

Information acquisition and mutual funds*

Diego García

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Abstract

We generalize the standard competitive rational expectations equilibrium (Hellwig (1980), Verrecchia (1982)) by studying the possibility that informed agents open mutual funds in order to sell their private information. We illustrate how mutual funds endogenously arise in equilibrium and we characterize the fund managers' optimal investment management fees under imperfect competition. In our model the household sector views the mutual funds as a new asset class: investing in a stock through an informed agent's fund is different from investing in the stock directly. The paper further analyzes the incentives to acquire information, and compares these to those that prevail when agents can only trade on their own account. We then study the relationship between the model's economic primitives and the investment management fees, the size of the mutual fund industry, the informativeness of the risky asset price, price volatility and the equilibrium fraction of informed agents.

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1 Introduction

Over the past two decades mutual funds have become one of the most popular investment vehicles for individual investors.¹ While mutual funds have received a great deal of attention in the literature, little has been done to formally explain the existence and the size of the mutual fund industry. We examine this issue by analyzing a rational expectations model of mutual fund formation. Our model captures several important properties of today's mutual fund industry. For example, fund managers in our model get compensated via contingent fees on their funds' values, households hold multiple mutual funds in their portfolios, and there is imperfect competition in the mutual fund sector.² Furthermore, mutual funds in our model represent a new asset class, i.e., investing through a mutual fund whose trading strategy is based upon private information is quite different than owning a stock directly. Lastly, our model is consistent with an economy in which a mutual fund manager's investment strategy outperforms the uninformed household sector's strategy, but net of information acquisition costs both types of agents are indifferent between acquiring and not acquiring information.

Our model includes an ex ante stage in which all agents optimally decide whether or not to acquire costly private information, i.e., we endogenously determine the fraction of informed agents in our economy. After this information acquisition stage, informed agents optimally choose between trading on their own accounts and establishing mutual funds. In the latter case, the informed agents market their optimal investment strategies to the uninformed household sector. Hence, we go beyond the information acquisition decision to study the optimal use of the acquired information. Essentially, our model generalizes Verrecchia (1982) to the case in which there is an endogenous mutual fund sector in the economy. We then use our model to study the relationship between the informativeness of price, the equilibrium fraction of informed agents, price volatility, the size of the mutual fund industry, and the investment management fees.

Our contribution to the literature is threefold. First, we take the fraction of informed agents and the mutual fund fees as exogenous parameters and we solve (in closed-form) for a

¹As reported by the Investment Company Institute, a trade association for the mutual fund industry, 52% of U.S. households owned mutual funds in 2001, up from 5.7% in 1980. At the same time, the number of mutual funds offered by the financial services industry has increased substantially. In fact, there are now more than 12,000 mutual funds including those offered by Fidelity, Vanguard, Putnam, Janus, and Dreyfus.

²Although we allow for an imperfectly competitive mutual fund industry, we assume that the stock market is perfectly competitive. The assumption of a perfectly competitive stock market is an important one – it implies that there are no decreasing returns to scale in our model, as in Berk and Green (2002).

rational expectations equilibrium in which uninformed agents are allowed to invest in mutual funds (Proposition 1). Essentially, this generalizes Hellwig (1980) to the case in which the uninformed “household sector” of the economy is allowed to invest in mutual funds. Second, we endogenize the existence of a mutual fund industry. Given that some agents have acquired private information, we show that the informed agents always have an incentive to establish mutual funds, thereby selling their private information to the uninformed household sector (Proposition 2). This potentially explains the rather large number of mutual funds that we observe in practice. Along these lines, we also explicitly characterize the optimal fees that are charged by the mutual fund managers. Lastly, we characterize the ex ante information acquisition decisions of the agents in our economy (Proposition 3). We then use our model to analyze how investment management fees and information acquisition relate to the economy’s primitives, i.e., to the precision of the manager’s private information, the agents’ risk aversion parameters, the aggregate supply of the risky asset, and the intensity of the competition in the mutual fund sector.

The mutual fund sector is treated as one of oligopolistic competition. We solve for the unique Nash equilibrium in which the managers take their competitors’ fees as given when choosing their own fees. In this equilibrium, the fees that are set by a given manager affect the prices of the (multiple) funds that are held by the household sector. In contrast to most of the oligopoly literature,³ we explicitly specify the household sector’s preferences and allow these agents to purchase products (i.e., mutual funds) from several firms. A fund manager’s pricing decision is therefore not only affected by the other managers’ fees, but also by the household sector’s equilibrium demand for mutual funds.⁴

Many of our results are intuitive. For example, we show that managers with more precise signals will generally charge higher investment management fees, and we show that fees are inversely related to competition.⁵ On the other hand, other results are less straightforward. Average risk aversion and the average signal precision have non-monotonic relationships with both the optimal fees and the fraction of informed agents in our economy. This non-

³For work closely related to the issues of information acquisition in the oligopoly literature see Vives (1988) and Hwang (1993).

⁴In the standard oligopoly game, agents value only one unit of the good in question. In the context of our mutual fund model, this appears to be a strong and undesirable assumption due to the diversification benefit that arises from investing in multiple funds.

⁵As reported by the Investment Company Institute, the average total cost for U.S. equity mutual funds declined from 226 bp in 1980 to 128 bp in 2001. Much of this decline was driven by the explosion in the number of offered funds and the increased level of competition that accompanied the explosion.

monotonicity stems from the dual role that these variables play in our model. The variables directly affect the managers' pricing decisions by impacting the households' marginal willingness to pay, but they also indirectly affect the managers' decision by altering the amount of information that is revealed by price. Lastly, competition in our model plays a similar dual role – a fund's pricing decision directly affects the prices of the other funds that are held by a household, but it also indirectly affects fund prices due to the information that is revealed through price. We also show that if the mutual fund sector is sufficiently competitive, the stock price in an economy with mutual funds will always be more informative than in the standard model (Verrecchia (1982)).

The analysis of mutual funds in the financial economics literature has a very long history.⁶ One area that has received considerable attention is the analysis of optimal contracts for mutual fund managers.⁷ Our research overlaps with this literature since our model offers some novel predictions for how mutual fund fees relate to the various economic primitives (e.g., risk tolerances, signal precisions, etc.). While most of this literature is in partial equilibrium in nature, the papers by Brennan (1993) and Cuoco and Kaniel (2000) study general equilibrium models in which there exists a mutual fund sector. Both papers start with the assumption of the existence of a mutual fund sector with a given incentive scheme (linear contracts with benchmarking in Brennan (1993) and call option contracts in Cuoco and Kaniel (2000)), and study the implications of the existence of these traders for equilibrium asset prices. Neither paper attempts to endogenize the mutual fund sector or to solve for the optimal fees of the fund managers. Using a similar preference structure to our own, Mamaysky and Spiegel (2001) study a general equilibrium setting in which agents are subject to endowment shocks. Mutual funds in their setting serve as a hedging vehicle, whereas in our model the existence of the mutual fund sector is driven instead by the profitability of trading on the basis of private information. Although the two models share several features, it should be noted that we explicitly model the contracts between fund managers and the household sector, thereby generating implications with respect to the funds' optimal fees, which is not the focus of Mamaysky and Spiegel (2001).

⁶The earliest work in this area goes back at least to Brown and Vickers (1963), Horowitz (1966) and Sharpe (1966).

⁷For early work in this area, see Ross (1974), Admati and Ross (1985), Bhattacharya and Pfleiderer (1985), Dybvig and Ross (1985a), Dybvig and Ross (1985b), Dybvig and Spatt (1986). Some recent work in the area includes Stoughton (1993), Brennan and Chordia (1993), Huberman and Kandel (1993), Heinkel and Stoughton (1994), Dow and Gorton (1997), Admati and Pfleiderer (1997), Carpenter (2000), Rajan and Srivastava (2000), Dybvig, Carpenter, and Farnsworth (2000), Das and Sundaram (2002), Christoffersen and Musto (2002), Germain (2003), Palomino and Prat (2003), Ou-Yang (2003).

Our paper appears to be the first to deal with optimal fees and the endogenous creation of mutual funds in a general equilibrium model with asymmetrically informed traders.

The closest paper to ours is perhaps Admati and Pfleiderer (1990), who analyze the indirect sale of information (e.g., through a mutual fund) by a monopolist.⁸ While our paper is related to their work,⁹ there are several important differences in both the modeling technique and the scope of the investigation. First, we assume that the fund manager charges a proportional (contingent) fee for his services, which is consistent with how the majority of mutual funds in the U.S. are structured.¹⁰ Second, we analyze the case of multiple mutual funds, i.e., our focus is on the imperfect competition between the mutual funds whereas Admati and Pfleiderer (1990) analyze only the monopolistic case. Third, each fund manager in our model has a zero impact on the stock price, i.e., each is negligible in size. In contrast, the monopolistic seller in Admati and Pfleiderer (1990) directly impacts the stock market.¹¹ Lastly, our model does not reduce to that of Admati and Pfleiderer (1990), even in the case of a monopolist. This makes the monopolist case of our paper of some interest on its own.¹²

The remainder of our article is organized as follows. For ease of exposition, we break the presentation of the model into several sections. First, section 2 discusses the structure of the model including the agents' preferences, beliefs, and private information. Essentially, this section outlines all of the model's economic primitives. Next, section 3 assumes that mutual funds are available to the household sector and solves for a rational expectations equilibrium of the Hellwig (1980) type. In doing so, we take both the fraction of informed agents in the economy and the mutual funds' contingent fees as given. Our equilibrium is summarized

⁸Admati and Pfleiderer (1986) study the direct sale of information, e.g., through a newsletter. See Simonov (2000) for the case of a duopolist selling information directly. Veldkamp (2003) studies dynamic properties of asset prices in the presence of markets for information.

⁹The literature that studies markets for information in non-competitive models is also related to ours. See Admati and Pfleiderer (1988), Fishman and Hagerty (1995) and Biais and Germain (2002).

¹⁰In contrast, Admati and Pfleiderer (1990) do not allow the fund manager to keep a fraction of the fund's payoff as compensation. Since they are interested in analyzing the choice between selling information directly (by creating, say, an investment newsletter) and selling it indirectly (by establishing a mutual fund), modeling contingent fees is not relevant in their paper. As we show in section 6.2, the introduction of a contingent fee does not alter the optimization problem faced by the fund managers. Another innocuous, but different, assumption in our model is that each fund manager endogenously chooses his fund's position in the stock market by maximizing his expected utility conditional on his information set. These assumptions are necessary in order to endogenize the information acquisition decision, in which we necessarily must assign preferences to the mutual fund managers. Since Admati and Pfleiderer (1990) are not concerned with information acquisition, they are able to work in a setting in which managers do not have explicit preferences.

¹¹This aspect of Admati and Pfleiderer (1990) is somewhat undesirable since the monopolist seller is assumed to act as a price taker even though his trading strategy affects the price (see, for example, the discussions in Hellwig (1980) or Admati and Pfleiderer (1988)).

¹²See section 6.2 for details.

in Proposition 1. Section 4 then endogenizes the contingent fees of the mutual funds, while still holding constant the fraction of informed agents in the economy. Our main result in this section is Proposition 2, which characterizes the optimal contingent fees and shows that informed agents always establish mutual funds rather than trade for their own accounts using their private information. This is followed by section 5, which endogenizes the fraction of agents that become informed (see Proposition 3). Section 6 then presents several extensions of our model, and section 7 offers concluding remarks and directions for future research. All of our proofs are collected in the Appendix.

2 A model of mutual fund formation

We analyze information acquisition and mutual fund formation in a noisy rational expectations setting. We approach this topic by modifying the normal-exponential framework of Grossman and Stiglitz (1980), Hellwig (1980), and Verrecchia (1982). Investors in our model can either remain uninformed (i.e., as a “household”), they can acquire costly private information and trade on their own account (i.e., as a “proprietary trader”), or they can acquire costly private information and offer investment management services to the household sector (i.e., as a “mutual fund manager”). In the latter case, the mutual fund manager earns a contingent investment management fee, i.e., the manager retains a fraction of the fund’s final value as his compensation. We discuss the exact nature of the contingent fees in more detail below.

There are two types of primitive assets available for trading, a riskless asset and a risky asset. The riskless asset pays zero interest and has a perfectly elastic supply. The risky asset, on the other hand, has a payoff of X , where X has a normal distribution with mean zero and variance σ_x^2 , i.e., $X \sim \mathcal{N}(0, \sigma_x^2)$. The per capita supply of the risky asset is $U \sim \mathcal{N}(0, \sigma_u^2)$ and is interpreted as the presence of noise traders in the economy. Alternatively, one could interpret U as representing endowment shocks rather than noise trading. All of our results remain unchanged under this alternative interpretation. In addition to the primitive assets, the investment opportunity set of the household sector includes mutual funds. The mutual funds are optimally established in equilibrium and are discussed in greater detail below. In addition, for simplicity, we do not allow informed agents to invest with other informed agents (e.g., mutual funds cannot hold positions in other mutual funds).

Agents in our model can acquire private information by paying a fixed cost $c > 0$. Upon

paying c , the i th agent observes the private signal $Y_i = X + \epsilon_i$, where $\epsilon_i \sim \mathcal{N}(0, \sigma_\epsilon^2)$. We let λ denote the fraction of agents in our economy that is informed. When this fraction is endogenously determined, we use the notation λ^* instead of λ . Agents in our model have rational expectations in the sense of Grossman (1976), i.e., they rationally use the information revealed by price when forming their posterior beliefs. Hence, even though some agents may remain uninformed by not acquiring private information, the uninformed agents *do* learn something about the private information of the informed agents by observing the risky asset price.

All random variables in our model are defined on a common probability space $(\Omega, \mathcal{F}, \mathbb{P})$. We assume that X , U , and the collection of signal errors $\{\epsilon_i\}$ are mutually independent. We also assume that all agents have CARA preferences with risk-aversion parameter τ , i.e., for a given final payoff W_i , the i th agent has the expected utility $\mathbb{E}[u(W_i)] = \mathbb{E}[-\exp(-\tau W_i)]$. We relax the homogeneous risk aversion assumption in section 6.1, but for now it allows us to focus on the informational aspects of the model. Lastly, we assume that all agents have zero initial wealth. By the standard properties of CARA utility, this assumption is without loss of generality.

The sequence of events in our model can be described by using a timeline with three dates. Date 0 is the fund formation stage of the model. At this date we analyze the agents' information acquisition decisions in order to determine the equilibrium fraction of informed agents in our economy. We also analyze the mutual fund managers' fee setting problems in order to determine the optimal contingent management fees. Date 1 is the trading stage of the model. Each manager observes Y_i and, given his private signal and the information revealed by price, the manager chooses his fund's investment in the risky and riskless assets in order to maximize his expected utility. Likewise, given the information revealed by price, each household chooses its investment strategy in order to maximize its expected utility. Finally, date 2 is the payoff stage of the model. At this date the risky asset pays X , the mutual funds distribute their payoffs, and all agents consume their final realized wealth levels.

We consider a large economy in which there is a continuum of agents.¹³ Given this assumption and letting $\lambda > 0$, it is apparent that a continuum of mutual funds will exist in equilibrium. Even though a household in principle would benefit from diversifying across all of these funds, it would be awkward and unrealistic to allow each household to invest in an

¹³We use a continuum of agents for tractability reasons - see the discussion in section 6.2.

infinite number of assets.¹⁴ We therefore assume that households can invest in only m mutual funds. The parameter m can be interpreted as the outcome of a costly search model in which each household must identify an appropriate set of mutual funds. Under this interpretation, the value of m would be decreasing in the contracting friction between the households and the mutual funds.

Given the symmetry of the model, we can equivalently think of a search model that produces groups of n agents, λ of which are informed. The uninformed agents in each group are then allowed to invest in the funds that are established by the $m = \lambda n$ informed agents. Given a group of size n and assuming that all households are served by each fund manager in the group, it is easy to see that each fund serves $h = m \left(\frac{1-\lambda}{\lambda}\right)$ households. In the sequel, we take m to be an integer and we ignore integer problems with h . Although we motivate a fixed m (or n) in terms of a costly search model, we do not formally model the search process. Instead, we take the fixed m as an exogenous constraint that is imposed on the agents in our model. Comparative statics with respect to m then allow us to analyze how mutual fund fees and the equilibrium fraction of mutual funds vary with respect to the household sector's ability to invest in additional funds.

The parameter m also measures the degree of competition in the mutual fund sector. This is a useful feature of our model since it allows us to go beyond the monopolistic case that is analyzed in Admati and Pfleiderer (1990). For $m > 1$, we assume that the managers within each group behave in a non-cooperative manner when choosing their investment management fees. Each manager takes the other $m - 1$ fees as given and chooses his own fee in order to maximize his ex ante expected utility. Hence, managers in our model are faced with non-trivial strategic pricing decisions when choosing their fees. In addition, for any value of m (i.e., including the monopolistic case of $m = 1$), there is a negative externality that arises due to the information that is revealed by the risky asset price. As will become apparent in section 4, the fund managers' optimal fees affect the informativeness of the risky asset price. In turn, this affects the household sector's marginal willingness to pay for investment management services. This negative externality plays an important role in our model.

In our economy, an informed agent is allowed to indirectly sell his private information to the uninformed agents within his group by establishing a mutual fund. The i th informed agent

¹⁴In fact, in section 6.2 we argue that a restriction on the number of funds that is included in the household sector's investment opportunity set is a necessary condition for the existence of an equilibrium.

(i.e., the i th fund manager) can set up a mutual fund whose payoff, Z_i , is given by

$$Z_i = P_i + \gamma_i(X - P_x); \quad (1)$$

where P_i denotes the price of the i th fund, γ_i is the trading strategy of the i th manager, and P_x is the price of the risky stock (whose payoff is X). Without loss of generality, we normalize the mutual fund's size to be equal to 1 unit. Hence, P_i represents the initial amount invested in the i th fund by the households that belong to the i th manager's group. Given the form of (1), note that we have assumed that P_i remains in the mutual fund and is invested by the fund manager.¹⁵ This corresponds to current practice in the mutual fund industry.

The i th fund manager keeps a fraction α_i of Z_i , i.e., the manager's compensation is given by $\alpha_i Z_i$. We refer to the variable α_i as being the i th manager's *contingent fee*. The compensation contract $\alpha_i Z_i$ covers many of the fee structures that are typically used by mutual funds today. In particular, we can think of α_i as representing the joint effect of any contingent deferred sales charge and investment management fees.¹⁶ Following Das and Sundaram (2002), we assume that the manager chooses the fee α_i . Later in the article we discuss how this assumption affects the equilibrium and we analyze how our results would differ if instead the household sector set the fee. Lastly, we assume that the contingent fee is set prior to any agent observing his private signal. Contingent fees in our model are therefore constants, i.e., they do not directly depend on any of the model's random variables and they do not convey any information.¹⁷

For ease of exposition, we also define the i th manager's *total fee* as the product of the contingent fee and the fund's price. In other words, the total fee for manager i is given by $\alpha_i P_i$. We motivate the concept of the total fee by examining the typical household's payoff from investing in the i th mutual fund. Since the manager keeps $\alpha_i Z_i$, the net fund payoff to

¹⁵While this differs from Admati and Pfleiderer (1990), it is an innocuous assumption (see pp. 906-907 of Admati and Pfleiderer (1990)).

¹⁶For simplicity, we use $\alpha_i Z_i$ as the manager's payoff. A slightly more general compensation scheme, however, would allow us to handle several other types of mutual fund fees. For example, using $\beta_i + \alpha_i Z_i$ would allow us to interpret β_i as representing the effect of any front-end sales charges and Rule 12b-1 fees. Rule 12b-1 of the U.S. Securities and Exchange Commission allows mutual funds to use fund assets to pay for the distribution costs (i.e., marketing and administration costs) of the fund.

¹⁷We assume that the signal precisions are common knowledge. In the case where agents have heterogeneous precisions, the absence of this assumption would open up the possibility that the managers might signal their quality via their contingent fees. In this case, unlike the symmetric model, the contingent fees might be informative to the household sector. See Huberman and Kandel (1993) and Das and Sundaram (2002) and the references cited therein for the effects of signalling in mutual fund markets.

the household (per unit investment in the mutual fund) is given by

$$(1 - \alpha_i)Z_i - P_i = \underbrace{(1 - \alpha_i)\gamma_i(X - P_x)}_{\text{net risky asset bet}} - \underbrace{\alpha_i P_i}_{\text{total fee}} . \quad (2)$$

As equation (2) illustrates, the net fund payoff can be separated into a fixed part and a variable part. The fixed part, given by $\alpha_i P_i$, represents the total fee paid to the fund manager by the household sector. The variable part, on the other hand, represents the household sector's portion of the mutual fund's risky asset bet. Essentially, the household sector pays a net amount equal to $\alpha_i P_i$ in exchange for exposure to the risky payoff $(1 - \alpha_i)\gamma_i(X - P_x)$. Hence, the true cost of purchasing 1 full unit of the i th mutual fund is $\alpha_i P_i$ rather than P_i . This follows from our assumption that the manager keeps the initial amount P_i in the fund.

In general, mutual funds offer many benefits to individual investors, including diversification and access to possibly superior financial market information. In our model it is a combination of these items that drives households to invest in mutual funds. For example, households can obtain equity market exposure in one of two ways – by directly investing in the stock market or by indirectly investing via a mutual fund. If the i th fund manager has private information about X , then his demand for the stock, γ_i , will depend on his private information. Hence, from the perspective of the household sector, γ_i is a random variable and the mutual fund's stock market bet represents a new asset class. This is easy to see by examining (2). While $(X - P_x)$ is normally distributed, the quantity $\gamma_i(X - P_x)$ will not be normally distributed since it involves a product of random variables. Households therefore diversify their risk by purchasing shares in the mutual fund even though the fund manager holds the same stock that the households buy directly. Essentially, in a world with asymmetric information, investing in a stock via a mutual fund is quite different than investing in that same stock directly.

More specifically, given the framework in our model, the i th fund's stock market bet $\gamma_i(X - P_x)$ is a noisy quadratic function of X . Since it depends on X^2 , which is always positive, households derive some benefit from investing in mutual funds.¹⁸ However, since the bet $\gamma_i(X - P_x)$ also depends on the error term ϵ_i that drives the fund manager's private information,

¹⁸While Brennan and Cao (1996) analyze a quadratic payoff in the standard rational expectations setting, our model differs from theirs in several respects. First, our quadratic function arises endogenously as the mutual fund's optimal payoff. Second, our quadratic function is noisy, i.e., it depends on the errors $\{\epsilon_i\}$. Lastly, our quadratic function depends on $\{\alpha_i\}$ which implies that the fund managers control the household sector's exposure to X^2 .

investing in mutual funds introduces additional noise into the household’s problem that is unrelated to fundamentals. Investing in mutual funds therefore involves a trade-off between the profitability of the informed managers’ investment strategies and the idiosyncratic risk that is associated with them. We explore this trade-off in the sequel.

3 A rational expectations equilibrium at the trading stage

We begin our analysis at date 1 by examining the trading stage of the model. For now, we fix $\alpha_i \in (0, 1]$ for all i and we take $\lambda \in (0, 1)$ as given. Later we endogenize these quantities (see sections 4 and 5). With α_i fixed, the i th manager’s optimization problem at date 1 can be written as

$$\max_{\gamma_i} \mathbb{E} \left[-e^{-\tau(\alpha_i Z_i - c)} \mid P_x, Y_i \right]; \quad (3)$$

where Z_i is given in (1). Expression (3) clearly illustrates how our model of mutual fund formation generalizes the work of Hellwig (1980). Specifically, our model reduces to a simplified version of Hellwig (1980) if we set $\alpha_i = 1$ for every informed agent. In this case, the household sector does not invest in any mutual fund (i.e., $P_i = 0$ for every i) and the informed agents use their private information to trade on their own account. In other words, an informed agent engages in “proprietary trading” for his own account when $\alpha_i = 1$. For $\alpha_i \in (0, 1)$, the informed agent establishes a mutual fund and manages money for the household sector.

Turning to the household’s investment problem, let θ_j denote the number of shares of the risky asset and let ϕ_{ij} denote the number of units of the i th mutual fund that are demanded by the j th household. Since there is only 1 unit outstanding of each mutual fund, ϕ_{ij} can be interpreted as the *fraction* of fund i that is held by household j . Using this notation, we can write the optimal investment problem of the j th household at date 1 as

$$\max_{\theta_j, \{\phi_{ij}\}_{i=1, \dots, m}} \mathbb{E} \left[-e^{-\tau W_j} \mid P_x \right]; \quad (4)$$

where

$$W_j = \theta_j (X - P_x) + \sum_{i=1}^m \phi_{ij} [Z_i (1 - \alpha_i) - P_i]. \quad (5)$$

Given problems (3) and (4), a rational expectations equilibrium at date 1 (i.e., at the trading stage) is formally defined as:

- (i) a collection of mutual fund trading strategies such that the i th manager's strategy, $\hat{\gamma}_i$, solves (3);
- (ii) a collection of household trading strategies such that the j th household's strategy, $\hat{\theta}_j$ and $\{\hat{\phi}_{ij}\}_{i=1,\dots,m}$, solves (4);
- (iii) a price function for the stock, $P_x : \Omega \rightarrow \mathbb{R}$, and a collection of price functions for the mutual funds, $P_i : \Omega \rightarrow \mathbb{R}$ for all i , such that all markets clear, i.e.,

$$\int_0^\lambda \hat{\gamma}_i di + \int_\lambda^1 \hat{\theta}_j dj = U; \quad (6)$$

$$\sum_{j=1}^h \hat{\phi}_{ij} = 1 \quad \text{for all } i. \quad (7)$$

Given the normal-exponential setup and the symmetrical nature of our model, we make several conjectures. All of these conjectures are formally verified to be true in equilibrium. First, we conjecture that the risky asset's equilibrium price at date 1 is a linear function of X and U , i.e., we conjecture that

$$P_x = bX - dU \quad (8)$$

where P_x denotes the risky asset's price. As in Hellwig (1980), the price coefficients b and d are endogenously determined in our model. Second, we conjecture that the prices of the established mutual funds are uninformative with respect to X . Lastly, due to the symmetry of the model (i.e., since all informed agents have identical risk aversion parameters and identical signal precisions), we conjecture that all fund managers have the same contingent fees.

In light of the above discussion, we are now ready to state our first result. Proposition 1 summarizes the rational expectations equilibrium at the trading stage of our model.

Proposition 1. *There exists a noisy rational expectations equilibrium at date 1 with the following properties:*

- (i) the optimal date 1 risky asset demand of manager i is

$$\hat{\gamma}_i = \frac{\mathbb{E}[X|P_x, Y_i] - P_x}{\alpha_i \tau \text{var}(X|P_x, Y_i)}; \quad (9)$$

(ii) the optimal date 1 risky asset demand of household j is

$$\hat{\theta}_j = \left[\frac{\mathbb{E}[X|P_x] - P_x}{\tau \text{var}(X|P_x)} \right] \left(1 + \frac{\text{var}(X|P_x)}{h \sigma_\epsilon^2} \sum_{k=1}^m \left(\frac{1 - \alpha_k}{\alpha_k} \right) \right) - \frac{1}{h} \sum_{k=1}^m (1 - \alpha_k) \mathbb{E}[\hat{\gamma}_k|P_x]; \quad (10)$$

(iii) the optimal date 1 demand of household j for mutual fund i is $\hat{\phi}_{ij} = \frac{1}{h}$;

(iv) the date 1 market clearing risky asset price is given by (8) with price coefficients

$$d = \frac{1 + \frac{\lambda}{\bar{\alpha} \tau^2 \sigma_\epsilon^2 \sigma_u^2}}{\left[\frac{\lambda}{\bar{\alpha} \tau \sigma_\epsilon^2} + \frac{1}{\tau} \left(\frac{1}{\sigma_x^2} + \left(\frac{\lambda}{\bar{\alpha} \tau \sigma_\epsilon} \right)^2 \frac{1}{\sigma_u^2} \right) \right]} \quad (11)$$

and

$$b = \left(\frac{\lambda}{\bar{\alpha} \tau \sigma_\epsilon^2} \right) d, \quad (12)$$

where $\lambda = \frac{m}{m+h}$ and $\bar{\alpha}$ denotes the average contingent fee across all managers in the economy;

(v) the date 1 equilibrium value of fund i is

$$P_i = \frac{\left(\frac{1}{\alpha_i \tau \sigma_\epsilon^2} \right) \left[\left(\frac{1 - \alpha_i}{\alpha_i} \right) - \frac{1}{h} \left(\frac{1 - \alpha_i}{\alpha_i} \right)^2 \right]}{\left[\text{var}(X|P_x)^{-1} + \frac{2}{h \sigma_\epsilon^2} \sum_{k=1}^m \left(\frac{1 - \alpha_k}{\alpha_k} \right) - \frac{1}{h^2 \sigma_\epsilon^2} \sum_{k=1}^m \left(\frac{1 - \alpha_k}{\alpha_k} \right)^2 \right]}. \quad (13)$$

For notational purposes, note that we can write the manager's demand in (9) in terms of the economic primitives by substituting

$$\text{var}(X|P_x, Y_i) = \left[\frac{1}{\sigma_x^2} + \frac{1}{\sigma_\epsilon^2} + \frac{b^2}{d^2 \sigma_u^2} \right]^{-1}; \quad (14)$$

and

$$\mathbb{E}[X|P_x, Y_i] = \left[\frac{Y_i}{\sigma_\epsilon^2} + \frac{P_x b}{d^2 \sigma_u^2} \right] \text{var}(X|P_x, Y_i). \quad (15)$$

Letting $\sigma_\epsilon^2 \rightarrow \infty$ in (14) and (15) produces expressions for $\mathbb{E}[X|P_x]$ and $\text{var}(X|P_x)$, which show up in the expressions for $\hat{\theta}_j$ and P_i in (10) and (13), respectively.

The above equilibrium have several interesting features. First, while the manager's demand

in (9) has the familiar mean-variance form, it is apparent that the aggressiveness of the fund manager's trading strategy is decreasing in his contingent fee. As α_i increases, the fund manager trades less aggressively by taking a smaller position (either long or short) in the risky asset. Second, the contingent fee has the effect of reducing the i th manager's effective risk aversion from τ (which would prevail in the Hellwig (1980) setting) to $\alpha_i\tau$. In turn, due to demand aggregation and market clearing, we find that the coefficients of the equilibrium price function depend on the quantity $\bar{\alpha}\tau$. For a given λ , this produces a larger value for the relative price coefficients, b/d , which measures the information content in prices, relative to what would prevail in Hellwig (1980), and drives up the informativeness of the risky asset price. We formally state this fact as a corollary.

Corollary 1. *For a fixed λ , the risky asset price in the equilibrium in which the informed agents offer mutual funds to the household sector is more informative than the risky asset price in the equilibrium in which the informed agents engage only in proprietary trading.*

Given the expression for $\hat{\theta}_j$ in (10), it is easy to see how the existence of mutual funds affects the typical household's stock market investment. There are two separate effects – a *risk aversion effect* and a *feedback effect*. To see this, note that the term in square brackets in (10) is the familiar mean-variance demand that shows up in Hellwig (1980). This is multiplied by the quantity

$$\left(1 + \frac{\text{var}(X|P_x)}{h\sigma_\epsilon^2} \sum_{k=1}^m \left(\frac{1 - \alpha_k}{\alpha_k}\right)\right). \quad (16)$$

We call this the risk aversion effect since we can think of the household's effective risk aversion as being equal to τ divided by the above quantity. Since the above quantity is always greater than 1, the risk aversion effect leads to aggressive trading, i.e., the typical household trades more aggressively than they otherwise would in an economy without mutual funds.

The aggressive trading that stems from the risk aversion effect is partially offset by the household's expectation of the managers' optimal trades. Indeed, the final term in (10) reveals that the typical household decreases its long position (or increases its short position) if it believes that mutual fund managers are taking long positions in the risky asset. Likewise, the typical household increases its long position (or decreases its short position) if it believes that mutual fund managers are taking short positions in the risky asset. We call this the feedback effect since the household's demand for the risky asset is at least partially determined by the

investment strategies of the mutual fund managers. Finally, note that as $\alpha_i \rightarrow 1$ for all i , both the risk aversion effect and the feedback effect vanish. In this case, all managers trade in a proprietary fashion for their own accounts and our model collapses to the standard Hellwig (1980) setting.

The optimal mutual fund demands in property (iii) of Proposition 1 arise due to efficient risk sharing among the h households in each group. In particular, due to the symmetrical nature of our model, the j th household holds $\frac{1}{h}$ of each of the m mutual funds in which it is allowed to invest. Later in the article (see Proposition 4) we discuss how our trading stage equilibrium would be altered if households were allowed to be heterogeneous.¹⁹

We note that b and d in (11)-(12) depend only on the average contingent fee in the economy, $\bar{\alpha}$, and not on any particular manager's fee, α_i . However, the equilibrium fund value in (13) explicitly depends on the fees of the m managers that comprise a group. These facts allow us to distinguish between strategic behavior in the mutual fund sector and strategic behavior in the stock market. Specifically, fund managers in our model are price takers with respect to their stock market investments but they behave strategically when setting their contingent fees. They correctly account for the fact other managers are also offering investment management services to the household sector and they also correctly account for the fact that their contingent fee affects the fund's payoff via their optimal stock demand, $\hat{\gamma}_i$.

Lastly, note that the expressions for b and d in (11)-(12) depend only on λ and do not directly depend on m and h . We can therefore scale m and h by the same constant without changing the equilibrium stock market level. However, this type of scaling will have a profound effect on mutual fund values since P_i in (13) depends on m and h individually. This dependence will in turn affect the equilibrium mutual fund fees, since those are determined by maximizing $\alpha_i P_i$, which as we show next is essentially the objective function of the i th manager. We explore both of these features more thoroughly in section 4 when we analyze the optimal fee setting problem of the mutual fund managers.

¹⁹If the h households in a group instead had heterogeneous risk-aversion, the j th household's equilibrium demand for mutual fund i would be equal to $\hat{\phi}_{ij} = \tau_j^{-1} / \sum_{k=1}^h \tau_k^{-1}$. In the symmetric case, this reduces to $\frac{1}{h}$.

4 Optimal fees

At date 0 each manager chooses his contingent fee α_i in order to maximize his indirect utility function. We denote the optimal fee of the i th manager as $\hat{\alpha}_i$ and we map the i th manager's optimal fee choice into a fund formation decision. For example, if $\hat{\alpha}_i = 1$, the i th manager trades for his own (proprietary) account. On the other hand, if $\hat{\alpha}_i < 1$, the manager establishes a mutual fund and markets his optimal investment strategy to the household sector. When solving the fund managers' problems at date 0 we assume that the managers have rational expectations. In particular, we assume that they know the form of the equilibrium given in Proposition 1. We also maintain the assumption that $\lambda \in (0, 1)$.

Using (1) we see that the payoff for the fund manager, $\alpha_i Z_i$, is of the form $\alpha_i P_i + \alpha_i \gamma_i (X - P_x)$. From (9) it is immediate that $\alpha_i \gamma_i$ is independent of α_i , so the i th manager's problem at date 0 reduces to

$$\max_{\alpha_i} \alpha_i P_i \equiv f(\alpha_i) \quad (17)$$

where P_i is given in (13). Obviously, since P_i depends on the entire collection of contingent fees, every manager behaves in a strategic (non-cooperative) manner when choosing his own contingent fee. We use a standard Nash equilibrium framework and present the solution to the fee setting problem in the next proposition.

Proposition 2. *The symmetric Nash (fee setting) equilibrium at date 0 is characterized by $\hat{\alpha}_1 = \hat{\alpha}_2 = \dots = \hat{\alpha}_m \equiv \hat{\alpha}$ where $\hat{\alpha}$ is the unique solution of the cubic equation*

$$k_3 \hat{\alpha}^3 + k_2 \hat{\alpha}^2 + k_1 \hat{\alpha} + k_0 = 0 \quad (18)$$

that satisfies $\hat{\alpha} \in (1/(1 + 0.5h), 1)$. The coefficients of the cubic equation are given by

$$\begin{aligned} k_0 &= \frac{2\eta^3(m-1)}{m^3\sigma_\epsilon^2} - \frac{2\eta\lambda^2}{\tau^2 m \sigma_u^2 \sigma_\epsilon^4} \\ k_1 &= \frac{\lambda^2}{\tau^2 \sigma_u^2 \sigma_\epsilon^4} + \frac{2\eta\lambda^2}{\tau^2 m \sigma_u^2 \sigma_\epsilon^4} - \frac{(5m-4)\eta^2}{m^2\sigma_\epsilon^2} - \frac{6\eta^3(m-1)}{m^3\sigma_\epsilon^2} \\ k_2 &= \frac{2\eta(m-1)}{m\sigma_\epsilon^2} - \frac{2\eta}{m\sigma_x^2} + \frac{2(5m-4)\eta^2}{m^2\sigma_\epsilon^2} + \frac{6\eta^3(m-1)}{m^3\sigma_\epsilon^2} \\ k_3 &= \frac{1}{\sigma_x^2} - \frac{2\eta(m-1)}{m\sigma_\epsilon^2} + \frac{2\eta}{m\sigma_x^2} - \frac{(5m-4)\eta^2}{m^2\sigma_\epsilon^2} - \frac{2\eta^3(m-1)}{m^3\sigma_\epsilon^2} \end{aligned}$$

where $\eta \equiv \frac{\lambda}{1-\lambda}$.

A remarkable property of the fee setting equilibrium is that $\hat{\alpha} \in (1/(1 + 0.5h), 1)$, i.e., $\alpha_i = 1$ is never optimal for an informed agent in our economy. In economic terms, this implies that every informed agent in the economy establishes a mutual fund and markets his optimal investment strategy to the household sector. In other words, rather than trade on their own accounts and keep the entire amount of their risky asset bets, the informed agents find it optimal to share their risky asset bets with the household sector in exchange for the total fees. Hence, our model provides one possible explanation for why mutual funds arise in practice.

In order to gain some intuition about the factors that drive the fees, we can use (13) to rewrite the i th manager's problem in (17) as

$$\max_{\rho_i} \frac{\rho_i(1 - \rho_i/h)}{\text{var}(X|P_x)^{-1} + \left[\frac{2}{h\sigma_\epsilon^2} \sum_{k=1}^m \rho_k - \frac{1}{h^2\sigma_\epsilon^2} \sum_{k=1}^m \rho_k^2 \right]} \quad (19)$$

where $\rho_i = (1 - \alpha_i)/\alpha_i$. The importance of the choice variable ρ_i can be seen by examining the net fund payoff per unit that is received by a typical household. In particular, we can expand expression (2) to get

$$(1 - \alpha_i)Z_i - P_i = \underbrace{\frac{\rho_i}{\tau\sigma_\epsilon^2} Y_i(X - P_x)}_{Y_i\text{-bet}} + \underbrace{q(P_x)(X - P_x)}_{P_x\text{-bet}} - \underbrace{\alpha_i P_i}_{\text{total fee}} \quad (20)$$

where $q(P_x)$ is a linear function of P_x but does not depend on Y_i . We refer to the first term on the right-hand side of (20) as the Y_i -bet since this portion of the total payoff from the fund is measurable with respect to the manager's information set, but not with respect to the household's. Note that the household sector can replicate the P_x -bet by trading directly in the stock market, so his marginal willingness to pay for the fund's shares is independent of $q(P_x)$. The quantity ρ_i controls the household sector's exposure to the portion of the total bet that cannot be replicated by trading directly in the stock market. Because of this, we denote ρ_i as the i th fund's *exposure* to the risky asset.

Since $Y_i = X + \epsilon_i$, we can further decompose the Y_i -bet into a term that depends on $X(X - P_x)$ and a term that depends on $\epsilon_i(X - P_x)$. The first term captures the benefit of investing in any mutual fund since it involves X^2 , which is always positive. Hence, even in a down stock market, the household sector receives some benefit from holding mutual funds.

The second term highlights the cost of investing in the i th mutual fund – namely, it exposes the household to the error term ϵ_i that drives the manager’s private signal. Since ϵ_i is independent of X , households are exposed to idiosyncratic risk when they invest in mutual funds. In addition, since a household is only allowed to purchase $m < \infty$ funds, the idiosyncratic risk is a residual risk that cannot be diversified away.

Returning to expression (19), it is apparent that several factors influence the fund manager’s total fee. From the above discussion, there is a direct relationship between the manager’s total fee and the fund’s exposure ρ_i . Evidently, the exposure that maximizes the manager’s total fee is not the exposure that is most preferred by the household sector. If instead we allowed the household sector to choose ρ_i subject to the constraint $P_i \geq 0$, the optimal exposure would be $\rho_i = h$. Recalling that $\rho_i = (1 - \alpha_i)/\alpha_i$, this implies that the household sector’s most preferred contingent fee is $\alpha_i = 1/(1 + h)$. It is easy to check that this value of α_i is not a solution of equation (18). However, this value of the contingent fee does achieve optimal risk sharing in ex-ante terms and is therefore referred to as the *first-best contingent fee*.

When the fund managers set their fees, they choose $\rho_i \in (0, h/2)$, i.e., they choose an exposure that is more than 50% less than the household sector’s optimal exposure. This is a direct result of each fund manager maintaining some market power within his group. The managers extract rents from the household sector by limiting the households’ exposure to the risky asset. This result is very similar to how a monopolist would restrict the demand for his product.²⁰ We also note that the manager’s optimal contingent fee is always strictly greater than the fee that would otherwise be chosen by the household sector. The household sector prefers $\alpha_i = 1/(1 + h)$ while the manager chooses $\alpha_i > 1/(1 + 0.5h) > 1/(1 + h)$.

The denominator of (19) reveals that there are two additional factors that affect a manager’s total fee. The first factor in the denominator is $\text{var}(X|P_x)^{-1}$, which is the precision of X given the household sector’s information set. Rather intuitively, a fund manager’s total fee is decreasing in the household’s precision, suggesting that households are less willing to pay for investment management services if they already have good quality information about the risky asset’s payoff. We also note that even though $\text{var}(X|P_x)^{-1}$ is independent of any particular manager’s fee, it does depend on $\bar{\alpha}$, which is the average contingent fee across all managers in the economy. In fact, recalling (14), we see that $\text{var}(X|P_x)^{-1}$ is a linear function of $1/\bar{\alpha}^2$, which arises because $\text{var}(X|P_x)^{-1}$ depends on $(\frac{b}{d})^2$. Hence, the managers’ contingent fees

²⁰This same effect is also present in the analysis of Admati and Pfleiderer (1988) for example.

have a direct effect on their trading aggressiveness (see equation (9)) and this affects the informativeness of P_x . In turn, as P_x becomes more informative, the household sector is less willing to invest through mutual funds and the total fee of each manager is reduced in equilibrium.

The second factor in the denominator of (19) is given by the term in square brackets. Since this term involves m , it allows us to assess the impact of mutual fund competition on the i th manager's total fee. In equilibrium, this term is always positive²¹ and it therefore introduces an additional negative effect on the manager's total fee. If we increase the number of fund managers in a group from m to $m + 1$, the term in square brackets increases and this lowers the equilibrium total fee of the i th manager in the group.

Turning next to the comparative statics of the model, the following corollary presents some of the properties of the optimal contingent fee, $\hat{\alpha}$.

Corollary 2. *For a fixed λ , the optimal contingent fee $\hat{\alpha}$ is: (i) increasing in σ_ϵ^2 for low values of σ_ϵ^2 and decreasing in σ_ϵ^2 for high values of σ_ϵ^2 ; (ii) increasing in τ ; (iii) increasing in σ_u^2 ; and, (iv) decreasing in m . Furthermore, the optimal contingent fee satisfies*

$$\lim_{\sigma_u^2 \downarrow 0} \hat{\alpha} = \lim_{\tau \downarrow 0} \hat{\alpha} = \lim_{\sigma_\epsilon^2 \downarrow 0} \hat{\alpha} = \lim_{\sigma_\epsilon^2 \uparrow \infty} \hat{\alpha} = \frac{1}{1 + 0.5h} \quad (21)$$

Property (i) arises due to the negative externality that a fund manager faces when information is revealed by price. For high values of σ_ϵ^2 , prices reveal very little information and hence this negative externality is small. In this case, an increase in σ_ϵ^2 makes mutual funds less desirable to the household sector and the fund managers respond by lowering their contingent fees. This increases ρ_i and gives the household a higher exposure to the risky asset, i.e., one that is closer to their preferred exposure. On the other hand, when σ_ϵ^2 is low, the risky asset price is very informative. On a relative basis, the contingent fees in this case are also low since the household sector is unwilling to pay very much for the managers' private information. An increase in σ_ϵ^2 decreases the informativeness of price and therefore mutual funds become more desirable to the household sector. The fund managers respond by increasing their contingent fees, thus reducing the household sector's exposure to the risky asset. This property is illustrated in the top panel of Figure 1. As shown in the figure, this effect is more pronounced for

²¹This follows from the simple observation that in equilibrium the i th fund manager will always choose ρ_i such that $\rho_i(1 - \rho_i/h) > 0$. Otherwise his total fee will be negative.

smaller values of m .

Properties (ii) and (iii) of the corollary can be easily understood by returning to expression (19). The only way that τ and σ_u^2 enter this expression is through the household sector's posterior precision, i.e., through $\text{var}(X|P_x)^{-1}$. As τ increases, the mutual fund managers in the economy trade less aggressively and therefore less information is revealed by P_x . In addition, as σ_u^2 increases there is more noise in the economy, which also reduces the informativeness of P_x . From our previous discussion, a decrease in $\text{var}(X|P_x)^{-1}$ implies that the households assign a higher value to the services of the fund managers. In turn, every fund manager is able to charge a contingent fee that is strictly greater than the first-best level. We illustrate these properties in the top panels of Figures 2 and 3.

Collectively, the top panels of Figures 1-3 illustrate property (iv). Upon being offered a new mutual fund in which to invest, the typical household will lower its marginal willingness to pay for any particular fund's services. This results in a lower value for $\hat{\alpha}$, i.e., one that is closer to the lower bound of $1/(1+0.5h)$. Also note that for m large the optimal fees do not converge to the first-best levels, but rather to the higher value $1/(1+0.5h)$. This is due to the fact that even when there are many mutual funds competing, each fund is a differentiated product, due to the fact that each signal is different, so a fund manager still retains some market power.

While the previous corollary addressed the properties of the optimal contingent fee, we now examine the properties of the total fee.

Corollary 3. *For a fixed λ , the total fee $\hat{\alpha}_i P_i(\hat{\alpha}_i)$ is: (i) increasing in σ_ϵ^2 for sufficiently low σ_ϵ^2 and decreasing in σ_ϵ^2 for high σ_ϵ^2 ; (ii) decreasing in τ for large τ and increasing in τ for small τ ; and, (iii) increasing in σ_u^2 . Furthermore, unlike $\hat{\alpha}$, the total fee may be increasing in m .*

Properties (i) and (iii) are identical to their counterparts found in Corollary 2 above. These properties arise due to the reasons discussed above and, in the interest of brevity, we do not repeat the same discussion here. On the other hand, Property (ii) is quite different than its counterpart in Corollary 2. As τ increases, we find that there are two effects on the fund manager's total fee. The first effect is a risk premium effect, i.e., as the risk aversion level increases the household sector is willing to pay less for their share of the mutual fund's risky bet. On the other hand, as discussed above, an increase in τ also affects the information revealed by price, which tends to make mutual funds more valuable. Corollary 3 indicates that

the latter effect dominates when τ is small while the former effect dominates when τ is large. Properties (i)-(iii) are illustrated in the bottom panels of Figures 1-3.

Our last claim in Corollary 3 concerns the relationship between the total fee and the level of competition in the mutual fund sector. While the contingent fee $\hat{\alpha}$ is always decreasing in m , we find that for some parameter values the total fee $\hat{\alpha}P(\hat{\alpha})$ may be *increasing* in m . For other parameter values, the total fee may be non-monotonic in m . These facts can be explained by noting that the number of households that is served by a typical mutual fund is equal to $h = m \left(\frac{1-\lambda}{\lambda} \right)$. Hence, h is proportional to m , and both affect the denominator of (13), albeit in different directions. We illustrate this feature of the model by examining the bottom panels of Figures 1-3. For a fixed value on the horizontal axis, we can move vertically through the plots to establish a relationship between the total fee and m . Note that sometimes this relationship is decreasing in m (e.g., see Figure 1 for low values of σ_ϵ^2) while at other times this relationship is increasing in m (e.g., see Figure 1 for large values of σ_ϵ^2). For other parameter values, the relationship is non-monotonic (e.g., see Figure 1 for $\sigma_\epsilon^2 = 0.8$).

Lastly, we can use our model to analyze the total amount of money that is spent by a typical household on investment management services. While a typical household pays $\hat{\alpha}_i P_i(\hat{\alpha}_i)$ for the i th manager's services, the same household spends a grand total of $m \hat{\alpha}_i P_i(\hat{\alpha}_i)$ on investment management services. Figure 1 illustrates several features of this latter quantity. For example, for very low values of σ_ϵ^2 , the total amount spent on investment management services when $m = 1$ is approximately equal to the total amount spent when $m = 2$ (i.e., $\hat{\alpha}_i P_i(\hat{\alpha}_i)$ for $m = 2$ is about one-half of that for $m = 1$). Hence, for at least some parameter values, we find that the total amount spent is insensitive to m . However, for other parameter values, we find an extreme disparity in the total amount spent on investment management services. This is nicely illustrated by examining Figure 1 for very large values of σ_ϵ^2 . In this case, the total amount spent on investment management services when $m = 2$ is more than twice the amount that is spent when $m = 1$.

5 Equilibrium information acquisition

While all of our previous results treated λ as a parameter, we now turn to the problem of endogenizing this quantity (i.e., we solve for λ^*). To accomplish this, we exploit the model's symmetry and we identify λ^* as the value of λ that equates the date 0 indirect utilities of

a typical fund manager and a typical household. Hence, in equilibrium, neither an informed agent nor an uninformed agent has an incentive to alter his information acquisition decision. Essentially, we can interpret λ^* as the unique outcome of a Nash equilibrium in which each agent optimally chooses whether or not to become informed. The next proposition states the main result in this section.

Proposition 3. *The equilibrium fraction of informed agents is given by the value of λ that solves the nonlinear equation*

$$e^{-\tau(f(\hat{\alpha})-c)} \sqrt{\frac{1}{P}} = e^{\frac{\tau\lambda}{(1-\lambda)}f(\hat{\alpha})} \sqrt{\frac{1}{D}} \quad (22)$$

where $f(\hat{\alpha})$ is expression (17) evaluated at $\hat{\alpha}$, $P = \text{var}(X|Y_i, P_x)^{-1}$, and

$$D = \text{var}(X|P_x)^{-1} + \frac{2\eta}{\sigma_\epsilon^2} \left(\frac{1-\hat{\alpha}}{\hat{\alpha}} \right) - \frac{\eta^2}{\sigma_\epsilon^2 m} \left(\frac{1-\hat{\alpha}}{\hat{\alpha}} \right)^2. \quad (23)$$

The equilibrium fraction of informed agents satisfies $\lambda^ \in (0, 1]$.*

As illustrated by (22), the solution to our mutual fund model with endogenous information acquisition is characterized by a single equation for λ that is parameterized by m , σ_ϵ^2 , σ_u^2 , σ_x^2 , c , and τ .²² However, given that $\hat{\alpha}$ itself is a nonlinear equation in λ , it does not seem possible to solve (22) in closed-form. To shed some light on our results, we compare our value of λ^* to the value that would otherwise arise in the absence of a mutual fund sector. Of course, this latter case is the one that is analyzed by Verrecchia (1982) and Diamond (1985) and hence our model can be viewed as a direct extension of their work. For example, since the agents in Verrecchia (1982) can only trade on their own account, it is possible none of the agents in his model will find it optimal to acquire private information. This situation would arise precisely when the cost to acquire information is extremely high. In our notation, this is when c takes on a very large value. In contrast, our model predicts that the corner solution $\lambda^* = 0$ is never possible. The intuition behind our claim is that even if it is very costly to acquire information, the total fee of a typical fund manager grows without bound as $\lambda \rightarrow 0$. Hence, there always exists a $\lambda^* > 0$ such that the informed agents can recoup their investment in private information even if c is large.

²²In principle, the solution to (18) can be substituted into (22) for $\hat{\alpha}$. This produces a single equation (for λ) that drives the entire equilibrium.

While $\lambda^* = 0$ never arises in our model, it *is* possible that all agents will become informed, i.e., $\lambda^* = 1$ may occur. The reason is that private information always has positive value to an agent. Hence, if the cost of acquiring information is sufficiently low, every agent in our model may optimally decide to purchase a private signal. We note that this is a “knife edge” case in which no mutual funds are established in equilibrium. Since $\lambda^* = 1$ produces $h = 0$, no informed agent can establish a mutual fund. Instead, all agents use their private information to trade on their own accounts. The case of $\lambda^* = 1$ is the *only* case in which no mutual funds are established in equilibrium.²³ For any $\lambda^* \in (0, 1)$, we have $h > 0$. In addition, from Proposition 2, we have $\hat{\alpha} \in (1/(1 + 0.5h), 1)$. Hence, $\lambda^* \in (0, 1)$ corresponds to the situation in which there is a positive fraction of informed agents that optimally choose to offer investment management services to the household sector. It is also worth noting that the set of parameter values that produces $\lambda^* = 1$ in our model can be quite different than the ones that produce a similar result in Verrecchia (1982). The top panel of Figure 4 nicely illustrates this fact.

We can also compare our model to that of Verrecchia (1982) by examining the informativeness of the risky asset price. While Corollary 1 addressed this issue for a fixed (i.e., exogenous) value of λ , our next corollary analyzes this issue for an endogenous λ .

Corollary 4. *Suppose that λ is determined by expression (22) and assume that the following condition holds*

$$e^{2\tau c} > 1 + \frac{\sigma_x^2}{\sigma_\epsilon^2}. \quad (24)$$

Then the risky asset price in the equilibrium in which there is a mutual fund sector is more informative than the risky asset price in the equilibrium in which there are no mutual funds.

Our corollary can be easily understood by examining the equilibrium in which there are no mutual funds. In this equilibrium, expression (24) is sufficient for all agents to optimally remain uninformed. In turn, no information is revealed by the equilibrium risky asset price. In contrast, suppose that we use the same set of parameter values but instead we introduce a mutual fund sector. As shown above, the introduction of a mutual fund sector implies that $\lambda^* > 0$. Hence, at least some agents acquire private information and the statement in the corollary immediately follows. This shows that the existence of mutual funds *may* produce a

²³It should be pointed out that this result does rely heavily on the standard setup of Hellwig (1980) and Verrecchia (1982) which this paper builds on. Given the results in the absence of markets for information in García and Urosević (2003), it seems that the upper bound on λ under which no funds are created will not bind in other classes of competitive models.

more informative price. This is likely to be the case when either information is costly, risk aversion is high, private signals are noisy, or the payoff variance is low.

On the other hand, when the parameters do not satisfy (24), it might be the case that the price is more informative without a mutual fund sector. We illustrate this fact with a numerical example. Suppose that $\sigma_x^2 = \sigma_u^2 = 1$, $\sigma_\epsilon^2 = 0.6$, $\tau = 3.1$, $m = 2$, and $c = 0.12$. Using these parameter values, we find that $\lambda^* = 0.5$ and $h = 2$, i.e., one-half of the agents acquire private information and each informed agent establishes a mutual fund that serves two households. In equilibrium, we also find that $b/d \approx 0.502$. On the other hand, if we do not allow agents to establish mutual funds, we find that the equilibrium fraction of informed agents is 1, i.e., all agents become informed. In this case, the equilibrium price coefficients satisfy $b/d \approx 0.538$. Since a higher value of b/d implies a lower value of $\text{var}(X|P_x)$, the risky asset price in the equilibrium without mutual funds is more informative. The bottom panel of Figure 4 shows that this result is robust, i.e., it holds for an open set of the model's parameters.

Exploring this topic a bit further, let us return to the expression for b/d in (12). Using this expression, it is easy to see that the risky asset price in the mutual fund equilibrium is the more informative of the two if $\lambda^* > \bar{\alpha}\lambda_V$, where λ_V denotes the equilibrium fraction of informed agents when no mutual funds exist.²⁴ For an exogenous value of λ , Corollary 1 showed that the risky asset price in the mutual fund equilibrium is always more informative. This effect is driven by the fact that $\bar{\alpha} < 1$. However, when λ is endogenously determined, the effect in Corollary 1 might be more than offset by the fact that different fractions of traders will become informed in the two equilibria (i.e., with and without mutual funds). In fact, as the above example demonstrates, the value of λ_V might be sufficiently large such that the risky asset price in the mutual fund equilibrium is actually less informative.

While the above discussion indicates that mutual funds may or may not produce a more informative price, we can partially restore the appeal of Corollary 1 by focusing on the level of competition between fund managers. This also sheds some light onto the potentially negative effect that imperfect competition in mutual fund sector can have on the informativeness of the equilibrium price. Along these lines, we have the following positive result.

Corollary 5. *For m large enough, the risky asset price in the equilibrium in which there is a*

²⁴An expression for the equilibrium fraction of informed traders in the absence of a mutual fund sector can be found in Lemma 3(c) of Diamond (1985). We can express this result using our notation as $\lambda_V = \tau\sigma_u\sigma_\epsilon\sqrt{\frac{1}{C(\tau)} - \frac{\sigma_\epsilon^2}{\sigma_x^2}}$ where $C(\tau) \equiv e^{2\tau c} - 1$.

mutual fund sector is more informative than the risky asset price in the equilibrium in which there are no mutual funds.

This result can be understood by noting that for large m , the average equilibrium contingent fee is very close to the lower bound, i.e., $\bar{\alpha} \approx 1/(1 + 0.5h)$. In addition, since h and m are proportional to one another, this implies that $\bar{\alpha} \approx 0$ for large m . Since $\lambda^* \in (0, 1]$ and λ_V is a constant, the condition $\lambda^* > \bar{\alpha}\lambda_V$ is always satisfied for large enough m . Hence, the introduction of a sufficiently large set of funds in the economy does lead to a more informative risky asset price. This shows that a crucial element of the above numerical example is the imperfect competition in the mutual fund sector.

Given the nonlinear nature of (18) and (22), few other general statements can be made about our economy. Hence, we conclude this section by numerically illustrating some of the properties of our model. Figures 5 through 7 plot the fraction of informed agents λ^* , a measure of price informativeness $\frac{1}{\sigma_u} \left(\frac{b}{d}\right)$, contingent fees $\hat{\alpha}_i$, total fees $\hat{\alpha}_i P_i(\hat{\alpha}_i)$, mutual fund assets per household $\frac{mP_i(\hat{\alpha}_i)}{h}$, and the stock price volatility $\sqrt{\text{var}(P_x)}$ as a function of σ_ϵ^2 , σ_u^2 and τ . As the top left panel of Figure 5 illustrates, λ^* initially increases with σ_ϵ^2 but then eventually trends downward as the quality of private information becomes very poor. The intuition for this result goes back to the discussion of the dual role of σ_ϵ^2 in total fees, from Corollary 3. For low values of σ_ϵ^2 , a small increase in this quantity reduces the information revealed by prices, thereby increasing the total fees charged and therefore the expected utility of the fund managers. Since the expected utility of the household sector drops with the increase in σ_ϵ^2 , it follows that λ^* must rise in equilibrium. This effect disappears for σ_ϵ^2 large, and mutual funds simply become less valuable as their information precision drops, which drives the equilibrium λ^* to decrease with σ_ϵ^2 . This effect tends to be more pronounced for smaller values of m . In addition, there is an apparent non-monotonicity of λ^* as a function of m due to the fact that the total fees had a non-monotonic relationship with m , as discussed in Corollary 3. The intuition for Figures 6 and 7 is similar, recalling that τ has a similar dual role to σ_ϵ^2 , as discussed in section 4.

The model also has implications for price informativeness, optimal contingent fees, size of the mutual fund industry and price volatility. For example, price informativeness is a decreasing function of all three variables σ_ϵ^2 , σ_u^2 and τ , as we would have expected. Nevertheless, price informativeness can decrease with m , as illustrated in Figure 6. This is due to the drop in λ

as a function of m , which can make prices less informative. The relationships between optimal fees and the three variables follow along the same lines as in section 4, with the additional effects that arise from an endogenous λ^* . As illustrated in the bottom panels of Figures 5 through 7, the model's implications with respect to price volatility and mutual fund assets per capital are fairly rich, due to the highly non-linear nature of the equilibrium.

6 Extensions

While the above model relies on several simplifying assumptions (e.g., identical risk aversion, identical signal precisions for the informed agents, etc.), we illustrate below how the model can be generalized to accommodate certain types of heterogeneity. In some cases, our model remains valid with little more than a change of notation. However, as we discuss in detail below, other types of heterogeneity present more of a challenge. In addition, in section 6.2 we compare our model more closely to that of Admati and Pfleiderer (1990). For a fixed (exogenous) value of λ , many of our results can be viewed as a generalized version of their model. We make this statement precise in the sequel by showing the exact changes that must be made to their framework in order to reproduce our results. This better connects our work to that of the existing literature.

6.1 Agent heterogeneity

We first generalize the model in sections 3 and 4 in order to accommodate different risk aversion coefficients among the agents. We also allow the informed agents to have different signal precisions. Maintaining the assumption that each group contains h households and m fund managers, we denote the j th household's risk aversion by τ_j for $j = 1, \dots, h$. For notational purposes, we let $\bar{\tau} = \left(\sum_{k=1}^h \frac{1}{\tau_k}\right)^{-1}$, i.e., $1/\bar{\tau}$ is the average risk tolerance of the household sector. Likewise, we denote the i th fund manager's risk aversion by τ_i and we denote the i th manager's signal precision by σ_i^2 , where $i = 1, \dots, m$. Hence, in this case, the i th fund manager observes the private signal $Y_i = X + \epsilon_i$, where $\epsilon_i \sim N(0, \sigma_i^2)$. While the groups themselves are identical to one another, we have altered our symmetric model in order to accommodate heterogeneity within a group.²⁵ For a fixed λ , the next proposition summarizes the rational

²⁵Note that this is not the most general case since we could also allow for heterogeneity across groups (including perhaps a different size for each group). However, due to the notational complexity that arises in this more

expectations equilibrium at the trading stage and the optimal fees that are chosen by each fund manager.

Proposition 4. *Allowing for heterogeneity within a group, the mutual fund equilibrium has the following properties:*

(i) *the equilibrium price of the risky asset is given by (8) where the coefficients satisfy*

$$\frac{b}{d} = \lambda \left(\frac{1}{m} \sum_{k=1}^m \frac{1}{\bar{\alpha}_k \tau_k \sigma_k^2} \right); \quad (25)$$

(ii) *the optimal demand of household j for mutual fund i is $\hat{\phi}_{ij} = \bar{\tau}/\tau_j$;*

(iii) *the optimal contingent fees are given by $\alpha_i = 1/(1 + \tau_i \sigma_i^2 \rho_i)$ where the collection $\{\rho_i\}$ solves the system of non-linear equations*

$$1 - 2\bar{\tau} \rho_i \sigma_i^2 = \frac{2\bar{\tau} \rho_i (1 - \bar{\tau} \sigma_i^2 \rho_i)^2}{\text{var}(X|P_x)^{-1} + 2\bar{\tau} \sum_{k=1}^m \rho_k - \bar{\tau}^2 \sum_{k=1}^m \sigma_k^2 \rho_k^2}; \quad i = 1, \dots, m \quad (26)$$

While the equilibrium involves a set of expressions that are somewhat similar to those stated previously, we note that there are subtle differences. For example, instead of the average contingent fee across all managers showing up in b/d , we now find that there are m distinct averages given by $\bar{\alpha}_k$ for $k = 1, \dots, m$. This is due to fact that there are now m distinct types of fund managers in the economy. We also note that b/d now explicitly depends on the average aggressiveness of the fund managers' trading strategies. To see this, recall that the trading strategy of the i th manager is of the form $\hat{\gamma}_i = (1/(\alpha_i \tau_i \sigma_i^2)) Y_i + q(P_x)$, where $q(P_x)$ is a linear function of P_x . The quantity $1/(\alpha_i \tau_i \sigma_i^2)$, which is the coefficient on Y_i , is a measure of how aggressive the manager trades with respect to his private signal. In turn, the quantity $1/(\bar{\alpha}_k \tau_k \sigma_k^2)$ is the average aggressiveness of the k th type of fund manager while $\frac{1}{m} \sum_{k=1}^m (1/(\bar{\alpha}_k \tau_k \sigma_k^2))$ is the average aggressiveness across all fund managers in the economy.

From property (ii) of the proposition, we note that the heterogeneous risk aversion of the household sector produces a different allocation of mutual funds from that in the homogeneous case. Each household now holds a fraction of each mutual fund that is equal to the household's risk tolerance dividend by the average risk tolerance, i.e., unlike the symmetric case, different

general case, we maintain the assumption of identical groups and only allow heterogeneity *within* a group.

households now hold different fractions. However, the relative holdings remain unchanged – the j th household still holds identical fractions of all m mutual funds. This can easily be seen by noting that the j th household’s demand for fund i does not explicitly depend on i .

We characterize the optimal fees in property (iii) as the Nash equilibrium of a game in which the i th manager has the payoff $\alpha_i P_i(\alpha_i)$, where $i = 1, \dots, m$. In particular, the i th manager takes the other managers’ fees as given and now solves

$$\max_{\rho_i} \alpha_i P_i(\alpha_i) = \frac{\rho_i(1 - \bar{\tau}\sigma_i^2\rho_i)}{\text{var}(X|P_x)^{-1} + 2\bar{\tau}\sum_{k=1}^m \rho_k - \bar{\tau}^2\sum_{k=1}^m \sigma_k^2\rho_k^2}. \quad (27)$$

Solving (27) for each $i = 1, \dots, m$ produces a collection of m first-order conditions. This collection is precisely the system of equations given in (26). In addition, it is easy to see that the optimal contingent fee must satisfy $\hat{\alpha}_i \in ((1 + 0.5(\tau_i/\bar{\tau}))^{-1}, 1)$, i.e., the fee is bounded below and $\alpha_i = 1$ is never optimal. As in the symmetric model, this latter fact implies that none of the informed agents trade on their own account. They instead prefer to establish mutual funds and offer investment management services to the household sector.

The above expressions shed further light onto the value that the household sector assigns to the mutual fund industry. It is straightforward to check that, holding the informativeness of price constant, the total fee of a fund manager is decreasing in $\bar{\tau}$. This is rather intuitive – as $\bar{\tau}$ increases, the household sector must be compensated in terms of a higher risk premium and this shows up as a lower total fee that is paid to the fund manager. It is also immediate from (26) and (27) that, holding the informativeness of price constant, both the total fee and the contingent fee are decreasing in the manager’s risk aversion, τ_i . Obviously, all three of the quantities α_i , σ_i^2 , and τ_i also play a role in affecting the informativeness of price since these quantities indirectly show up in $\text{var}(X|P_x)$. Lastly, we note the comparative statics that arise in our heterogeneous model are quite similar to those that were already discussed in section 4.

The heterogeneous economy also allows us to state the following intuitive result concerning the cross-sectional relationship between a manager’s information precision and his optimal contingent fee.²⁶

Corollary 6. *Suppose that all agents have identical risk aversion. Then the manager with the*

²⁶To prove the corollary, note that the nonlinear system of equations in Proposition 4 implies, under the conditions of the Corollary, that

$$\frac{\sigma_k^2}{\sigma_i^2} = \frac{t(\alpha_k)}{t(\alpha_i)} \quad (28)$$

highest information precision has the highest contingent fee, the manager with the next highest information precision has the next highest contingent fee, and so forth.

While there are other related topics for investigation, due to the complexity of the model with full heterogeneity we only mention (and do not pursue) these topics here. For example, it would be interesting to examine the information acquisition stage in a setting in which agents have heterogeneous risk aversion and have the ability to purchase signals with different precisions at different costs. It is tempting to conjecture that only the most risk tolerant agents will become informed and establish funds, as in the standard model (Verrecchia (1982)). However, given the complexity of the expressions in Proposition 4, it seems difficult to make this statement analytically precise. Nevertheless, we mention it as a possible extension of our results. Second, it would be interesting to investigate an equilibrium in which two sets of households have access to overlapping collections of mutual funds. For example, suppose that one household has access to a collection of m_1 mutual funds while a second household has access to a partially overlapping collection of m_2 mutual funds. If the fund managers have heterogeneous signal precisions, the households' equilibrium holdings of the mutual funds do not reduce to the optimal risk sharing holdings that are given in Proposition 4. Instead, a system of nonlinear equations must be solved in order to obtain the households' equilibrium mutual fund holdings.

The same tractability problem arises if the household sector has heterogeneous information precision. For example, this would arise if the households happened to be endowed with heterogeneous prior information. It would also arise if we allowed informed agents to invest with other informed agents. Recall that for $\lambda^* \in (0, 1)$, our current framework includes only two types of agents, informed mutual fund managers and uninformed households. By allowing informed agents to forego establishing a mutual fund but to invest with other informed agents, the number of different types of agents is expanded from two to three: uninformed households that hold mutual funds, informed mutual fund managers that do not hold positions in other funds, and informed proprietary traders that hold mutual funds. While this is potentially a

where the function $t(z)$ is

$$t(z) = \frac{\left(\frac{1-z}{z}\right) \left[1 - \left(\frac{1-z}{z}\right)\right]^2}{\left[1 - 2\left(\frac{1-z}{z}\right)\right]}$$

Noting that $\frac{dt(z)}{dz} < 0$, we conclude from expression (28) that $\sigma_k^2 > \sigma_i^2$ implies that $\alpha_k < \alpha_i$. We can therefore rank the agents according to their information precisions and this ranking will coincide with an otherwise identical ranking that is based on the agents' contingent fees.

more realistic scenario, it also produces two types of mutual fund investors (i.e., informed and uninformed) and the tractability of our model is greatly diminished. We leave the study of these extensions for future research.

6.2 Alternative formulations

As we pointed out in the introduction, our model is related to that of Admati and Pfleiderer (1990). In fact, as we will show, our model for a fixed λ can be viewed as a generalized version of their model. Briefly, recall that Admati and Pfleiderer (1990) consider the case of a single informed agent, i.e., they deal with the case of a monopolistic seller of information. The monopolist sells his information to a continuum of agents who have CARA preferences. Their framework and our framework differ along four dimensions. First, since they consider only the monopolist case with a single informed agent, the monopolist’s private signal shows up in the risky asset’s equilibrium price. As discussed by Hellwig (1980), this introduces a certain “schizophrenia” to the model – namely, the monopolist behaves as a price taker with respect to the risky asset even though he knows that his signal shows up in its price. Our framework eliminates this problem by relying on a continuum of identical groups. We can therefore analyze the monopolistic case by setting $m = 1$ while still maintaining the “mental health” of our informed agents. Second, since our model is valid for any $m < \infty$, we go beyond the monopolistic case to study how imperfect competition itself affects the mutual fund sector. Third, we assign CARA preferences to the fund managers in order to endogenize their trading strategies. Lastly, our assignment of preferences allows us to solve for λ^* and address the topic of endogenous information acquisition. It also allows us to endogenize the fund formation decision, i.e., informed agents establish mutual funds if $\hat{\alpha}_i < 1$ and otherwise they trade on their own accounts.

To better see the connection between the two models, consider the following generalized version of Admati and Pfleiderer (1990). Fix λ and assume that all households have the same risk aversion parameter τ . Suppose that there are a total of N agents in the economy, λN of which are informed. Moreover, assume that each household can invest in a given set of m mutual funds, i.e., form groups of $m + h$ agents where h denotes the number of households in each group. Hence, as in our model, there is an infinite number of groups as $N \rightarrow \infty$. Next, index the m mutual funds in a group by $i = 1, \dots, m$ and suppose that the i th fund’s payoff per unit is $Z_i = Y_i(X - P_x)$. This latter assumption is consistent with Admati and Pfleiderer

(1990) in which the fund's payoff is modeled as a linear function of the manager's private signal. We note that this payoff arises due to an exogenously specified trading strategy for the manager, i.e., the trading strategy is fixed ex ante. Lastly, letting ϕ_i denote the number of units of fund i purchased by a typical household, suppose that the i th manager chooses the fund price P_i to maximize his profits $\phi_i P_i$. We note that the i th manager chooses P_i prior to observing Y_i and that the households can purchase any number of units of the mutual fund at this price. Of course, the households can also invest directly in the stock market. Lastly, note that our definition of ϕ_i is slightly different than that used in the prior sections of the article, but consistent with that of Admati and Pfleiderer (1990).

Using this framework, it is possible to show that the i th fund manager solves

$$\max_{\phi_i} \phi_i P_i(\phi_i) = \frac{\phi_i(1 - \bar{\tau}\sigma_i^2\phi_i)}{\text{var}(X|P_x)^{-1} + 2\bar{\tau}\sum_{k=1}^m \phi_k - \bar{\tau}^2\sum_{k=1}^m \sigma_k^2\phi_k^2};$$

with

$$\text{var}(X|P_x)^{-1} = \frac{1}{\sigma_x^2} + (m\bar{\phi})^2 \frac{1}{\sigma_u^2};$$

and $\bar{\tau} = \tau/h$. Comparing this expression to (27) we see that they are indeed equivalent optimization problems, i.e., our model with contingent fees reduces to the generalized version of the Admati and Pfleiderer (1990) model. This equivalence (for a fixed λ) shows that the actual timing of our model is not crucial. In fact, the household sector and the fund managers could contract before the trading stage in our model without affecting any of the results.

The above extension relies on our grouping procedure that involves m managers and h households. If we extended the model of Admati and Pfleiderer (1990) along the lines of our own paper, i.e., by allowing for contingent fees, by endowing the fund managers with CARA preferences and by consider an arbitrary number of information sellers, we run into an apparent tractability problem – namely, the equilibrium is characterized by the solution to a nonlinear system of equations.²⁷ This makes the model much more challenging to study. We have looked at several numerical examples and the intuition and the results discussed in the body of our paper appear to be robust.

The main impact of our grouping procedure can be seen by examining a limiting version

²⁷The model essentially reduces to that discussed on pages 479-485 of Hellwig (1980) if we replace the informed agent's risk aversion τ_i with his effective risk aversion $\alpha_i\tau_i$. Further note that Hellwig (1980)'s characterization of an equilibrium is tractable for the case of a single informed agent, i.e., for the case studied by Admati and Pfleiderer (1990).

of our model. In particular, in order to have a competitive mutual fund model in the spirit of Hellwig (1980), it is necessary that each household in our model be allowed to invest in only a finite number of mutual funds. To see this, consider what happens in our model as we let $m \rightarrow \infty$. In this case, the number of mutual funds held by a typical household grows without bound and our equilibrium fails to exist. Essentially, if the household sector can buy an infinite number of mutual funds, it can perfectly diversify the error terms $\{\epsilon_i\}$ that drive the managers' private signals (recall the discussion following expression (20)). In this case, the aggregate payoff of the mutual fund sector is a non-noisy quadratic function of X and the coefficient on X^2 is equal to the fund sector's aggregate risky asset exposure. Since the presence of the X^2 term is the main driver of the household sector's demand for mutual funds, we find that households are willing to invest extremely large amounts in any mutual fund that delivers a large risky asset exposure. In turn, this delivers a large coefficient on X^2 . Technically, P_i is increasing in ρ_i in the limit as $m \rightarrow \infty$. In addition, $\alpha_i \rightarrow 0$ in such a way that $\alpha_i P_i(\alpha_i) \rightarrow \infty$ for all i and our nonexistence claim immediately follows. While it seems natural to consider the case of $m \rightarrow \infty$, the above discussion suggests that it should be addressed in a setting that includes some type of additional contracting cost (i.e., while a household may have access to an infinite number of funds, it may optimally choose to invest in only a subset of these). We leave this as a topic for future research.

We also leave open the possibility of considering a more complex group formation model. First, our model is summarized by the parameter m , which is specified exogenously. One could modify the current setup by allowing for a formal matching model (search game). In this case, we would expect that m could be endogenized, i.e., m could be expressed as a function of the model's primitives. This might also address the potential integer problems that arise for h in the current setup of our model. Second, even within our current framework, we note that several modifications to the grouping procedure are possible. For example, rather than exogenously fix m , we could instead impose a constraint on the overall group size, i.e., we could fix $m + h$ and then allow λ^* to determine m and h individually. If we ignore integer problems, this variation of the model is mathematically equivalent to the one that we discuss in the paper. However, if we impose an integer constraint on m (which seems natural since m enters the household's problem in (4)), the two models will produce different results.

7 Concluding remarks

We study the fund formation decision of rational informed investors in order to offer an explanation for why we observe so many mutual funds in practice. Our findings indicate that the creation of mutual funds in equilibrium is more the rule than the exception. As opposed to trading solely for their own accounts, in our model informed investors are always better off by establishing mutual funds and marketing their investment strategies to the public. While a fund manager's contingent fee allows him to retain only a fraction of his optimal stock market bet, the manager's trading strategy also depends on his contingent fee. These two effects offset and, since the manager's fee also applies to the initial assets under management, the manager derives increased utility from managing money. Households are also better off when mutual funds are established. Since the fund managers have private information, a stock market bet that is achieved by purchasing a mutual fund is very different than a direct stock market bet. Essentially, mutual funds offer a new asset class to the investing public. Our model makes this statement precise.

We endogenize the information acquisition decision of the informed agents in order to compare our mutual fund model to the case in which a mutual fund sector does not exist (see, e.g., Verrecchia (1982) and Diamond (1985)). This allows us to make precise statements about the equilibrium relationship between the level of information acquisition and the model's economic primitives. It also facilitates our discussion of information revelation – namely, we outline conditions under which our mutual fund model produces a more informative risky asset price. These conditions are directly related to the trading aggressiveness of the fund managers (i.e., the informed agents' effective risk aversion is lower if they establish mutual funds). Lastly, we use our model to study the equilibrium relationship between information precision, contingent management fees, total fees, and mutual fund size (e.g., assets under management).

Our model has several natural extensions. For example, given the multi-asset nature of actual financial markets, an extension of the model to the case of multiple risky assets appears to be a worthy topic for future research. In this case, agents might purchase signals on only a subset of the available risky assets and therefore establish specialized mutual funds. This type of model would facilitate an examination of the relationship between fund specialization (i.e., mutual fund styles) and contingent fees. While this topic has received considerable

attention in the empirical literature, there are few theoretical results in this area. A second natural extension is to study the existence and formation of mutual fund families. Essentially, one could examine the case in which managers share information and possibly collude in the setting of their fees. This potentially could explain the structure of the mutual fund industry, i.e., in practice we tend to observe large fund companies (such as Fidelity and Vanguard) that each manage an array of mutual funds. This type of analysis could potentially shed some light onto the equilibrium relationship between the size of a mutual fund family and their contingent fees, among other items.

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Appendix

Proof of Proposition 1: Using the standard properties of Gaussian random variables, it is straightforward to evaluate the conditional expectation in expression (3). Solving the i th manager's problem at date 1 produces the familiar mean-variance expression in (9).

To evaluate the j th household's problem, we let V denote the variance-covariance matrix of $(X, \epsilon_1, \dots, \epsilon_m)$ conditional on price, and μ_x and μ_i the expected values of X and ϵ_i conditional on price. Further define $\gamma_i \equiv r_i Y_i + q_i P_s$, where r_i and q_i are readily obtainable from (9). The household's problem in expression (4) can be expressed as

$$\max_{\theta, \phi} \quad \tau M(\theta, \phi) + \frac{1}{2} \log(|B(\phi)|) - \frac{\tau^2}{2} g(\theta, \phi)^\top B(\phi)^{-1} V g(\theta, \phi).$$

where

$$\begin{aligned} M(\theta, \phi) &= \theta(\mu_x - P_x) - \sum_{i=1}^m (1 - \alpha_i) \phi_i P_i + (\mu_x - P_x) \left(\sum_{i=1}^m (1 - \alpha_i) \phi_i [q_i P_x + r_i(\mu_x + \mu_i)] \right) \\ B(\phi) &= I + 2\tau V A(\phi); \\ g(\theta, \phi) &\equiv f(\theta, \phi) + 2A(\phi)\mu; \\ f(\theta, \phi) &= \begin{pmatrix} \theta + P_x \sum_{i=1}^m (q_i - r_i)(1 - \alpha_i) \phi_i \\ -P_x r_1 (1 - \alpha_1) \phi_1 \\ \vdots \\ -P_x r_m (1 - \alpha_m) \phi_m \end{pmatrix} \\ A_{ij}(\phi) &\equiv \begin{cases} \sum_{i=1}^m r_i (1 - \alpha_i) \phi_i & : i = j = 1; \\ r_i (1 - \alpha_i) \phi_i / 2 & : i = 1, j \neq 1 \text{ or } j = 1, i \neq 1; \\ 0 & : \text{otherwise} \end{cases} \end{aligned}$$

where $I \in \mathbb{R}^{(m+1) \times (m+1)}$ denotes the identity matrix, $f(\theta, \phi), g(\theta, \phi) \in \mathbb{R}^{m+1}$, and $B(\phi), A(\phi) \in \mathbb{R}^{(m+1) \times (m+1)}$. Let $C \equiv B(\phi)^{-1} V$. Then the first-order conditions yield

$$\frac{\mu_x - P_x}{\tau} = \sum_{j=1}^{m+1} C_{1j}(\phi) g_j(\phi) \quad (29)$$

$$M_{\phi_i} + \frac{1}{2} (1 - \alpha_i) r_i \text{trace}(C(\phi) D_i) = \frac{\tau}{2} \left(2g_{\phi_i}^\top - g^\top C D_i \right) C g, \quad i = 1, \dots, m; \quad (30)$$

where H_{ϕ_i} denotes the derivative of the function H with respect to ϕ_i . Some basic manipulations of (29) and (30) yield

$$M_{\phi_i} + \frac{1}{2}(1 - \alpha_i)r_i\text{trace}(C(\phi)D_{\phi_i}) = P_x q_i(1 - \alpha_i) + r_i(1 - \alpha_i)(\mu_x + \mu_i); \quad i = 1, \dots, m;$$

so that

$$P_i = \frac{1}{2} \frac{(1 - \alpha_i)r_i}{\alpha_i} \text{trace}(C(\phi)D_{\phi_i}) = \frac{(1 - \alpha_i)r_i}{\alpha_i} (C_{11} + C_{1,i+1}).$$

The standard matrix inversion lemma and some simple algebra yields (13).

Given the symmetry of the model, market clearing in the mutual fund sector implies the mutual fund demands are $\hat{\phi}_{ij} = \frac{1}{h}$. Substituting these equilibrium demands into (29) yields, after some simple manipulations, the optimal stock demands in (10).

We extract the price coefficients b and d from the market clearing condition for the risky asset (6). Using the previous expressions for the optimal demands of both informed and uninformed agents, the left-hand side of (6) can be written as

$$\frac{m}{\tau\bar{\alpha}} \left(\frac{X}{\sigma_\epsilon^2} + \frac{bP_x}{d^2\sigma_u^2} - P_x V \right) + \frac{h}{\tau} \left(\frac{bP_x}{d^2\sigma_u^2} - P_x V \right) + \frac{mP_x(1 - \bar{\alpha})}{\tau\bar{\alpha}} \left(\frac{1}{\sigma_x^2} + \frac{b(b-1)}{d^2\sigma_u^2} \right).$$

Solving for P_x verifies the functional form in (8) and produces a system of equations for b and d . Solving this system yields the expressions in (11)-(12). This completes the proof. \square

Proof of Proposition 2: Taking the fees of the other $m-1$ managers as given and noting that the price coefficients b and d are independent of any individual manager's fees, the first-order condition for (17) gives the equation

$$1 - \frac{2\rho_i}{h} - \left(\frac{2}{h\sigma_\epsilon^2} \right) \frac{\rho_i \left(1 - \frac{\rho_i}{h}\right)^2}{\text{var}(X|P_x)^{-1} + \frac{2}{h} \sum_{k=1}^m \rho_k - \frac{1}{h^2\sigma_\epsilon^2} \sum_{k=1}^m \rho_k^2} = 0 \quad (31)$$

where $\rho_i \equiv (1 - \alpha_i)/\alpha_i$. Substituting for P_i and $\frac{b}{d}$, letting $\alpha_1 = \alpha_2 = \dots = \alpha_m \equiv \alpha$, and noting that $\bar{\alpha} = \alpha$, the above expression reduces to

$$\frac{\left(\frac{1-\alpha}{\alpha\sigma_\epsilon^2} \right) \left(1 - \frac{1}{h} \left(\frac{1-\alpha}{\alpha} \right)\right)^2}{\left[\frac{1}{\sigma_x^2} + \frac{1}{\sigma_u^2} \left(\frac{\lambda}{\tau\alpha\sigma_\epsilon^2} \right)^2 + \frac{2m}{h\sigma_\epsilon^2} \left(\frac{1-\alpha}{\alpha} \right) - \frac{m}{h^2\sigma_\epsilon^2} \left(\frac{1-\alpha}{\alpha} \right)^2 \right]} = 1 - \frac{2}{h} \left(\frac{1-\alpha}{\alpha} \right) \quad (32)$$

Some simple algebraic manipulation produces the cubic equation that is presented in Propo-

sition 2. The second-order condition for a maximum is easily verified by differentiating the left-hand side of (31) with respect to α_i . It is straightforward to check that the sign of the second derivative of $\alpha_i P_i$ is the same as the sign of $(1/\tau - \alpha_i P_i/h)$. Since the first-order condition in (32) implies that $\tau \alpha_i P_i < h$, the second-order condition is verified.

Next, recalling the definitions of k_0 , k_1 , k_2 , and k_3 , let us define the function $G(\alpha)$ as

$$G(\alpha) = k_3 \alpha^3 + k_2 \alpha^2 + k_1 \alpha + k_0$$

It is straightforward to check that $G(1) < 0$ and that $G(1/(1+h/2)) > 0$. Since $G(\alpha)$ is a continuous function of α , this shows that a solution in the interval $(1/(1+h/2), 1)$ exists. Moreover, it can be shown that $\frac{dG}{d\alpha} < 0$ for every point that lies in the interval $(1/(1+h/2), 1)$. This immediately leads to uniqueness.

Lastly, note that in general $G(\alpha)$ has three roots. For some parameter values $G(\alpha)$ has two complex roots and one real root, while for other parameter values $G(\alpha)$ has three real roots. Obviously, in the former case, we take the sole real root as the solution to $G(\alpha)$ and this real root lies in the interval $(1/(1+h/2), 1)$. In the case of three real roots, one of the roots is negative and we therefore discard this root. While the remaining two roots may lie in $[0, 1]$, only one of these two roots produces a non-negative total fee. Hence, by a process of elimination we conclude that the unique real root that lies in $(1/(1+h/2), 1)$ is the economically meaningful root. \square

Proof of Proposition 3: The i th manager's indirect utility function at date 1 is

$$\mathbb{E} \left[-e^{-\tau \alpha_i (P_i + \hat{\gamma}_i (X - P_x)) + \tau c} \middle| P_x, Y_i \right] = -e^{-\tau (\alpha_i P_i - c)} e^{-\frac{(\mathbb{E}[X|P_x, Y_i] - P_x)^2}{2 \text{var}(X|P_x, Y_i)}} \quad (33)$$

where $\mathbb{E}[X|P_x, Y_i]$ and $\text{var}(X|P_x, Y_i)$ are given in (14)-(15). Taking the expectation of both sides of (33) produces the manager's date 0 indirect utility function, i.e.,

$$E \left[-e^{-\tau \alpha_i (P_i + \hat{\gamma}_i (X - P_x)) + \tau c} \right] = -e^{-\tau (\alpha_i P_i - c)} \sqrt{\frac{1}{P\Lambda}} \quad (34)$$

where

$$\Lambda = d^2 \sigma_u^2 + (b-1)^2 \sigma_x^2. \quad (35)$$

Some tedious algebra reveals that the date 1 indirect utility of household j is given by

$$\mathbb{E} \left[-e^{-\tau \hat{W}_j} \middle| P_x \right] = -\sqrt{\frac{S}{D}} e^{-\frac{(\mathbb{E}[X|P_x] - P_x)^2}{2\text{var}(X|P_x)} + \frac{\tau}{h} \sum_{k=1}^m \hat{\alpha}_k P_k} \quad (36)$$

where \hat{W}_j denotes the household's optimal wealth, $S \equiv \text{var}(X|P_x)^{-1}$. Taking the expectation of both sides of (36) produces the household's date 0 indirect utility function, i.e.,

$$\mathbb{E} \left[-e^{-\tau \hat{W}_j} \right] = -\sqrt{\frac{1}{D\Lambda}} e^{\frac{\tau}{h} \sum_{k=1}^m \hat{\alpha}_k P_k} \quad (37)$$

where Λ is given in (35). Recalling that the manager's date 0 indirect utility function is given by (34), we can now equate (34) and (37) in order to identify λ^* . This completes the proof. \square

Proof of Proposition 4: The demand functions for the informed traders are given by the usual mean-variance form (9). The uninformed agents' optimization problem can be solved by the same approach as in Proposition 1. In particular, it is straightforward to check that the first-order conditions to household j optimization problem yield

$$\alpha_i P_i = \frac{\rho_i (1 - \phi_{ij} \sigma_i^2 \rho_i)}{\text{var}(X|P_x)^{-1} + 2 \sum_{k=1}^m \phi_{kj} \rho_k - \sum_{k=1}^m \phi_{kj}^2 \sigma_k^2 \rho_k^2}; \quad (38)$$

where $\rho_i \equiv (1 - \alpha_i)/(\alpha_i \sigma_i^2 \tau_i)$. An immediate calculation shows that the demands $\hat{\phi}_{ij}$ both satisfy the market clearing condition (7) and the above first-order conditions. Using the optimal demands for the informed agents and the market clearing condition for the stock one can easily obtain (25). Finally, by the same arguments as in Proposition 3 we have that each fund manager solves $\max_{\alpha_i} \alpha_i P_i$, which yields the first-order conditions (26). This completes the proof. \square

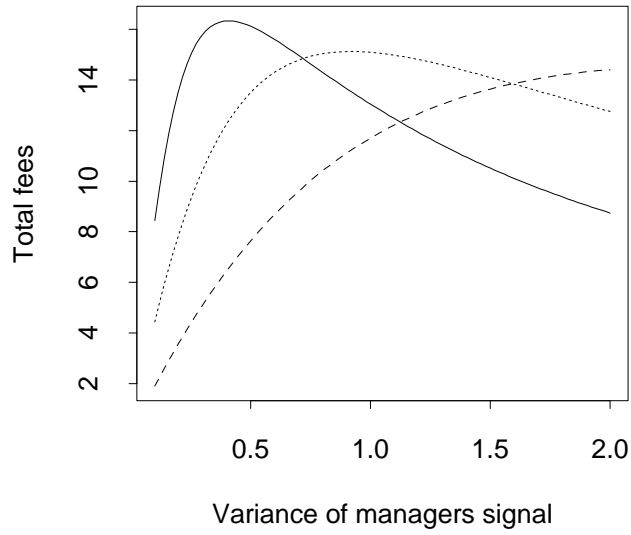
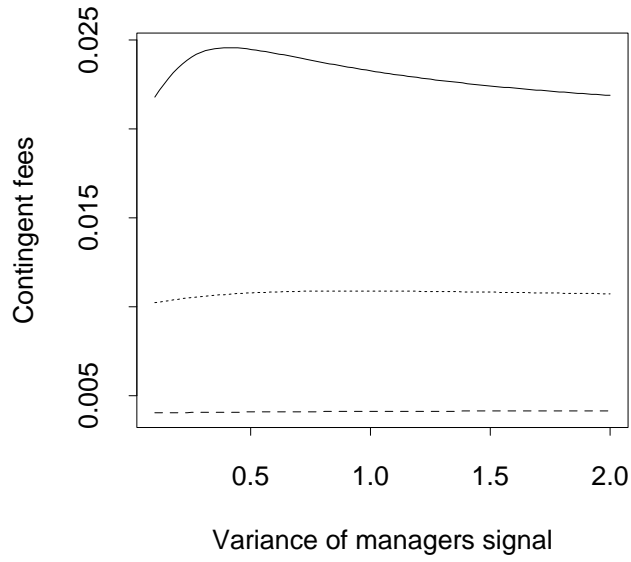


Figure 1: The figure shows $\hat{\alpha}_i$ and $\hat{\alpha}_i P(\hat{\alpha}_i)$ as a function of σ_ϵ^2 . The solid line corresponds to $m = 1$, the dotted line to $m = 2$, and the dashed line to $m = 5$. Other parameter values: $\tau = 1$, $\sigma_u^2 = 1$, $\sigma_x^2 = 1$, $\lambda = 0.01$.

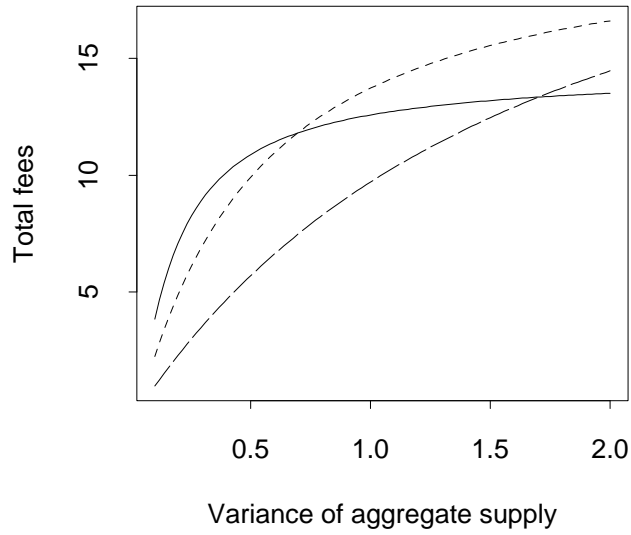
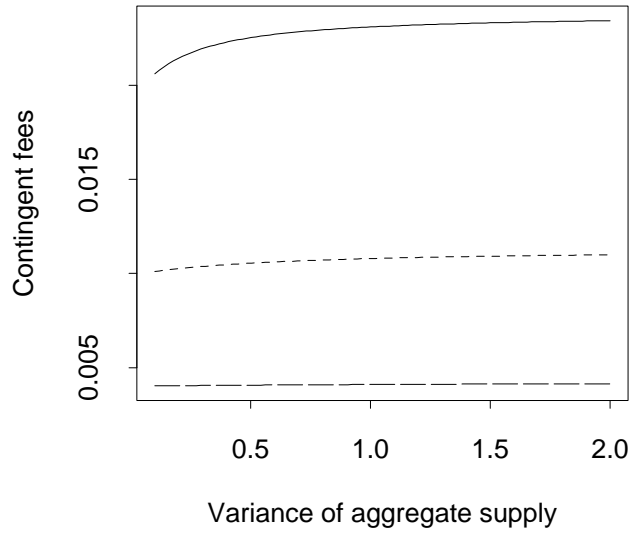


Figure 2: The figure shows $\hat{\alpha}_i$ and $\hat{\alpha}_i P(\hat{\alpha}_i)$ as a function of σ_u^2 . The solid line corresponds to $m = 1$, the dotted line to $m = 2$, and the dashed line to $m = 5$. Other parameter values: $\sigma_\epsilon^2 = 1$, $\tau = 1$, $\sigma_x^2 = 1$, $\lambda = 0.01$.

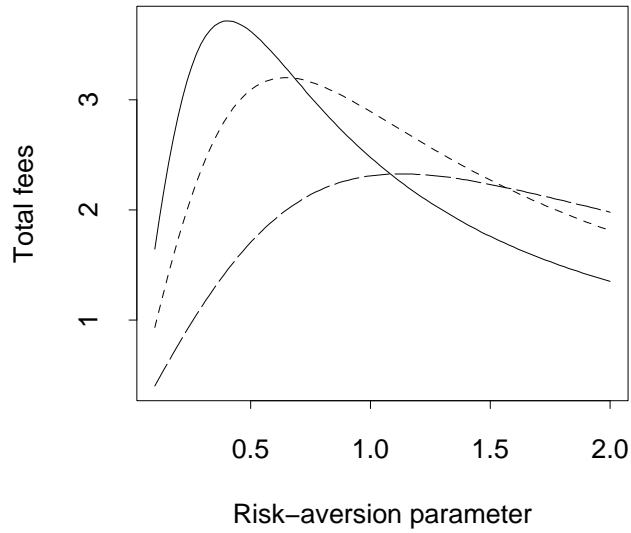
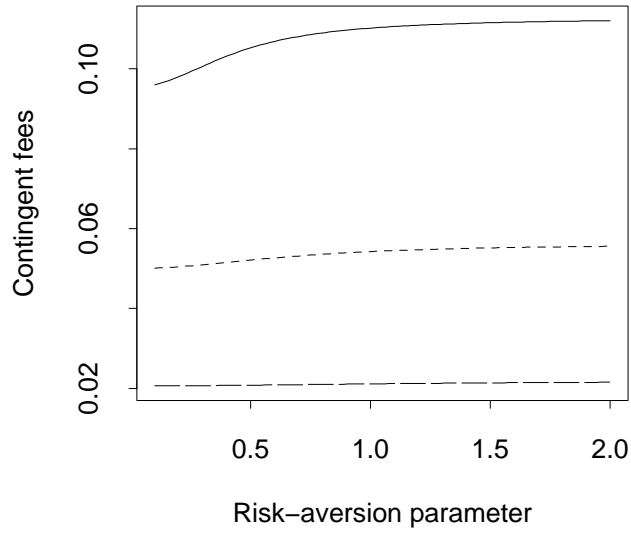


Figure 3: The figure shows $\hat{\alpha}_i$ and $\hat{\alpha}_i P(\hat{\alpha}_i)$ as a function of τ . The solid line corresponds to $m = 1$, the dotted line to $m = 2$, and the dashed line to $m = 5$. Other parameter values: $\sigma_u^2 = 1$, $\sigma_\epsilon^2 = 1$, $\sigma_x^2 = 1$, $\lambda = 0.05$.

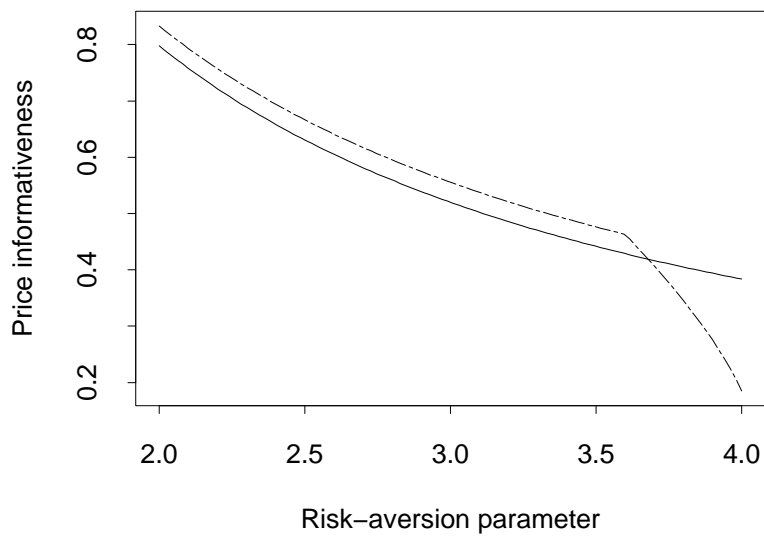
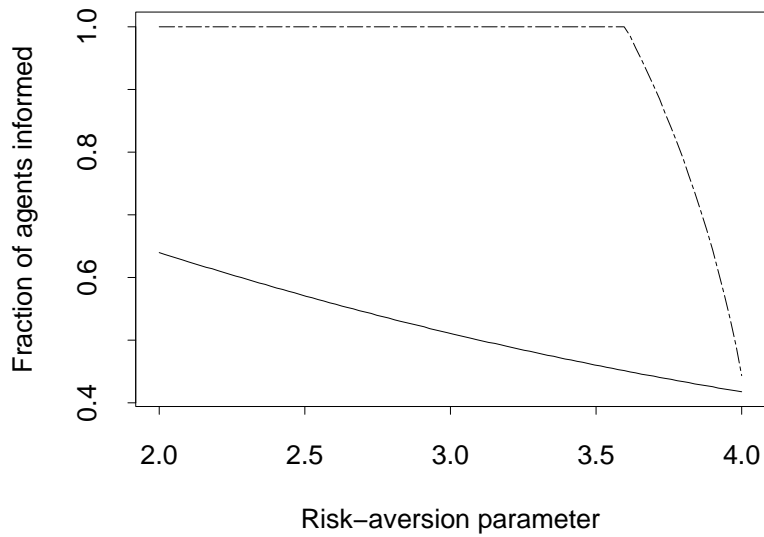


Figure 4: The figure shows λ^* and $(b/d)/\sigma_u$ as a function of τ . The solid line corresponds to $m = 5$ while the dotted line corresponds to the model without mutual funds. Other parameter values: $\sigma_u^2 = 1$, $\sigma_\epsilon^2 = 0.5$, $\sigma_x^2 = 1$, $c = 0.18$.

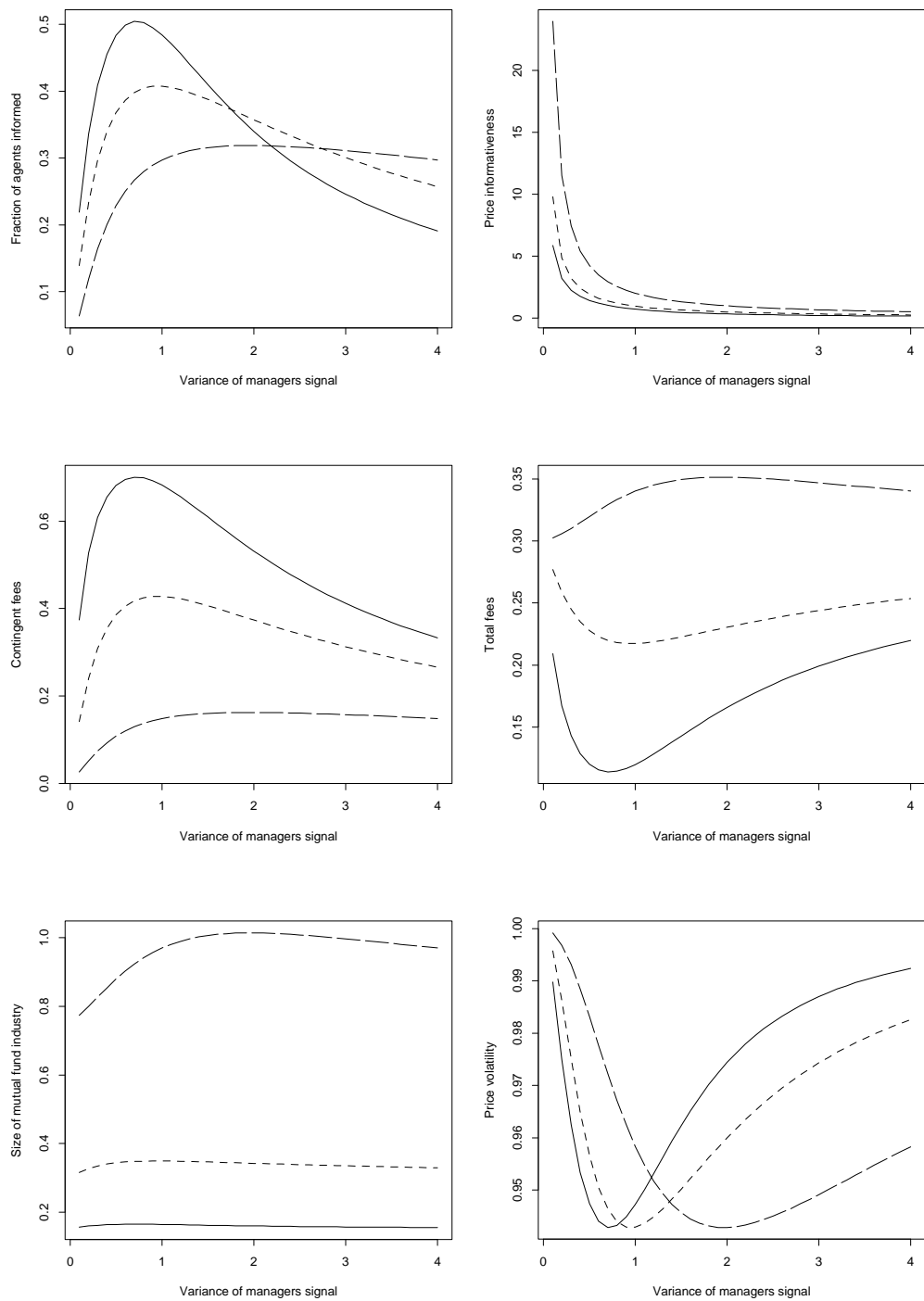


Figure 5: The figure shows λ^* , $(b/d)/\sigma_u$, $\hat{\alpha}_i$, and $\hat{\alpha}_i P(\hat{\alpha}_i)$ as a function of σ_ϵ^2 . The solid line corresponds to $m = 1$, the dotted line to $m = 2$, and the dashed line to $m = 5$. Other parameter values: $\tau = 1$, $\sigma_u^2 = 1$, $\sigma_x^2 = 1$, $c = 0.3$.

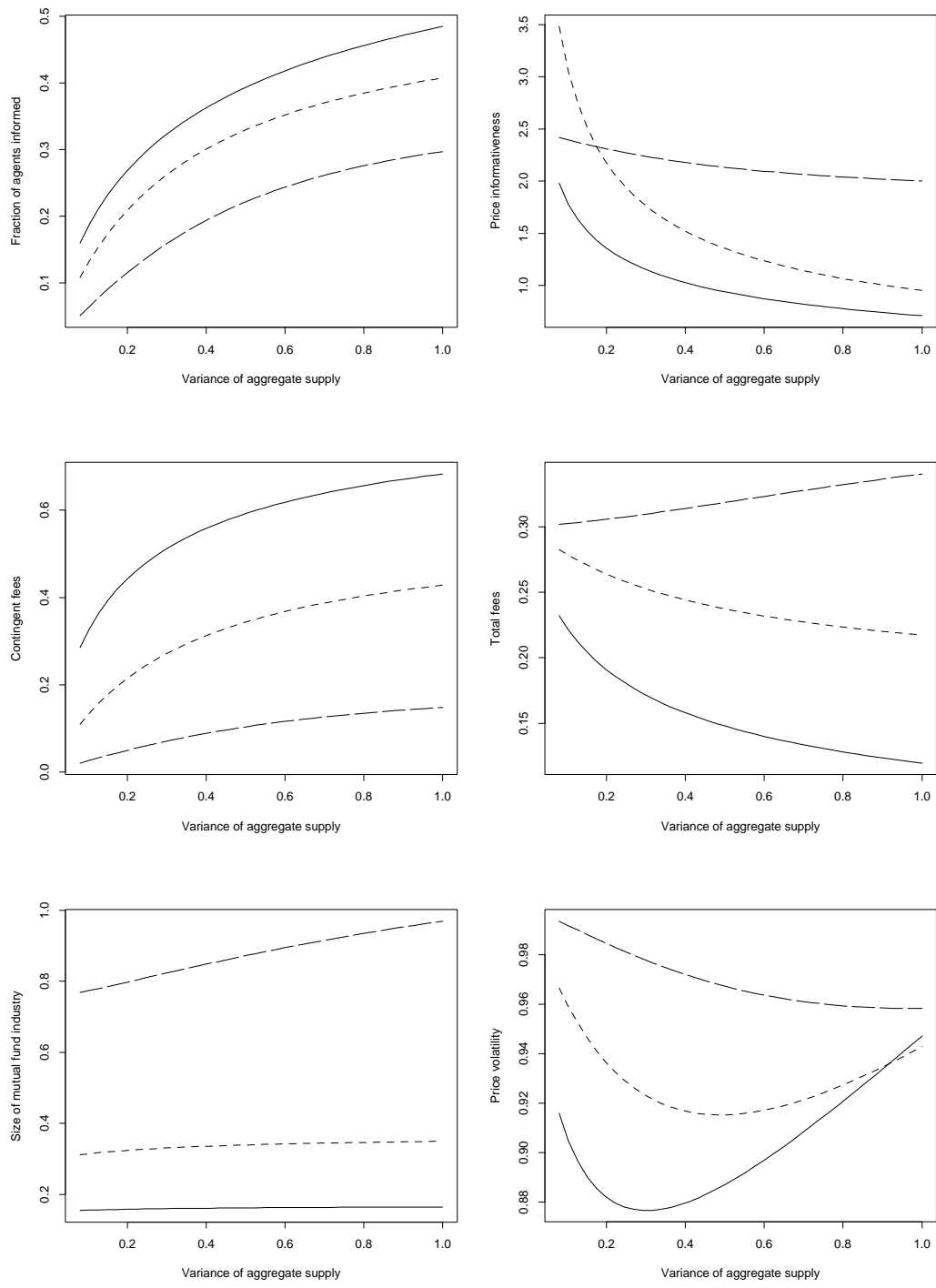


Figure 6: The figure shows λ^* , $(b/d)/\sigma_u$, $\hat{\alpha}_i$, and $\hat{\alpha}_i P(\hat{\alpha}_i)$ as a function of σ_u^2 . The solid line corresponds to $m = 1$, the dotted line to $m = 2$, and the dashed line to $m = 5$. Other parameter values: $\tau = 1$, $\sigma_\epsilon^2 = 1$, $\sigma_x^2 = 1$, $c = 0.3$.

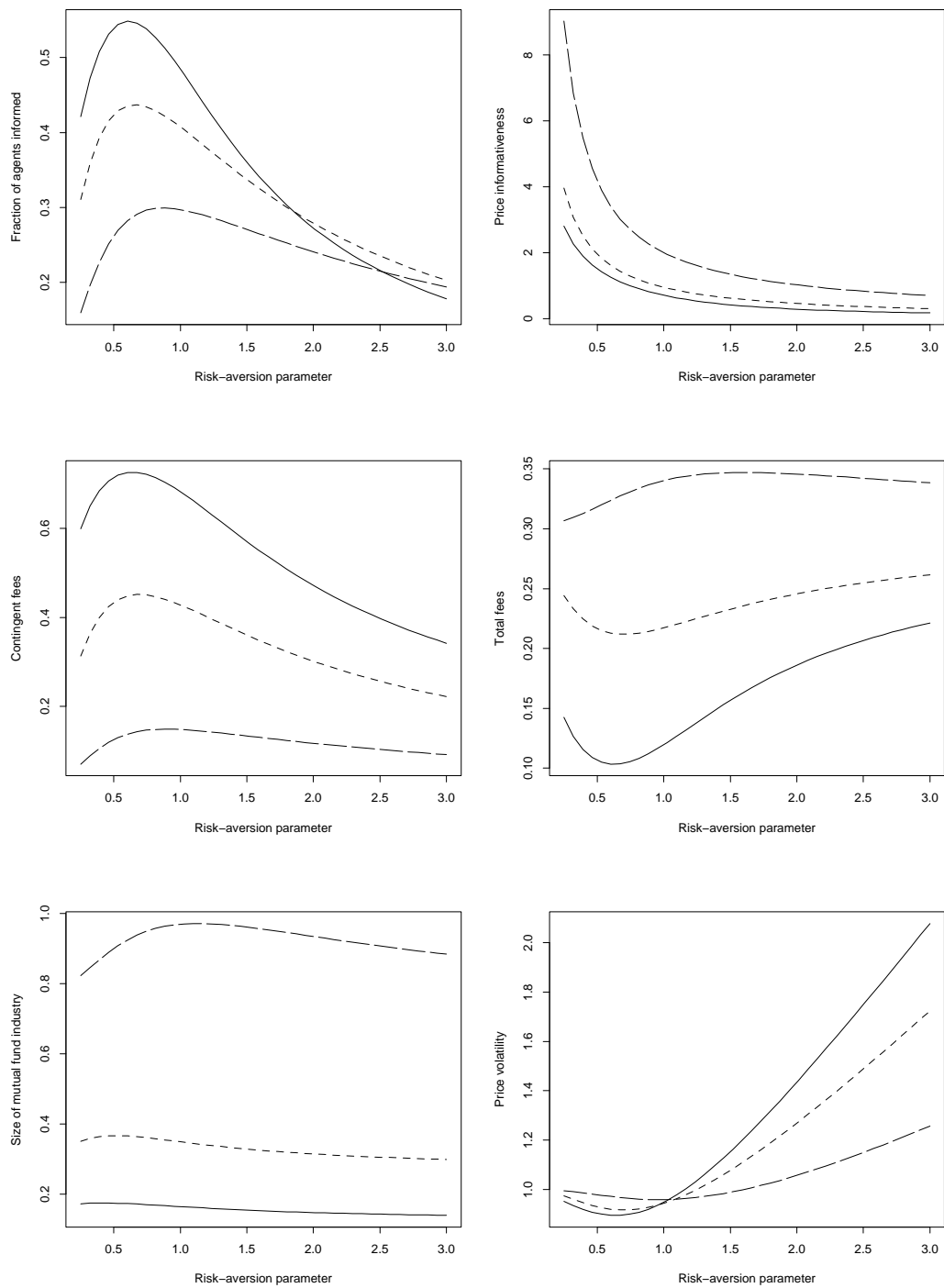


Figure 7: The figure shows λ^* , $(b/d)/\sigma_u$, $\hat{\alpha}_i$, and $\hat{\alpha}_i P(\hat{\alpha}_i)$ as a function of τ . The solid line corresponds to $m = 1$, the dotted line to $m = 2$, and the dashed line to $m = 5$. Other parameter values: $\sigma_u^2 = 1$, $\sigma_\epsilon^2 = 1$, $\sigma_x^2 = 1$, $c = 0.3$.