction than in the soft close format for a wide range of parameter values.

\(^5\)Note that the signal that we discuss here is the seller’s choice of the selling mechanism. This is different from bids by other bidders, which can also be signals of the quality of the product.

\(^6\)http://fraiseauction.com/why-auction/
4. Finally, the presence of experts in markets with hard close auctions and posted prices allows the seller of high quality items to credibly signal the quality of the item by selling in the auction and separating themselves from sellers of low quality items who sell using posted prices, under certain conditions.

In what follows, we review related literature. Section 2 introduces the main model, Section 3 solves the equilibria of the model with hard close, and Section 4 compares them with the corresponding equilibria of the auction with a soft close, which does not allow for sniping. In Section 5, we analyze the sellers’ game of choosing among selling formats. We conclude the paper in Section 6. All proofs are relegated to Appendix A.

Related Literature

Our work relates to the literature on online auctions with common values and hard close, intermediaries’ incentives to reveal product quality information, sellers’ strategies to signal product quality, and the advantages and disadvantages of auctions versus posted prices. In the following, we review the related literature on each topic.

Bajari and Hortacsu (2003) argue that last-minute bidding is an equilibrium in a stylized model of eBay auctions with common values. They develop and estimate a structural econometric model of bidding in eBay auctions with common value and endogenous entry. Wilcox (2000) and Rasmusen (2006) use common values to model sniping and bidders’ behavior on eBay auctions. Wilcox (2000) shows that sniping increases as buyers’ experience increases. Furthermore, the increase in the sniping behavior of the more experienced bidders is more pronounced for the type of items that are more likely to have a common value component. Similarly, a model with no common value as in Yoganarasimhan (2013) demonstrates no sniping behavior. Rasmusen (2006) considers a model where bidders incur a cost for learning the common value of the item. As a result, those who acquire the information snipe to hide their information from other bidders. Similar to the previous literature, sniping

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7The literature on trying to explain sniping in online auctions is vast, other than previously mentioned
emerges as an equilibrium strategy in our model as well. However, our focus is the effect of the presence of experts on non-experts', sellers' and the platform’s strategies and revenues, which is crucially missing in the earlier literature. In contrast to earlier work of Ockenfels and Roth (2006) who show an example in which seller revenue is lower at the equilibrium for hard close than in the soft close case, in our model, we show that the hard close format increases revenue compared to the soft close format. More specifically, we provide an explanation as to why online auction companies such as eBay retain the auction format that allow for sniping from a revenue perspective that takes into account the aggressive bidding behavior of the non-experts.

In this paper, we show that an intermediary could benefit from withholding information about the quality of the items in an auction. This is in contrast with the well-known linkage principle by Milgrom and Weber (1982). The linkage principle argues that the auction house always benefits from committing to reveal all available information. The intuition behind the principle is that revealing the information can mitigate the winner’s curse, and motivates the buyers to bid more aggressively. We arrive at the contrast due to buyers’ heterogeneity in terms of their information about the quality value of the item, as modeled by their expert status. More specifically, the result of Milgrom and Weber (1982) is established when valuation of bidders depend symmetrically on the unobserved signals of the other bidders, a condition that is not satisfied in our setup.\footnote{Withholding information, under certain circumstances, has also been shown to increase social welfare, by Zhang (2013), in the context of product labeling. Gal-Or et al. (2007) show that, under certain conditions, a buyer benefits from withhold information in procurement schemes.} Withholding information, under certain circumstances, has also been shown to increase social welfare, by Zhang (2013), in the context of product labeling. Gal-Or et al. (2007) show that, under certain conditions, a buyer benefits from withhold information in procurement schemes.

Many researchers in marketing have studied signaling unobserved quality under information asymmetry. Moorthy and Srinivasan (1995) and Soberman (2003) show that sellers can papers, see also Hossain and Morgan (2006), Ockenfels and Roth (2006), Hossain (2008), Wintr (2008), and Ely and Hossain (2009).\footnote{Failure of the linkage principle has also been argued in a few other papers in the auction theory literature. For example, Perry and Reny (1999), Chapter 8.1 of Krishna (2002), and Fang and Parreiras (2003) show the failure in setups with multiple items, ex-ante asymmetries and budget constraints, respectively.}
use warranties such as money-back guarantees to signal the quality of their items. Bhardwaj et al. (2005) show that by letting the customers to request information about an item, rather than revealing it without solicitation, a seller can signal the quality of his item. Mayzlin and Shin (2011) show that uninformative advertising, as an invitation for search, can be used to signal product quality. Li et al. (2009) investigate auction features such as pictures and reserve price that enable sellers to reveal more information about their credibility and product quality, and empirically examine how different types of indicators help alleviate uncertainty. Finally, Subramanian and Rao (2015) show that, by displaying daily deal sales, a platform can leverage its sales to experienced customers to signal its type and attract new customers. This is relevant to our result as in both papers, the existence of experts (or experienced customers) can help the sellers to extract more revenue from the non-expert customers. However, the higher revenue is achieved using very different tools, displaying daily deal sales versus selling in auctions, in the two papers. Compared to the previous literature, we introduce a new dimension for sellers to signal the quality of their items. In particular, for product categories with a common value component where assessing the common value needs expertise (e.g., antiques category), we show that selling via auction can signal that the item has a high common value.

Finally, we review the related literature that compare auctions to posted price selling mechanisms. Einav et al. (2013) propose a model to explain the shift from Internet auctions to posted prices and consider two hypotheses: a shift in buyer demand away from auctions, and general narrowing of seller margins that favors posted prices. By using eBay data, they find that the former is more important. There is a significant economics literature that compares auctions to posted price mechanisms. Notably, Wang (1993) compares auctions with posted prices and shows that auctions become preferable when buyers valuations are more dispersed. In another important paper, Bulow and Klemperer (1996) have shown that additional revenue by attracting one more bidder is greater than setting the optimal reserve price, hence in a sense establish that “value of negotiating skills is small relative to value of
additional competition.” In an empirical work, Bajari et al. (2009) conclude that the choice of sales mechanism may be influenced by the characteristics of the product being sold. To the best of our knowledge, our paper is the first work that considers the signaling effects of the choice of the mechanism on buyers’ beliefs. Specifically, we show that the choice of selling mechanism can be used by seller of high-quality items as a signal of their item’s quality.

2 Model

We consider a model with two buyers and one item. We assume that there are two types of buyers, experts and non-experts, and each buyer is an expert with probability $p$. Due to anonymity of online marketplaces, we assume that each buyer does not know whether his opponent is an expert or not.9

In our model, the items sold in online auctions have differing levels of “quality value,” which may reflect the condition of a used good, or the relative efficacy of a product among its competitors. Note that this value is similar to a common value in that its benefit accrues equally to both expert bidders (who can accurately predict quality value) and non-expert bidders (who do not know the quality value). We assume that the quality value, denoted by a binary random variable $C$ with realizations 0 and $c$, is only known by experts and it is the same for both experts and non-experts (therefore it can be described as a common value). Moreover, the items sold in online auctions also have differing levels of “private value,” which may reflect bidders’ private tastes for the items, or whether they have immediate needs for the items. Each bidder may have a different private value. We assume that the private value, denoted by a binary random variable $V$ with realizations 0 and $v$, is learned privately both by experts and non-experts.

The total value of the item for a bidder is the sum of the quality value and an additional private value component. More specifically, we assume that $C$ has a binary distribution:

9On eBay and most other auction platforms, identities of bidders are revealed only after an auction ends. Furthermore, bidders can easily hide their type by creating and using a new account online.
Pr (C = c) = q (high common value) and Pr (C = 0) = 1 − q (low common value), also
V (for each bidder) has a binary distribution: Pr (V = v) = r (high private value) and
Pr (V = 0) = 1 − r (low private value). We assume that c, v, p, q and r are common
knowledge. Moreover, buyer private value types are privately known by all buyers and
the realization of C is privately known by experts only (non-experts only know the prior
probability distribution). The total value of the item for each bidder is simply C + V where
C is the quality value of the item and V is the buyer specific private value.

We model the online auction with a hard close as a two-stage bidding game where the
second stage represents the very last opportunity to submit a bid (the sniping window),
while the first stage represents the whole time window preceding the close. Even though in
practice the period before the sniping window is a dynamic game, we model it (Stage 1) by
allowing each bidder to submit a single bid: To reconcile this with reality, we can think of
the highest bid that a bidder submitted before the sniping window as the first stage bid.
Bidders can observe competitors’ bids of Stage 1, and respond to it in Stage 2; however, they
do not have enough time to respond to competitors’ bids of Stage 2. It is worth mentioning
that we can derive all of our results with a more realistic dynamic game model of the first
stage. However, while being a bit more involved, it does not add any further insight to
our analysis so we use the simpler two-stage formulation here. Moreover, motivated by the
the fact that bidding in the sniping window has the risk of losing the bid due to erratic
internet traffic, we assume that a bid in stage 2 goes through only with probability 1 − δ,
for sufficiently small δ.

The timing of the model is as follows (see also Figure 1). Before Stage 1, each buyer
knows his own type (expert or non-expert), but not the type of the other buyer. If a buyer
is an expert, he also knows the common value (whether C = 0 or C = c). All buyers also
know their buyer specific private values (whether V = 0 or V = v). In Stage 1, both buyers
simultaneously submit their bids. After Stage 1 and before Stage 2, both buyers observe the

\footnote{We can consider a dynamic auction in the time interval [0, 1) and sniping at time 1.}
other buyer’s bid, and may be able to infer their opponent’s type (and values). In Stage 2, both buyers simultaneously decide if they want to increase their bid from Stage 1, and if so by how much. In other words, bids of Stage 2 have to be greater than or equal to bids of Stage 1. Stage 2 bids are received by the auctioneer with probability \(1 - \delta\). If the bid of Stage 2 is lost for a bidder (with probability \(\delta\)), the auctioneer continues to use the bid of Stage 1 for that bidder. After Stage 2, the item is given to the buyer with the highest bid at the price of the second highest bid. If there is a tie between a low-type bidder and a high-type bidder, then the item goes to the high-type bidder.

In auctions with soft close, there are possibly an infinite number of stages. If a bid is submitted at any stage, bidders can submit another bid in the next stage. The game ends when no bid is submitted in some stage. We also consider posted prices in Section 4. In this game, seller posts a price \(z\) and bidders then decide whether to buy at this price. The trade takes place at the posted price \(z\) if and only if at least one bidder is interested in the item. If both bidders want the item, each of them gets the item with probability 0.5.

3 Effect of Experts on Buyer Strategies

In this section, we characterize the equilibria of the auction game. In equilibrium, experts sometimes use sniping to protect their information about the common value of the item. We

\[\text{\footnote{For a full description and motivation of the tie-breaking rule, please see Appendix A}}\]
show that non-experts with high private value bid aggressively—even above their expected valuation—in order to compete with experts.

### 3.1 Experts Induce Sniping.

Based on the relation of the parameters $c, v, p, r,$ and $q,$ we split the set of possible values into nine mutually exclusive and collectively exhaustive ranges. In the first four ranges, we have that $cq + v < c$ and $v < cq,$ in the next two, we have $cq + v < c$ and $v \geq cq,$ in the next two, we have $cq + v \geq c$ and $v < cq,$ and in the last range, we have $cq + v \geq c$ and $v \geq cq.$ We call an expert/non-expert with high/low private value a high/low expert/non-expert. Note that $cq + v$ is the expected value of a high non-expert for the item, while $cq$ is the expected value of a low non-expert.

Consider the function $f(c, p, r, q) = c \cdot \frac{(1-p)(1-q)r}{2pq(1-r)+(1-p)r}.$ Let $m_1 = f(c, p, r, q), m_2 = f(c, p, 1-r, 1-q), M_1 = f(c, p, 1, q) = c \cdot (1-q),$ and $M_2 = f(c, p, 1, 1-q) = c \cdot q.$ Note that one can easily verify that $m_1 \leq M_1$ and $m_2 \leq M_2.$ The nine different cases we consider are as follows: $v \in [0, \min\{m_1, m_2\}), v \in [m_1, \min\{m_2, M_1\}), v \in [m_2, \min\{m_1, M_2\}), v \in [\max\{m_1, m_2\}, \min\{M_1, M_2\}), v \in [M_2, m_1), v \in [\max\{m_1, M_2\}, M_1), v \in [M_1, m_2), v \in [\max\{m_2, M_1\}, M_2),$ and $v \in [\max\{M_1, M_2\}, +\infty).$

To describe an equilibrium, we use the notation $(s_1, s_2, s_3, s_4),$ which means that a high expert follows the strategy $s_1,$ a low expert follows the strategy $s_2,$ a high non-expert the strategy $s_3,$ and a low non-expert the strategy $s_4.$ For the bidding strategies of each type we use the following notation:

- For a high expert, consider the following strategies:
  
  - $s_1^{HE}$: If $C = 0,$ he bids $v$ in the first stage and does nothing in the second stage. If $C = c,$ he bids $cq + v$ in the first stage and bids $c + v$ in the second stage (sniping strategy).
  
  - $s_2^{HE}$: If $C = 0,$ he bids $v$ in the first stage and does nothing in the second stage.
If $C = c$, he bids $c$ in the first stage and bids $c + v$ in the second stage (sniping strategy).

- For a low expert, consider the following strategies:
  - $s^{LE}$: If $C = 0$, he does nothing. If $C = c$, he bids $cq + v$ in the first stage and $c$ in the second stage (sniping strategy).
  - $t^{LE}$: If $C = 0$, he does nothing. If $C = c$, he bids $c$ in the first stage and nothing in the second stage (truthful strategy).

- For a high non-expert, consider the following strategies:
  - $m^{HNE}$: He bids $cq + v$ in the first stage. If he sees a bid other than $0, v, cq, v$ in the first stage, he bids $c + v$ in the second stage. Otherwise, he bids $c$ in the second stage with probability $1 - a$, where $a = 1 - \frac{2p(1-r)qv}{(1-p)(1-r)(c-(cq+v))}$ (mixed strategy).
  - $a^{HNE}$: He bids $c$ in the first stage. If he sees a bid other than $0, v, cq, c$ in the first stage, he bids $c + v$ in the second stage. Otherwise, he does nothing in the second stage (aggressive strategy).
  - $t^{HNE}$: He bids $cq + v$ in the first stage. If he sees a bid other than $0, v, cq, c, cq + v$ in the first stage, he bids $c + v$ in the second stage (truthful strategy).

- For a low non-expert, consider the following strategies:
  - $m^{LNE}$: He bids $v$ in the first stage. He bids $cq$ in the second stage with probability $1 - g$, where $g = \frac{2p(1-r)v}{(1-p)(1-r)(1-p)(1-r)}$ (mixed strategy).
  - $u^{LNE}$: He bids $v$ in the first stage and nothing in the second stage (underbidding strategy).
  - $t^{LNE}$: He bids $cq$ in the first stage and nothing in the second stage (truthful strategy).
We describe equilibrium bidding strategies for buyers in the nine cases in the following lemma, whose proof is relegated to Appendix A.

**Lemma 3.1.** For the auction model described in Section 2, the buyers’ equilibrium bidding strategies are given below.

1. If \( v \in [0, \min\{m_1, m_2\}) \), the set of strategies \((s_{HE}^1, s^{LE}, m^{HNE}, m^{LNE})\) forms an equilibrium.

2. If \( v \in [m_1, \min\{m_2, M_1\}) \), the set of strategies \((s_{HE}^2, t^{LE}, a^{HNE}, m^{LNE})\) forms an equilibrium.

3. If \( v \in [m_2, \min\{m_1, M_2\}) \), the set of strategies \((s_{HE}^1, s^{LE}, m^{HNE}, u^{LNE})\) forms an equilibrium.

4. If \( v \in [\max\{m_1, m_2\}, \min\{M_1, M_2\}) \), the set of strategies \((s_{HE}^2, t^{LE}, a^{HNE}, u^{LNE})\) forms an equilibrium.

5. If \( v \in [M_2, m_1) \), the set of strategies \((s_{HE}^1, s^{LE}, m^{HNE}, t^{LNE})\) forms an equilibrium.

6. If \( v \in [\max\{m_1, M_2\}, M_1) \), the set of strategies \((s_{HE}^2, t^{LE}, a^{HNE}, t^{LNE})\) forms an equilibrium.

7. If \( v \in [M_1, m_2) \), the set of strategies \((s_{HE}^1, t^{LE}, a^{HNE}, m^{LNE})\) forms an equilibrium.

8. If \( v \in [\max\{m_2, M_1\}, M_2) \), the set of strategies \((s_{HE}^1, t^{LE}, a^{HNE}, u^{LNE})\) forms an equilibrium.

9. If \( v \in [\max\{M_1, M_2\}, +\infty) \), the set of strategies \((s_{HE}^1, t^{LE}, t^{HNE}, t^{LNE})\) forms an equilibrium.

Lemma 3.1 characterizes the equilibria of the game in nine cases. In some of these cases, if the common value is high, an expert “pretends” to be a non-expert by mimicking a non-expert’s strategy in Stage 1, and then snipes with his true value in Stage 2 (strategies...
In other words, experts use sniping to hide their valuable information from non-experts. This is in line with the findings of Roth and Ockenfels (2002).

3.2 Impact of Experts on Non-experts’ Strategy.

A high non-expert’s optimal strategy depends on the value of $v$. If $v$ is sufficiently high ($v \geq M_1$), a high non-expert’s expected value for the item is higher than $c$. In this case, high non-experts always win the competition against low experts. When $v$ is smaller than $M_1$, the situation is more interesting. By bidding their expected value against experts, high non-experts only win when the common value is low. Therefore, high non-experts have to bid higher than their expected value (strategies $m^{HNE}, a^{HNE}$) in order to win a high-common-value item against low experts. Note that bidding above the expected value does not necessarily mean that they have to pay more than their expected value because the auction format is second price. The only risk is that if two high non-experts compete with each other, they may both bid above their expected value, and end up paying more than their expected value. In this case, a non-expert’s payoff could be negative. The following proposition discusses the conditions under which non-experts bid more than their expected value.

![Figure 2: Probability that a high non-expert overbids as $v$ increases for $p = 0.3$, $r = 0.5$, $q = 0.1$, and $c = 1$.](image1)

![Figure 3: Probability that a high non-expert overbids as $p$ increases for $v = 0.5$, $r = 0.5$, $q = 0.1$, and $c = 1$.](image2)
Proposition 1. If the expected value of a high non-expert for the item is less than the common value of the item (i.e., $cq + v < c$), the high non-expert may bid more than their valuation for the item in equilibrium. The probability of over-bidding increases as the fraction of experts in the market (i.e., $p$) increases.

Proposition 1 shows that if the value of $v$ is high enough (i.e., $v \in [m_1, M_1]$), non-experts always take the risk of over-paying, and bid above their expected value in order to win against experts. However, if $v < m_1$, a non-expert over-bids only with some probability (depicted in Figure 2). This mixed strategy allows the non-experts to mitigate the risk of over-paying due to competition with another non-expert. Furthermore, Proposition 1 shows that as the probability that the opponent is an expert increases, a non-expert’s willingness to take the risk and bid above his expected value increases (depicted in Figure 3).

4 Effect of Experts on Platform Strategies

An important assumption in Proposition 1 is that experts can hide their information by sniping. The platform can eliminate sniping by extending the duration of the auction whenever a bid is submitted (soft close). In this case, non-experts always have enough time to respond to experts’ bids, and therefore, do not have to bid above their expected valuation.

We show that, under certain conditions, non-experts’ aggressive behavior leads to higher revenue for the platform to the extent that the platform benefits from allowing sniping (by having a hard close). In other words, the existence of experts and their ability to hide their information force the non-experts to bid more aggressively, and ultimately lead to higher revenue for sellers and for the platform. This result also relates to platform strategies regarding the revelation of information. In Section 4.3, we show the breakdown of the linkage principle by showing that the platform may benefit from withholding quality information from the buyers when the buyers are heterogeneous in their level of expertise.
4.1 An auction with soft close.

We now consider a model in which sniping is not possible. One way to prevent sniping is by extending the duration of the auction by a few minutes every time there is a bid near the current end time of the auction. This auction is called an auction with a soft close and was used by the now defunct Amazon Auctions. A way to model this is by starting with a game that has only one stage and every time there is a bid during the current stage, the auction extends for one more stage. In other words, every time someone makes a bid, the other buyers can see it and respond to it. First, we characterize the equilibrium the model for auctions with soft close. Then we compare seller’s revenue and the platform’s revenue across the two models. The goal is to see which ending rule results in better revenues for the sellers (and therefore for the platform).

As before, for the bidding strategies of each type of buyer, we use the following notation:

- For a high expert, consider the following strategy:
  - $t^{'HE}$: If $C = 0$, he bids $v$ in the first stage and nothing later. If $C = c$, he bids $c + v$ in the first stage and nothing later (truthful strategy).

- For a low expert, consider the following strategy:
  - $t^{'LE}$: If $C = 0$, he does nothing. If $C = c$, he bids $c$ in the first stage and nothing later (truthful strategy).

- For a high non-expert, consider the following strategy:
  - $t^{'HNE}$: He bids $cq + v$ in the first stage. If he sees a bid of $c$ or $c + v$ at some point and $cq + v < c$, he bids $c$ in the next stage (truthful strategy).

- For a low non-expert, consider the following strategies:
  - $m^{'LNE}$: He bids $v$ in the first stage. In the second stage, he bids $cq$ with probability $1 - w$, where $w = \frac{2p(1-q)v}{(1-p)(1-r)(cq-v)}$, and nothing later (mixed strategy).
– $u^{\text{LNE}}$: He bids $v$ in the first stage and nothing later (underbidding strategy).
– $t^{\text{LNE}}$: He bids $c q$ in the first stage and nothing later (truthful strategy).

**Lemma 4.1.** In a platform with soft close:

1. If $v \in [0, m_2)$, the set of strategies $(t^{\text{HE}}, t^{\text{LE}}, t^{\text{HNE}}, u^{\text{LNE}})$ forms an equilibrium.
2. If $v \in [m_2, M_2)$, the set of strategies $(t^{\text{HE}}, t^{\text{LE}}, t^{\text{HNE}}, u^{\text{LNE}})$ forms an equilibrium.
3. If $v \in [M_2, +\infty)$, the set of strategies $(t^{\text{HE}}, t^{\text{LE}}, t^{\text{HNE}}, t^{\text{LNE}})$ forms an equilibrium.

Note that when we have soft close, high non-experts bid their expected value. If they see a bid of $c$ or $c + v$, they infer that the opponent is expert and the common value is high. In that case, they increase their bid to $c$ to win the item at price $c$ against a low expert.

### 4.2 Effect of Experts on Platform Revenue.

Using Lemma 4.1, it is easy to see that with soft close, experts always reveal the value of a high-common-value item to non-experts. This increases the non-experts willingness to pay and in some cases leads to higher revenue for the seller.

However, when there is a soft close, non-experts do not have to bid above their valuation. This reduces the competition and can hurt sellers’ revenue as well as the platform’s revenue. In Lemma 4.2 we see that sellers can benefit from hard close under certain conditions. We use this lemma to analyze the platform’s incentive in having a hard close.

**Lemma 4.2.** Suppose that $v < M_1$, then

- The seller of an item with low common value always benefits from hard close.
- The seller of an item with high common value benefits from hard close if $p$ is sufficiently large and $\delta$ is sufficiently small.
Lemma 4.2 shows that the seller of an item with low common value always benefits from hard close. This is intuitive because hard close causes sniping, which prevents the flow of information from experts to non-experts. Therefore, when there is hard close, non-experts are more likely to over-pay for an item with low common value. The interesting part is that even the seller of an item with high common value benefits from hard close if \( p \) is high enough and \( \delta \) is low enough. This is because when there is hard close, non-experts know that they will not be able to infer the common value, and therefore, have to bid more aggressively to win the item. As we observe in Proposition 1, this aggressive bidding behavior increases as \( p \) increases. If \( p \) is sufficiently large, the positive effect of this aggressive bidding behavior on seller’s revenue can dominate the negative effect of lack of information flow, and therefore, the revenue of the seller of a high-quality item can be higher with hard close than with soft close. The condition that \( \delta \) should be small is also needed, because if the probability that a sniping bid will fail is large, allowing sniping will not have a significant change in seller’s revenue. Using the same argument, we can see that the platform can also benefit from having a hard close when \( p \) is sufficiently large (and \( \delta \) is sufficiently small). This result is formalized in Proposition 2.

**Proposition 2.** If the expected value of the high non-experts for the item is less than the common value of the item (i.e., \( cq + v < c \)), and the fraction of experts in the market (i.e., \( p \)) is sufficiently large, the platform’s revenue from hard close is higher than that from soft close (for sufficiently small \( \delta \)).

The result of Proposition 2 is depicted in Figure 4. The region where hard close provides higher revenue appears when \( v \) is sufficiently large compared to \( c \), and \( p \) is sufficiently large. This is because higher \( v \) and higher \( p \) both lead to non-experts’ aggressive bidding, as we saw in Figure 2 and Proposition 1.
4.3 Experts and the Breakdown of the Linkage Principle.

Finally, we discuss the connection between hard close and revelation of information in the marketplace. Note that hard close allows the experts to protect their information about the value of the item. We know that the platform sometimes benefits from hard close. This could suggest that the platform may also benefit from withholding information about the value of the item. This is an important implication because it is in contrast with the well-known “linkage principle” in auction theory (Milgrom and Weber, 1982).

The linkage principle states that auction platforms (e.g. auction houses) benefit from committing to reveal all available information about an item, positive or negative. The platform revealing the information reduces the downside risk of winning the item, also known as the winner’s curse. But we show that there is also a downside in revealing the information in the presence of heterogeneous bidders, and the platform may sometimes benefit from committing to not revealing the information.
Our result shows that when bidders are asymmetric in terms of their information about the value of the product, bidders with less information have to bid more aggressively, otherwise, they only win the item when bidders with more information do not want the item (i.e., the common value is low). This aggressive behavior incentivizes the platform to withhold any information about the quality value of an item. This result is formalized in the following corollary.

**Corollary 1.** *When there is a hard close, the platform suffers (i.e. has lower revenue) from committing to reveal the common value to the buyers for moderate values of* $p$ *and* $\frac{v}{c}$.

We should note that the region in Figure [1] where the hard close format provides higher revenue is the same as the region in Corollary [1] in which the platform prefers to withhold the common value information.

So far we discussed the effect of the existence of experts on non-experts’ and the platform’s decisions. In the next section, we analyze the effect on sellers’ choice of selling mechanism. In particular, we show that existence of experts can help the sellers of items with high common value to signal the value of their items to non-experts.

## 5 Effect of Experts on Seller Strategies

In this section, we show that the existence of experts in the market could help the sellers to signal the quality/common value of their item to non-experts. We look at sellers’ choice of selling mechanism between an auction and a posted price sale. We call the seller of an item with high common value a high-type seller, and the seller of an item with low common value a low-type seller. A seller is high-type with probability $q$ where $q$ is common knowledge. A seller naturally knows his own type; experts also know the seller’s type (since they know the common value of items being offered). But non-experts do not know the seller’s type. We investigate whether a seller can signal his type using the selling mechanism (auction versus posted price). In particular, we derive conditions for existence of a separating equilibrium.
We show that existence of enough experts in the market is a necessary condition for a separating equilibrium to exist; furthermore, when the fraction of experts in the market, $p$, is sufficiently large, a separating equilibrium exists only for moderate values of $\xi$.

A seller sets his selling mechanism $M$ (posted price or auction). In case of posted price, $M$ also includes the price. For a mechanism $M$, we assume that all non-experts have the same belief about a seller who uses $M$. In general, non-experts’ belief about a mechanism is the probability that they think a seller using that mechanism is high-type. However, since we only consider pure strategy Nash equilibria of the game, non-experts’ belief about a mechanism is limited to three possibilities: Low ($L$), High ($H$), and Unknown ($X$). In belief $L$, non-experts believe that a seller using mechanism $M$ is always a low-type seller. In belief $H$, non-experts believe that a seller using mechanism $M$ is always a high-type seller. Finally, in belief $X$, non-experts cannot infer anything about the seller’s type and believe that the seller is high-type with probability $q$.

Non-experts have beliefs about each mechanism $M$. In equilibrium, the beliefs must be consistent with sellers’ strategies. In particular, if both types of sellers use the same mechanism in (a pooling) equilibrium, non-experts’ belief for that mechanism must be $X$. If the two types of sellers use different mechanisms in (a separating) equilibrium, non-experts’ belief for the mechanism used by the low-type seller must be $L$ and for the mechanism used by the high-type seller must be $H$. Furthermore, in an equilibrium, given the non-experts’ beliefs, sellers should not be able to benefit from changing their strategies.

Note that sniping is relevant only when buyers’ belief about some mechanism $M$ is $X$. Therefore, in a separating equilibrium, the platform’s decision on whether to use soft close or hard close does not affect buyers’ equilibrium behavior or sellers’ strategies. In other words, the following analysis applies to both soft close and hard close cases.

In general, signaling games can have infinitely many equilibria, supported by different out-of-equilibrium beliefs in the game. Therefore, proving just the existence of an equilibrium with certain characteristics may not be a strong result. To further strengthen the support
for our result that selling in auction can be used by high-type sellers as a signal of quality, we show that, under certain conditions, such equilibrium is the only separating equilibrium that survives “Intuitive Criterion” refinement. Intuitive Criterion, introduced by Cho and Kreps (1987), is an equilibrium refinement that requires out-of-equilibrium beliefs to place zero weight on types that can never gain from deviating from a fixed equilibrium outcome. Intuitive Criterion has been used in various signaling papers in the marketing literature including, but not limited to, Simester (1995), Desai and Srinivasan (1995) and Jiang et al. (2011).

Proposition 3 below shows that when the fraction of experts in the market is sufficiently large and the value of \( \frac{v}{c} \) is moderate, there exists a unique separating equilibrium in which a high-type seller chooses an auction and a low-type seller chooses posted price as their respective selling mechanisms. A proof and related analysis are provided in the Appendix. Figure 5 shows the regions in which this separating equilibrium exists and is unique as a function of \( p \) and \( \frac{v}{c} \). It is interesting to note that existence and uniqueness of this equilibrium does not depend on the value of \( q \).

Let us define

\[
\nu_1 = \min \left( \frac{(1-p)(1-p(1-2r(1-r)))}{2r(1-r)}, \frac{(1-p)^2}{2(1-p(1-p))(1-r)r} \right)
\]

\[
\nu_2 = \min \left( \frac{(1-pr)^2}{r(p(2-pr)-r)}, \frac{1}{2r(1-r)} \right)
\]

\[
\nu_3 = \min \left( \frac{1-r}{2r}, \frac{(1-p)(2-r(1-p))}{r(4 + (2-p(2-p)r^2 - 2r(3-p)))} \right)
\]

**Proposition 3.** If \( \frac{v}{c} \in [\nu_1, \nu_2] \), there exists a separating equilibrium in which a high-type seller uses an auction and a low-type seller uses a posted price \( v \). Furthermore, if \( \frac{v}{c} \in (\nu_1, \nu_3) \), this is the only separating equilibrium that survives the Intuitive Criterion refinement. Finally, there exists no separating equilibrium in which a low-type seller uses an auction.

The proof and a more elaborate discussion of Proposition 3 is relegated to the Appendix.
Figure 5: The graph shows the existence and uniqueness of a separating equilibrium in which the high-type seller uses auction and the low-type seller uses posted price, assuming $r = \frac{1}{4}$.

The intuition behind the proof of Proposition 3 is as follows. First, note that in general, an auction is more favorable to a high-type seller than a low-type seller. This is because, in auctions, price is determined by bidders, and expert bidders do not bid high when the seller is low-type. This allows the high-type seller to separate himself from the low-type seller by selling in an auction. But for this separating equilibrium to exist, the low-type seller’s incentive to mimic has to be sufficiently low and the high-type seller’s incentive to separate has to be sufficiently high. These two forces give us the conditions for existence (and uniqueness) of this equilibrium.

In a separating equilibrium, even non-experts know that the low-type seller is low-type. Hence, non-experts are willing to pay at most $v$ for the item sold by the low-type seller. Therefore, the low-type seller’s incentive to mimic increases as $v$ or $p$ decrease. If $p$ and $v$ are sufficiently small, since the low-type seller’s incentive to mimic is sufficiently large, a separating equilibrium does not exist. This is captured by condition $\frac{v}{c} \geq \nu_1$ in Proposition 3.
and is represented by the left contour in Figure 5.

On the other hand, as $\frac{v}{c}$ increases, the common value matters less, and the high-type seller’s incentive to signal his type (and to separate himself) decreases. When $\frac{v}{c}$ is large enough, we show that the high-type seller chooses to sell via an auction only if $p$ is sufficiently small. This gives us the second condition for existence of this separating equilibrium, namely, $\frac{v}{c} \leq \nu_2$. The condition for uniqueness of the equilibrium, $\frac{v}{c} \leq \nu_3$, follows a similar intuition. Basically, it requires the high-type seller to have a stronger incentive for separation by selling through auction.

6 Conclusion

In this paper we examined important questions for the buyers, sellers and the platform for an online market supporting auctions and posted prices; and we answered questions about optimal behavior for each of them using the well-documented presence of expertise among the bidders as the key underlying assumption. In particular, we studied the impact of the presence of expert bidders in online markets using a simple model of auctions with hard close and posted-prices. Motivated by large number of used items sold in online markets such as eBay.com, we supposed that items have differing levels of “quality” (which we model as common values), and different bidders have different capacities (which we model as expertise) to predict the quality. Bidders with low expertise may be affected by bids earlier in the auction, as these can be interpreted as signals for quality of the item. In our model, sniping emerges as an equilibrium strategy for experts to hide their information about the quality of the item in hard close auctions.

Our results provide several important managerial implications.

- We show that, as a consequence of sniping behavior in equilibrium by the experts in hard close auctions, non-expert buyers with less information have to bid aggressively, i.e. more than their expected value. This result highlights the compensatory behavior
adopted by the large majority of bidders (non-experts) that arises endogenously in these common marketplaces.

- Surprisingly, due to aggressive behavior of non-experts, the platform’s revenue can be higher in hard close auctions (where sniping is prevalent) when compared to soft close auctions (when such sniping is not allowed). This is a new explanation as to why many online auction sites use the hard close rather than the soft close format.

- Another interesting implication of non-experts’ aggressive behavior is that, the platform can benefit in its revenue from committing to hide the information. This result has important managerial implications as it suggests that when buyers are heterogeneous in terms of their information about the value of the item, the linkage principle does not always hold.

- In a platform with the choice of selling in an auction or posted price mechanism, a seller may be able to signal the high quality (or authenticity) of his item to the buyers by selling in an auction and thus separate himself from low quality item sellers as long as there are enough experts (who know the quality of the item) in the market. This provides useful guidance to vendors in such markets, where the magnitude and extent of these decisions can be moderated based on the degree and extent of the presence of expert buyers in the mix.

- In the equilibrium of seller’s choice among selling mechanisms, posted prices will be used more and more as the common values (and therefore, the expertise) become less important. Thus, our model provides one plausible explanation for the slow evolution in this direction seen in online markets.

Collectively, our work sheds a comprehensive light on the important differences that arise when knowledgeable or expert buyers are introduced in online marketplaces, and leads to useful guidelines for all participants in such markets.
References


A Appendix

Analyses and Proofs of Section 3

First we define and motivate the tie-breaking rule we use in the proof of Lemma 3.1.
**Tie-breaking rule:** If there is a tie between a low-type bidder and a high-type bidder, then the item goes to the high-type. If the two bidders are of the same type but of different expertise levels, then the item goes to the non-expert. Finally, if the two bidders are of the same type and of the same expertise level, then the winner is determined by a fair coin toss.

There are three interesting cases for which the tie-breaking rule has an effect in the equilibria described in the lemma. The first is when a high non-expert faces a low expert who knows that $C = c$, and they both bid $c$. In that case, the high non-expert is willing to bid above $c$ to break the tie and take the item, because his valuation is higher than $c$, but the low expert cannot do the same since his valuation is $c$. Therefore, the tie-breaking rule favors the bidder who would win anyway in a realistic situation (where the set of allowable bids is discretized and bidders were allowed to reveal their preferences about slight deviations based on their value).

The second case is when a high expert who knows that $C = 0$ faces a low non-expert, and they both bid $v$. In that case, the low non-expert does not want to win the item, because his valuation is below $v$. Therefore, he has an incentive to bid a bit below $v$, while the high expert does not want to do the same since his valuation is $v$. Thus, again the tie-breaking rule favors the bidder who would win anyway (with a discretized set of bids).

The third case is when a high expert faces a high non-expert, they both bid $cq + v$ in the first stage, the second bid of the high expert doesn’t go through so his highest recorded bid is $cq + v$, and the high non-expert does not make a second bid. In this case, the high expert actually makes a higher bid in the second stage but he is unlucky because his bid did not go through. Also, the high non-expert decides to not make a second bid. If the tie-breaking rule was not favoring the non-expert, then he would be willing to bid a bit above $cq + v$ in the second stage to win the item, while the high expert would not be able to do anything since his second bid did not go through. Therefore, once again the tie-breaking rule favors the bidder who would win with discretized sets of bids that let bidders reveal their deviation preferences based on their values.
Since in our model the set of bids is continuous and not discrete, we use the tie-breaking rule above to avoid unnecessary complications in case of a tie in the proof. However, as it is clear from the cases described above, the rule is justified by the fact that it favors the bidder whose private value is larger and thus more realistic than breaking ties at random.

**Proof of Lemma 3.1**

*Proof.* We will group the nine equilibria into four cases. These are $v < \min\{M_1, M_2\}$ (equilibria 1, 2, 3, and 4), $M_2 \leq v < M_1$ (equilibria 5 and 6), $\max\{M_2, M_1\} \leq v$ (equilibrium 9), and $M_1 \leq v < M_2$ (equilibria 7 and 8).

**First, assume that** $v < \min\{M_1, M_2\}$. This means that $cq + v < c$ and $v < cq$. Consider the following general set of strategies:

- **High Expert:** If $C = 0$, bids $v$ in the first stage and does nothing in the second stage. If $C = c$, bids $cq + v$ in the first stage and bids $c + v$ in the second stage.

- **Low Expert:** If $C = 0$, does nothing. If $C = c$, he bids $cq + v$ in the first stage and $c$ in the second stage.

- **High Non-Expert:** Bids $cq + v$ in the first stage. If he sees a bid other than $0, v, cq$, or $cq + v$ in the first stage, he bids $c + v$ in the second stage. Otherwise, he bids $c$ in the second stage with probability $1 - a$.

- **Low Non-Expert:** Bids $cq$ in the first stage with probability $b$ and $v$ with probability $1 - b$. If his bid was $v$, he bids $cq$ in the second stage with probability $1 - g$.

The probabilities $a, b, g$ are not determined yet. For now, we just assume that $a > 0$. We will examine if anyone has incentive to change strategy and at the same time try to determine the probabilities and the conditions for which the above is an equilibrium. These conditions will give us the proof that equilibria 1 and 3 are correct. Later, we will relax the assumption on $a$ and examine what happens when $a = 0$; this will lead us to conditions that
equilibria 2 and 4 are correct, and will conclude the proof of the four equilibria in the first case.

1) High Expert with $C = 0$: His valuation is $v$ and now he bids $v$ in the first stage. If he does nothing in the first stage and he bids $v$ in the second stage, then there is some probability that his bid will not go through with a payoff of 0, and in the case it goes through, his payoff would be the same in all cases as if he had bid $v$ in the first stage (against a low non-expert, his payoff is 0 in both cases). Therefore, it is optimal for him to follow this strategy.

2) High Expert with $C = c$: His valuation is $c + v$ and now he bids $cq + v$ in the first stage and bids $c + v$ in the second stage. We consider three alternative strategies which dominate all the rest, and we prove that he doesn’t have any incentive to deviate to one of them.

One strategy is to bid $v$ in the first stage and $c + v$ in the second stage. This strategy has a different result for him only if his bid in the second stage doesn’t go through. In that case, by having a bid of $v$ instead of a bid of $cq + v$ can only decrease his payoff.

Another strategy is to bid 0 in the first stage and $c + v$ in the second. The behavior of the rest of the bidders will not change, but his payoff will decrease because he can lose in some cases whereas the bid of $cq + v$ would give him a positive payoff.

The last strategy is to bid $c + v$ (or $c$) in the first stage and nothing (or $c + v$) in the second. However, if we assume that his bid will go through in the second stage, with the alternative strategy the result would be the same in all cases except in the case he faces a high non-expert, where his payoff strictly decreases. Therefore, since $\delta$ is sufficiently small, it is better to bid in the second stage.

3) Low Expert with $C = 0$: His value is 0 and he does nothing, which is optimal for him.

4) Low Expert with $C = c$: His value is $c$. The payoff if he bids $c$ in the first stage is

\[ A(\delta) = pr((1 - \delta)0 + \delta(c - (cq + v))) + \]
\[ + p(1 - r)((1 - \delta)0 + \delta(c - (cq + v))) + \]
\[ \text{opponent is high expert} \]
\[ \text{opponent is low expert} \]

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The payoff with the current strategy is

\[ B(\delta) = \begin{cases} 
(1 - \delta) \cdot \left[ pr((1 - \delta)0 + \delta(c - (cq + v))) + \\
+ p(1 - r)((1 - \delta)0 + \delta(c - (cq + v))) + \\
+ (1 - p)(1 - r)(b(c - cq) + (1 - b)(c - v) + (1 - g)(1 - \delta)(c - cq) + (1 - g)\delta(c - v)) \right] + \\
\end{cases} \]

It holds that \( B(0) - A(0) = (1 - p)r(a(c - (cq + v))) > 0 \) (for \( a > 0 \)), which means that for sufficiently small \( \delta \), \( B(\delta) > A(\delta) \), i.e. the current strategy is better.

The alternative is to bid something else in the first stage other than \( cq + v \) or \( c \), and \( c \) in the second, but this doesn’t increase their payoff.

5) High Non-Expert: His expected valuation is \( cq + v \). Bidding something else other than \( cq + v \) in the first stage will not change the bidding behavior of the opponent to something better for him, therefore he prefers to bid \( cq + v \) in the first stage rather than wait.

In the second stage, it doesn’t matter what they do if they see a bid of 0 or \( v \) or \( cq \), since the result cannot change. If they see a bid other than 0, \( v \), \( cq \), or \( cq + v \) (like \( c \) or \( c + v \), something that doesn’t happen in the equilibrium, they assume that the common value is high which means that their valuation is \( c + v \), so they bid \( c + v \). The reason is that the only
one who might have incentive to deviate from the current strategies is an expert with $C = c$ who tries to bluff in some way to hide the common value.

If they see a bid of $cq + v$, then they know that their opponent is a high expert with $C = c$, or a low expert with $C = c$, or a high non-expert. Their payoff by doing nothing in the second stage is

$$A_2 = \frac{prq}{prq + p(1-r)q + (1-p)r} \left( (1-\delta)0 + \delta(c + v - (cq + v)) + \frac{p(1-r)q}{prq + p(1-r)q + (1-p)r} \left( (1-\delta)v + \delta(c + v - (cq + v)) \right) + \frac{(1-p)r}{prq + p(1-r)q + (1-p)r} \left( a0 + (1-a)(1-\delta)0 + (1-a)\delta0 \right) \right)$$

while their payoff by bidding $c$ is

$$B_2 = \frac{prq}{prq + p(1-r)q + (1-p)r} \left( (1-\delta)0 + \delta(c + v - (cq + v)) + \frac{p(1-r)q}{prq + p(1-r)q + (1-p)r} \left( (1-\delta)v + \delta(c + v - (cq + v)) \right) + \frac{(1-p)r}{prq + p(1-r)q + (1-p)r} \left( a0 + (1-a)(1-\delta)0 + (1-a)\delta0 \right) \right) + \delta \cdot A_2.$$ 

By bidding $c + \epsilon$ for some small $\epsilon > 0$, his payoff can only decrease. By bidding $c - \epsilon$, the payoff is the same as if they stay with the bid of $cq + v$ (according to the tie-breaking rule, if two bidders are both high, the non-expert wins). It holds that

$$B_2 - A_2 = (1-\delta) \left[ \frac{p(1-r)q}{prq + p(1-r)q + (1-p)r} (1-\delta)v + \frac{(1-p)r}{prq + p(1-r)q + (1-p)r} (1-a)(1-\delta)\frac{cq + v - c}{2} \right].$$

and we want this to be equal to 0 to permit mixing these strategies, which will give us an
expression for the mixing probability \( a \). This is
\[
a = 1 - \frac{2p(1-r)qv}{(1-p)r(c-(cq+v))}.
\]
This is always \( \leq 1 \). We assumed also that \( a > 0 \), which is equivalent to
\[
v < \frac{c(1-p)(1-q)}{2p(1-r)(q+(1-p)r)} = m_1.
\]
Therefore, we need this condition to have an equilibrium in this case.

If \( 1 - \frac{2p(1-r)qv}{(1-p)r(c-(cq+v))} \leq 0 \), which is equivalent to \( v \geq m_1 \) and corresponds to \( a = 0 \), we need a different set of strategies and we consider this case later.

6) Low Non-Expert: His expected valuation is \( cq \). His payoff if he bids \( cq \) in the first stage is
\[
A_3 = p r (q0 + (1 - q)(-v)) + \begin{cases} 
\text{opponent is high expert} & \text{pr}(q0 + (1 - q)(-v)) + \\
\text{opponent is low expert} & + p(1 - r)(q0 + (1 - q)0) + \\
\text{opponent is high non-expert} & + (1 - p)r(0) + \\
\text{opponent is low non-expert} & + (1 - p)(1 - r)(b0 + (1 - b)(g(cq - v) + (1 - g)(1 - \delta)0 + (1 - g)\delta(cq - v))) 
\end{cases}
\]
His payoff if he bids \( v \) in the first stage and follows the current strategy in the second stage is
\[
B_3 = p r (q0 + (1 - q)(g0 + (1 - q)(1 - \delta)(-v) + (1 - g)\delta0)) + \begin{cases} 
\text{opponent is high expert} & pr(q0 + (1 - q)(g0 + (1 - q)(1 - \delta)(-v) + (1 - g)\delta0)) + \\
\text{opponent is low expert} & + p(1 - r)(q0 + (1 - q)0) + \\
\text{opponent is high non-expert} & + (1 - p)r(0) + \\
\text{opponent is low non-expert} & + (1 - p)(1 - r) \left[b0 + (1 - b)(g^2 \frac{cq - v}{2}) + (1 - g)g((1 - \delta)(cq - v) + \delta \frac{cq - v}{2}) + \\
\text{opponent is low non-expert} & + g(1 - g)((1 - \delta)0 + \delta \frac{cq - v}{2}) + (1 - g)^2((1 - \delta)^20 + (1 - \delta)\delta(cq - v) + \delta(1 - \delta)0 + \delta^2 \frac{cq - v}{2}) \right].
\end{cases}
\]
Now, in the second stage, if a low non-expert with a bid of \( v \) sees any bid other than \( v \) from
the opponent, bidding \( cq \) or nothing in the second stage doesn’t affect his payoff. If he sees a bid of \( v \), then he knows that the opponent is either a high expert with \( C = 0 \) or a low non-expert. If he does nothing in the second stage, his payoff is

\[
A_4 = \frac{pr(1 - q)}{pr(1 - q) + (1 - p)(1 - r)(1 - b)}(0) + \frac{(1 - p)(1 - r)(1 - b)}{pr(1 - q) + (1 - p)(1 - r)(1 - b)}(g \frac{cq - v}{2} + (1 - g)(1 - \delta)0 + (1 - g)\delta \frac{cq - v}{2}),
\]

while if he bids \( cq \), the payoff is

\[
B_4 = (1 - \delta) \left[ \frac{pr(1 - q)}{pr(1 - q) + (1 - p)(1 - r)(1 - b)}(-v) + \frac{(1 - p)(1 - r)(1 - b)}{pr(1 - q) + (1 - p)(1 - r)(1 - b)}(g(cq - v) + (1 - g)(1 - \delta)0 + (1 - g)\delta(cq - v)) \right] + \delta A_4.
\]

It must hold that \( A_4 = B_4 \) to permit mixing these strategies, from which we get an expression for the mixing probability \( g \) which is

\[
g = \frac{2pr(1 - q)v}{(1 - p)(1 - r)(1 - b)(cq - v)} - \delta.
\]

This expression is non-negative for sufficiently small \( \delta \) and it is < 1 iff

\[
v < \frac{c(1 - p)(1 - r)(1 - b)q}{2pr(1 - q) + (1 - p)(1 - r)(1 - b)}.
\]

For \( b = 0 \) and the corresponding \( g \), we get \( A_3 \leq B_3 \) (for \( g < 1 \)), therefore the current strategy of the low expert is optimal and we get an equilibrium. For this reason, we set
\[ b = 0. \] The above condition then becomes

\[ v < \frac{c(1 - p)(1 - r)q}{2p(1 - q) + (1 - p)(1 - r)} = m_2. \]

If \( v \geq m_2 \), then we set \( g = 1 \) (which corresponds to strategy \( u^{LNE} \)).

This ends the proof for equilibria 1 and 3.

When \( a = 0 \), the strategy for the low expert we considered above is not always optimal. This happens when \( v \geq m_1 \). More specifically, since he knows that the high non-expert will bid \( c \) in the second stage for sure, he has no reason to wait until the second stage to bid, and bids \( c \) from the first stage. With the same logic, since a high non-expert knows for sure that he will bid \( c \) in the second stage, it is even better to bid \( c \) from the first stage. Moreover, when a high non-expert sees a bid of \( c \) in the first stage, he doesn’t know for sure what the opponent is, so he doesn’t increase his bid. This will change also the strategy for the high expert with \( C = c \). In the first stage, he prefers to bid \( c \) instead of \( cq + v \), because a bid of \( cq + v \) would reveal that he is a high expert and \( C = c \). So, the equilibrium when \( v \geq \frac{c(1 - p)r(1 - q)}{2p(1 - q)r + (1 - p)r} = m_1 \) is as follows.

- **High Expert:** If \( C = 0 \), bids \( v \) in the first stage and does nothing in the second stage.
  If \( C = c \), bids \( c \) in the first stage and bids \( c + v \) in the second stage.

- **Low Expert:** If \( C = 0 \), does nothing. If \( C = c \), he bids \( c \) in the first stage and nothing in the second stage.

- **High Non-Expert:** Bids \( c \) in the first stage. If he sees a bid other than \( 0, v, cq \) or \( c \) in the first stage, he bids \( c + v \) in the second stage. Otherwise, he does nothing in the second stage.

- **Low Non-Expert:** Bids \( v \) in the first stage. He bids \( cq \) in the second stage with probability \( 1 - g \).

The proofs for the high expert, the low expert and the low non-expert are the same.
We need to check if the high non-expert has any reason to change strategy. An alternative strategy for him would be the one he had before, i.e. to bid $cq + v$ in the first stage and $c$ in the second with some probability. So, suppose that he had bidden $cq + v$ in the first stage and he sees a bid of $c$. His payoff by doing nothing in the second is $0$, while the payoff to bid $c$ in the second stage is

\[
B' = \frac{(1 - \delta)}{\text{bid goes through}} \cdot \left[ \frac{prq}{prq + p(1-r)q + (1-p)r}((1-\delta)0 + \delta(c + v - (c))) + \right. \\
+ \frac{p(1-r)q}{prq + p(1-r)q + (1-p)r}(c + v - c) + \\
+ \frac{(1-p)r}{prq + p(1-r)q + (1-p)r} \left( \frac{cq + v - c}{2} \right) + \\
+ \delta \cdot 0.
\]

This is $\geq 0$ for $v \geq \frac{c(1-p)r(1-q)}{2prq\delta + 2p(1-r)q + (1-p)r}$, which is true since

\[
v > \frac{c(1-p)r(1-q)}{2p(1-r)q + (1-p)r} \geq \frac{c(1-p)r(1-q)}{2prq\delta + 2p(1-r)q + (1-p)r}.
\]

Therefore, he is better off by bidding $c$ rather than $0$ in the second stage. This means that by bidding in the first stage he can increase his payoff. All other possible strategies are trivially dominated by those we considered above.

This ends the proof for equilibria 2 and 4.

Summarising the first case, when $a > 0$ (i.e. $v < m_1$) and $g < 1$ (i.e. $v < m_2$), we get the equilibrium $(s_1^{HE}, s^{LE}, m^{HNE}, m^{LNE})$, when $a = 0$ (i.e. $v \geq m_1$) and $g < 1$ (i.e. $v < m_2$), we get the equilibrium $(s_1^{HE}, t^{LE}, a^{HNE}, m^{LNE})$, when $a > 0$ (i.e. $v < m_1$) and $g = 1$ (i.e. $v \geq m_2$), we get the equilibrium $(s_2^{HE}, t^{LE}, a^{HNE}, u^{LNE})$, and when $a = 0$ (i.e. $v \geq m_1$) and $g = 1$ (i.e. $v \geq m_2$), we get the equilibrium $(s_2^{HE}, t^{LE}, a^{HNE}, u^{LNE})$.

Assume now that $M_2 \leq v < M_1$. This means that $cq \leq v$ and $cq + v < c$. Consider the
following set of strategies:

- High Expert: If $C = 0$, bids $v$ in the first stage and does nothing in the second stage. If $C = c$, bids $cq + v$ in the first stage and bids $c + v$ in the second stage.

- Low Expert: If $C = 0$, does nothing. If $C = c$, he bids $cq + v$ in the first stage and $c$ in the second stage.

- High Non-Expert: Bids $cq + v$ in the first stage. If he sees a bid other than $0, v, cq, or cq + v$ in the first stage, he bids $c + v$ in the second stage. Otherwise, he bids $c$ in the second stage with probability $1 - a$.

- Low Non-Expert: Bids $cq$ in the first stage and nothing in the second.

We now investigate if anyone has incentive to change strategy. For $a > 0$, the arguments for all types of bidders are the same as in the previous case except for the low non-expert.

The expected valuation of a low non-expert is $cq$. Now he bids $cq$ in the first stage and his expected payoff is $0$. The only way to get the item is only if he faces another low non-expert, in which case they both bid $cq$ and there is a tie. But even in this case he has to pay $cq$, so his payoff is $0$. He cannot achieve a better payoff, since it is never optimal to bid something above his expected valuation.

This ends the proof for equilibrium 5.

Similarly as in the previous case, the equilibrium when $v \geq \frac{c(1-p)r(1-q)}{2p(1-r)q+(1-p)r} = m_1$ (which means $a = 0$) is

- High Expert: If $C = 0$, bids $v$ in the first stage and does nothing in the second stage. If $C = c$, bids $c$ in the first stage and bids $c + v$ in the second stage.

- Low Expert: If $C = 0$, does nothing. If $C = c$, he bids $c$ in the first stage and nothing in the second stage.
• High Non-Expert: Bids $c$ in the first stage. If he sees a bid other than $0, v, cq, c$ in the first stage, he bids $c + v$ in the second stage. Otherwise, he does nothing in the second stage.

• Low Non-Expert: Bids $cq$ in the first stage and nothing in the second.

This ends the proof for equilibrium 6.

Summarising the second case, when $a > 0$ (i.e. $v < m_1$), we get the equilibrium $(s_{1HE}, s^{LE}, m_{HNE}, t^{LNE})$, and when $a = 0$ (i.e. $v \geq m_1$), we get the equilibrium $(s_{2HE}, t^{LE}, a^{HNE}, t^{LNE})$.

Next, suppose that \( \max\{M_2, M_1\} \leq v \). This means that \( \max\{cq, c(1 - q)\} \leq v \). We consider the following set of strategies:

• High Expert: If $C = 0$, bids $v$ in the first stage and does nothing in the second stage. If $C = c$, bids $cq + v$ in the first stage and bids $c + v$ in the second stage.

• Low Expert: If $C = 0$, does nothing. If $C = c$, he bids $c$ in the first stage and nothing in the second stage.

• High Non-Expert: Bids $cq + v$ in the first stage. If he sees a bid other than $0, v, cq, c, cq + v$ in the first stage, he bids $c + v$ in the second stage.

• Low Non-Expert: Bids $cq$ in the first stage and nothing in the second.

This is the simplest case. Both high and low non-experts have nothing to lose by bidding their expected valuation, therefore they do so from the first stage. The low non-expert has no reason to hide his identity, therefore he bids his valuation from the first stage. The same is true for a high expert with $C = 0$. Finally, the high expert with $C = c$ bids the highest possible he can in the first stage without revealing that he is a high expert, which is a bid of $cq + v$, and then he bids $c + v$ in the second stage. If he bids $c + v$ from the first stage, then his payoff strictly decreases because of the possibility that the opponent is a high non-expert.

This ends the proof for equilibrium 9.
Summarising the third case, we get the equilibrium \((s_1^{HE}, t^{LE}, t^{HNE}, t^{LNE})\).

Finally, suppose that \(M_1 \leq v < M_2\). This means that \(c(1 - q) \leq v < cq\). We consider two cases:

- If \(v < \frac{c(1-p)(1-r)q}{2pr(1-q)+(1-p)(1-r)} = m_2\), the following is an equilibrium
  - High Expert: If \(C = 0\), bids \(v\) in the first stage and does nothing in the second stage. If \(C = c\), bids \(cq + v\) in the first stage and bids \(c + v\) in the second stage.
  - Low Expert: If \(C = 0\), does nothing. If \(C = c\), he bids \(c\) in the first stage and nothing in the second stage.
  - High Non-Expert: Bids \(cq + v\) in the first stage. If he sees a bid other than \(0, v, cq, c\), or \(cq + v\) in the first stage, he bids \(c + v\) in the second stage.
  - Low Non-Expert: Bids \(v\) in the first stage. He bids \(cq\) in the second stage with probability \(1 - g\), where \(g = \frac{2pr(1-q)v}{(1-p)(1-r)(cq-v)-\delta}\).

- If \(v \geq \frac{c(1-p)(1-r)q}{2pr(1-q)+(1-p)(1-r)} = m_2\), the following is an equilibrium
  - High Expert: If \(C = 0\), bids \(v\) in the first stage and does nothing in the second stage. If \(C = c\), bids \(cq + v\) in the first stage and bids \(c + v\) in the second stage.
  - Low Expert: If \(C = 0\), does nothing. If \(C = c\), he bids \(c\) in the first stage and nothing in the second stage.
  - High Non-Expert: Bids \(cq + v\) in the first stage. If he sees a bid other than \(0, v, cq, c\), or \(cq + v\) in the first stage, he bids \(c + v\) in the second stage.
  - Low Non-Expert: Bids \(v\) in the first stage and nothing in the second.

For the experts and the high non-expert, the proofs are similar to the previous case. For the low non-expert, the proof is similar to the second case.

This ends the proof for equilibria 7 and 8.
Summarising the fourth case, when \( g < 1 \) (i.e. \( v < m_2 \)), we get the equilibrium \((s_1^{HE}, t^{LE}, t^{HNE}, m^{LNE})\), and when \( g = 1 \) (i.e. \( v \geq m_2 \)), we get the equilibrium \((s_1^{HE}, t^{LE}, t^{HNE}, u^{LNE})\).

**Proof of Proposition 1**

*Proof.* This result comes directly from Lemma 3.1. We can see that when \( m_1 \leq v < M_1 \), non-experts overbid all the time, and when \( v < m_1 \), they overbid with some probability. We saw in the proof of Lemma 3.1 that the probability of over-bidding is

\[
1 - a = \frac{2p(1-r)q_v}{(1-p)r(c-(cq+v))}.
\]

It is easy to see that this is an increasing function on \( p \).

**Analyses and Proofs of Section 4**

*Proof of Lemma 4.1*

*Proof.* With soft close, an expert is going to bid his true valuation at some point, because anything less than the true valuation will result in a lower payoff. If there is a non-expert opponent he is going to respond to that, therefore the expert may as well bid truthfully from the first stage. More specifically, the strategies for the experts will be as follows

- **High Expert:** If \( C = 0 \), bids \( v \) in the first stage and nothing later. If \( C = c \), bids \( c + v \) in the first stage and nothing later (strategy \( t'_{HE} \)).

- **Low Expert:** If \( C = 0 \), does nothing. If \( C = c \), he bids \( c \) in the first stage and nothing later (strategy \( t'_{LE} \)).

For the high non-expert, the strategy is simple as well. He will bid his expected valuation in the first stage, which is \( cq + v \). If the opponent bids \( c \) or \( c + v \) in the first stage (or at some later point), he will understand that he is an expert and that \( C = c \), therefore if \( cq + v < c \) he will bid \( c \) in the next stage (the minimum possible bid that maximizes his payoff). This is strategy \( t'_{HNE} \).
If \( cq \leq v \) (i.e. \( v \geq M_2 \)), then a low non-expert will bid his expected valuation in the first stage, which is \( cq \), and then he will not do anything (strategy \( t^{LNE} \)). Because, even if for example sees a bid of \( c \) and realize that the common value is high, by bidding \( c \) and winning the item, his payoff is still 0.

If \( v < cq \) (i.e. \( v < M_2 \)), then a low non-expert doesn’t want to bid \( cq \) from the beginning because if the opponent is a high expert and \( C = 0 \), he will end up with negative payoff. So, what he does is that he bids \( v \) in the first stage, i.e. the maximum he can without the risk above, and waits. If he sees a bid other than \( v \) from the opponent, he will lose anyway, so it doesn’t matter what strategy he will follow next, and we assume he will follow the same strategy as if he sees a bid of \( v \). If he sees a bid of \( v \), then he bids \( cq \) in the second stage with probability \( 1 - w \). No matter what happens in the second stage, he does nothing in the third stage. We need now to calculate the probability \( w \).

First of all, if he does nothing in the second stage and he sees a bid of \( cq \), he realizes that the opponent is another low non-expert, but there is no reason to bid something higher because his expected payoff will be 0. If the opponent doesn’t bid as well, then the auction ends, and there is no third stage. Therefore, his payoff if he sees a bid of \( v \) in the first stage and he does nothing in the second, is

\[
\frac{pr(1-q)}{pr(1-q) + (1-p)(1-r)} (0) + \frac{(1-p)(1-r)}{pr(1-q) + (1-p)(1-r)} \left( w \frac{cq-v}{2} + (1-w)0 \right).
\]

If he bids \( cq \) in the second stage, his payoff is

\[
\frac{pr(1-q)}{pr(1-q) + (1-p)(1-r)} (-v) + \frac{(1-p)(1-r)}{pr(1-q) + (1-p)(1-r)} \left( w(cq-v) + (1-w)0 \right).
\]

We need these two expressions to be equal, from which we get

\[
w = \frac{2pr(1-q)v}{(1-p)(1-r)(cq-v)}.
\]
This is always non-negative, and it is $< 1$ iff

$$v < \frac{c(1-p)(1-r)q}{2pr(1-q) + (1-p)(1-r)} = m_2.$$ 

Therefore, if $v < m_2$, the low non-expert follows the strategy $m_{LNE}$.

If $v \geq \frac{c(1-p)(1-r)q}{2pr(1-q) + (1-p)(1-r)} = m_2$ (and $v < M_2$), then it is sub-optimal to bid $cq$, therefore we set $w = 1$ (strategy $u_{LNE}$).

**Proof of Lemma 4.2**

**Proof.** For a low seller, hard close is always better, because the bid of every bidder is greater than or equal to his bid when there is soft close.

For a high seller, we know from Proposition 1 that as $p$ increases, high non-experts bid more and more aggressively. This makes the revenue go higher as $p$ increases, when there is hard close. Therefore, to show the result, it is enough to show that for $p \approx 1$ and $\delta = 0$ the revenue with hard close is better than the revenue with soft close. Then, this will mean that there is an interval $[k, 1]$ for $p$ and an interval $[0, h(p)]$ for $\delta$, for which hard close is better.

When $p \approx 1$ and $\delta = 0$, it holds that $m_1 \approx m_2 \approx 0$, therefore there are only four relevant equilibria in Lemma 3.1 (cases 4, 6, 8, and 9) and two in Lemma 4.1 (cases 2 and 3). Case 9 from Lemma 3.1 corresponds to Case 3 from Lemma 4.1, and we can see that in both these equilibria all bidders bid exactly the same, therefore the expected revenue will be the same. Cases 4, 6, 8 from Lemma 3.1 correspond to Case 2 from Lemma 4.1 and we can see that all bids are the same in both models except the bids of the high non-expert, which are sometimes higher with hard close (cases 4 and 6). Therefore, overall the expected revenue is higher for a high seller when there is a hard close.

This is also illustrated in Figure 6, which shows which policy gives higher revenue to the high seller in different regions of the parameter space. Notice that this is slightly different from Figure 4, which refers to platform’s revenue.
Figure 6: The regions show whether hard close provides higher revenue for a high seller (for $r = 0.5$ and $q = 0.1$).

**Proof of Proposition 2**

*Proof.* This result follows directly from Lemma 4.2. Assume that $\delta$ is sufficiently small. Since a low seller always benefits from hard close, and a high seller benefits for large $p$, the expected platform’s revenue is better with hard close for sufficiently large $p$. □

**Proof of Corollary 1**

*Proof.* When the platform reveals the common value to everyone, all bidders bid their true valuation. Therefore, in the region in which the aggressive bidding of high non-experts makes hard close better than soft close for the platform (the middle region in Figure 4), the platform prefers to hide the common value so that the high non-experts keep bidding higher than their true valuation. □
Analyses and Proofs of Section 5

We use the following notation to explain the results of this section: Let $\pi^B_T(M)$, where $T \in \{L, H\}$ and $B \in \{L, H, X\}$ denote the expected profit of a seller who uses mechanism $M \in \{A, (B, z)\}$ (where $A$ denotes auction, and $(B, z)$ denotes posted price where the price is $z$), has type $T$, and non-experts believe has type $B$. Let $M^\text{pool}$ be the mechanism that both types of sellers use in a pooling equilibrium. Let $M^{H,\text{sep}}$ and $M^{L,\text{sep}}$ be the mechanisms that high-type and low-type sellers use in a separating equilibrium, respectively. Finally, let $\Pi^\text{pool}_T$ and $\Pi^{\text{sep}}_T$, where $T \in \{L, H\}$, be the pooling equilibrium and separating equilibrium profits of a seller with type $T$, respectively. By definition, we have

$$\Pi^\text{pool}_L = \pi^X_L(M^\text{pool})$$
$$\Pi^\text{pool}_H = \pi^X_H(M^\text{pool})$$
$$\Pi^{\text{sep}}_L = \pi^L_L(M^{L,\text{sep}})$$
$$\Pi^{\text{sep}}_H = \pi^H_H(M^{H,\text{sep}}).$$

The revenue of a high- or low-type seller in an auction, where non-experts have belief high or low, is given in the following formulas. Recall that $p$ is the probability of being expert, and $r$ is the probability of having high value.

$$\pi^H_H(A) = c + r^2 v$$

$$\pi^L_L(A) = r^2 v$$

$$\pi^L_H(A) = \begin{cases} 
  cp^2 + rv(2(1-p)p(1-r) + r) & \text{if } v \leq c \\
  cp(2(1-p)r + p - 2(1-p)r^2) + r^2 v & \text{if } v > c.
\end{cases}$$
\[ \pi^H_L(A) = \begin{cases} c(1-p)^2 + rv(2(1-p)p(1-r) + r) & \text{if } v \leq c \\ c(1-p)(p(2(1-r)r - 1) + 1) + r^2v & \text{if } v > c. \end{cases} \]

Similarly, the revenue of a high- or low-type seller using posted price with price \( z \), in each of the four cases, is

\[ \pi^H_H(B, z) = \begin{cases} z & \text{if } z \leq c \\ (2r - r^2)z & \text{if } c < z \leq c + v \\ 0 & \text{otherwise} \end{cases} \]

\[ \pi^L_H(B, z) = \begin{cases} (1 - (1 - p)^2(1 - r)^2)z & \text{if } v \leq c \text{ and } z \leq v \\ (2p - p^2)z & \text{if } v \leq c \text{ and } v < z \leq c \\ (2pr - p^2r^2)z & \text{if } v \leq c \text{ and } c < z \leq c + v \\ (1 - (1 - p)^2(1 - r)^2)z & \text{if } v > c \text{ and } z \leq c \\ (2r - r^2)z & \text{if } v > c \text{ and } c < z \leq v \\ (2pr - p^2r^2)z & \text{if } v > c \text{ and } v < z \leq c + v \\ 0 & \text{otherwise} \end{cases} \]

\[ \pi^H_L(B, z) = \begin{cases} (1 - p^2(1 - r)^2)z & \text{if } v \leq c \text{ and } z \leq v \\ (2(1-p) - (1-p)^2)z & \text{if } v \leq c \text{ and } v < z \leq c \\ (2r(1-p) - r^2(1-p)^2)z & \text{if } v \leq c \text{ and } c < z \leq c + v \\ (1 - p^2(1 - r)^2)z & \text{if } v > c \text{ and } z \leq c \\ (2r - r^2)z & \text{if } v > c \text{ and } c < z \leq v \\ (2r(1-p) - r^2(1-p)^2)z & \text{if } v > c \text{ and } v < z \leq c + v \\ 0 & \text{otherwise} \end{cases} \]
\[ \pi^L_L(B, z) = \begin{cases} (2r - r^2)z & \text{if } z \leq v \\ 0 & \text{otherwise} \end{cases} \]

**Proof of Proposition 3**

*Proof.* We prove the proposition in three parts. In part A, we show that there is no separating equilibrium in which the high-type seller uses posted price and the low-type seller uses auction. In part B, we show that when \( v \in [\nu_1, \nu_2] \), there exists a separating equilibrium in which the high-type seller uses auction and the low-type seller uses posted price \( v \). Finally, in part C, we show that for \( v \in (\nu_1, \nu_3) \), this is the only equilibrium that survives Intuitive Criterion refinement.

**Part A:** Note that \( \pi^L_L(A) < \pi^L_L(B, v) \) which means that conditioned on the type of sellers being revealed, the low-type seller always prefers posted price \( v \) to auction. Therefore, the low-type seller never uses an auction in a separating equilibrium.

**Part B:** Note that for a separating equilibrium in which the high-type uses auction and the low-type uses posted price to exist, the following two conditions are necessary and sufficient:

\[ \pi^L_L(B, z) \leq \pi^H_L(A) \]
\[ \pi^H_L(A) \geq \pi^L_H(B, z) \]

The first condition guarantees that the low-type seller cannot benefit from deviating and the second condition guarantees that the high-type seller cannot benefit from deviating. \( \pi^L_L(B, z) \) is optimized at \( z = v \), and is equal to \( (2r - r^2)v \). Having this less than or equal to \( \pi^H_L(A) \), and using basic calculus, gives us the condition \( \frac{v}{c} \geq \nu_1 \). Similarly, solving the second inequality for \( v \) gives us condition \( \frac{v}{c} \leq \nu_2 \). If non-expert buyers’ beliefs are \( L \) for posted prices and \( H \) for auction, then \( \nu_1 \leq \frac{v}{c} \leq \nu_2 \) is also sufficient for existence of this equilibrium.

**Part C:** Finally, we show that if \( \nu_1 \leq \frac{v}{c} \leq \nu_3 \), the separating equilibrium in which the high-type uses auction and the low-type uses posted price is the only pure strategy
separating Nash equilibrium that survives Intuitive Criterion refinement. Assume for sake of
contradiction that there exists another separating equilibrium. We already know from Part
A of this proof that the low-type cannot be using auction. Therefore, both types must be
using posted price (with different prices) in this equilibrium. Using the same argument as in
Part B of the proof, we know that the low-type must be using posted price \(v\). Suppose that
the high-type is using posted price \(\zeta\). For this to be a separating equilibrium, the low-type
should not benefit from deviating and mimicking the high-type: \(\pi_L(B,v) \geq \pi_L(B,\zeta)\). Using
basic calculus, we can show that this implies the following condition on \(\zeta\). We must have
\[\zeta \leq \frac{(r-2)v}{(p-1)(pr-r+2)}\]. Let \(\pi^* = \pi_H(B,\zeta)\) be the profit of the high-type seller (in the hypothetical
separating equilibrium) subject to this constraint.

If \(\pi_H(A) > \pi^*\), then the high-type seller benefits from deviating to auction unless non-
exerts’ belief about auction is \(L\). But note that if \(\frac{v}{c} > \nu_1\), non-exerts’ belief about
auction cannot be \(L\) according to Intuitive Criterion refinement. Specifically, since the
high-type benefits from deviating to auction and the low-type does never benefits from
deviating to auction even if buyers’ belief in auction is \(H\), according to Intuitive Criterion
refinement, buyers’ belief in auction should be \(H\). Therefore, if \(\pi_H(A) > \pi^*\) the high-type
benefits from deviating to auction and the hypothetical equilibrium cannot exist. Using
basic calculus, condition \(\pi_H(A) > \pi^*\) reduces to \(\frac{v}{c} \leq \nu_3\). Therefore, for \(\frac{v}{c} \in (\nu_1, \nu_3)\), the
separating equilibrium in which the high-type uses auction and the low-type uses posted
price is the only pure strategy separating Nash equilibrium that survives Intuitive Criterion
refinement. \(\square\)